Huse,Henley, and Fisher Respond: Here we show how the exponents $\zeta=\frac{2}{3}$ for the transverse fluctuations in interface position and $\chi=\frac{1}{3}$ for the fluctuations in the free energy can be derived exactly for an interface in a random potential in two dimensions at any temperature. We do this by relating the problem of the interface to the damped Burgers's equation[1] in one dimension with random forcing, the scaling behavior of which has been analyzed by Forster, Nelson, and Stephen[2]

In the continuum limit with Hamiltonian[3]

$$H=\int dx\,[\tfrac{1}{2}\sigma(\partial y/\partial x)^2+V(x,y)], \tag{1}$$

the weight $W(x,y)$ of a path or interface ending at (x,y) satisfies the equation

$$\frac{\partial W(x,y)}{\partial x}=\frac{k_{\rm B}T}{2\sigma}\frac{\partial^2 W(x,y)}{\partial y^2}+\frac{1}{k_{\rm B}T}V(x,y)\,W(x,y), \tag{2}$$

where σ is the interface stiffness, $y(x)$ is the location of the interface, and the correlations in the random potential are

$$\langle V(x,y)\,V(x',y')\rangle=\Delta\delta(x-x')\delta(y-y'). \tag{3}$$

This is the continuum version of Kardar's[4] recursion relation for the weights in the lattice solid-on-solid (SOS) model. If we define $u(x,y)=[\partial F(x,y)/\partial y]/\sigma$, where the free energy is $F(x,y)=-k_{\rm B}T\ln W(x,y)$, Eq. (2) becomes

$$\frac{\partial u(x,y)}{\partial x}=\frac{k_{\rm B}T}{2\sigma}\frac{\partial^2 u(x,y)}{\partial y^2}-u(x,y)\frac{\partial u(x,y)}{\partial y}-\frac{1}{\sigma}\frac{\partial V(x,y)}{\partial y}, \tag{4}$$

which is Burgers's equation[1] with a diffusion constant or damping proportional to T and conservative random forcing, $\partial V/\partial y$. When (4) is viewed as a nonlinear diffusion equation, x serves as the time coordinate and y as the space coordinate, and $u(x,y)$ is the drift velocity. That $u(x,y)$ is indeed a velocity, which scales as distance over time (y/x), is necessary because of the Galilean invariance of (4). The free energy $F(x,y)$ has a term that is linear in x. Since $u=\partial F/\partial y$, however, the fluctuations in F about this average value scale as y^2/x. The fluctuations in F scale as x^χ and y scales as x^ζ, and so this implies $\chi=2\zeta-1$. This exponent relation was pointed out by Huse and Henley[3] and can also be seen by examining the gradient-squared term in the Hamiltonian (1).

The forced Burgers's equation (4) obeys a fluctuation-dissipation theorem[5] as a consequence of which its steady-state distribution is simply

$$P\{u(x,y)\}\propto\exp[-\tfrac{1}{2}\lambda\int dy\,u^2(x,y)], \tag{5}$$

with $\lambda=\sigma k_{\rm B}T/\Delta$. This invariant distribution implies that

$$\langle[F(x,y)-F(x,y')]^2\rangle=\sigma|y-y'|/\lambda, \tag{6}$$

and, hence, $2\chi=\zeta$. The two exponent relations together dictate $\zeta=\frac{2}{3}$ and $\chi=\frac{1}{3}$, which are equivalent to the exponents derived by Forster, Nelson, and Stephen[2] for (4). The analysis of Forster, Nelson, and Stephen[2] implies that, for a given λ, the same fixed point governs the behavior of (4) at large distance and time scales for all Δ, including in the limit $T\to 0, \Delta\to 0$ at fixed λ. This limiting case of the Burgers's equation with neither forcing nor damping is exactly integrable.[1] The scaling exponents discussed above were first obtained by Burgers,[1] who studied the evolution in this integrable limit of random initial conditions with a distribution similar to (5).

Kardar and Nelson[6] recently solved a model of parallel interfaces with disorder and hard-core repulsion, from which they indirectly obtained the exact exponents ζ and χ. A similar scaling behavior has also been found by van Beijeren, Kutner, and Spohn,[7] for a hard-core lattice-gas model of one-dimensional conduction.

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