Roughening By Impurities At Finite Temperatures

In a recent Letter[1] Huse and Henley examined roughening of interfaces by random coupling energies. In particular, on the basis of numerical simulations, they concluded that at zero temperature, in two dimensions, fluctuations y of an interface scale with its length x as $y \sim x^{\zeta}$, with $\zeta \simeq \frac{2}{3}$. Also the fluctuations in energy gain ΔE of the rough wall from the weaker bonds scale as x^{χ} with $\chi \simeq \frac{1}{3}$. Do these exponents (presumably universal) apply to a different model of interfaces; and are they applicable to finite temperatures (in principle, entropy effects could alter the scaling)? Numerical simulations at finite temperature indicate that the answers are in the affirmative.

Interfaces consider here are defined on a lattice, and at zero temperature, and in the absence of impurities are straight lines through $y=0$. At a finite temperature T or because of impurities the interface fluctuates, and its configuration is described by the integer heights $y(x)$. Overhangs and islands are ignored, and only configurations with $|y(x+1)-y(x)|=0$ or 1 are allowed (the solid-on-solid or SOS model). $|y(x+1)-y(x)|=1$ corresponds to a broken bond in the x direction, with an energy cost E_0 (a Boltzman weight $\gamma = e^{-E_0}$). The bonds $\mu(x,y)$ in the y direction are independent random variables of variance σ^2. The interface begins at $x=0$, $y=0$; and at finite T, the total weight $W(x,y)$ of paths connecting (0,0) to (x,y) is calculated recursively from

$$W(x+1,y) = \exp[-\mu(x,y)][W(x,y) + \gamma W(x,y-1) + \gamma W(x,y+1)],$$

with $W(0,y) = \delta_{y,0}$. For each realization of randomness, the $T=0$ optimal value $\mathrm{op}(y(x))$ corresponds to the path terminating at x with the largest weight, while at finite T the mean $\langle y(x)\rangle$ is obtained by averaging over the weights $W(x,y)$. Numerical results are plotted in Fig. 1 for $\sigma^2 = \frac{5}{6}$, with $\gamma = 1.0$ and $\gamma = 0.1$. The quenched random averaging in $y_0 = \overline{[\mathrm{op}(y(x))^2]}^{1/2}$ and $y_m = \overline{[\langle y(x)\rangle^2]}^{1/2}$ is performed by summing over 400 different realizations. Dashed lines are fits by power laws $y \sim x^{\zeta}$. The exponents for y_0 and y_m are 0.64 and 0.67 for $\gamma = 1.0$, and 0.62 and 0.67 for $\gamma = 0.1$. They are in agreement with $\zeta \simeq \frac{2}{3}$ of Ref. 1 suggesting that ζ is indeed universal and applicable to finite T. In contrast to nonrandom systems, fluctuations are smaller at finite T (although the scaling is the same). Since there are more paths to smaller y, entropy effects tend to reduce $\langle y\rangle$. Therefore, roughness of the interface at short length scales is smoothed out at finite T. At infinite temperatures, disorder $(\sigma \sim 1/T)$ disappears, and fluctuations characteristic of a uniform interface ($\langle y\rangle = 0$, $\langle y^2\rangle = x$) are expected. The nonrandom exponent of $\frac{1}{2}$ describes smoother fluctuations. Dimensional arguments[2] indicate that crossover to impurity-dominated roughening occurs at length scales larger than $x_d \simeq \gamma/\sigma^2$. Also important are the fluctuations in energy gain $\Delta E(x)$ from random bonds. Huse and Henley find $\Delta E(x) \sim x^{\chi}$, with $\chi \simeq \frac{1}{3}$. A similar exponent was obtained for the optimal paths in the above SOS model. The analogue of $\Delta E(x)$ at finite T is the constrained free energy $\Delta F(x,y)$ for paths from (0,0) to (x,y). Again $\Delta F(x,\langle y\rangle)$ scales as x^{χ} with $\chi \simeq \frac{1}{3}$; i.e., the free energy is dominated by the energy, and entropy effects are secondary. Extending the results to finite T is actually a necessary step in an indirect proof presented elsewhere[2] that the exponent ζ is exactly $\frac{2}{3}$, and provides a connection with the direct proof in the following Comment.

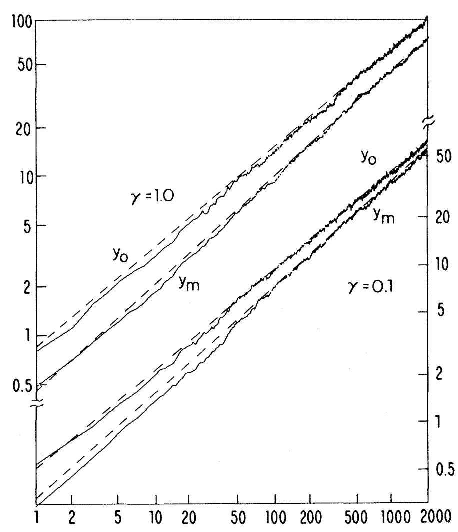

FIG. 1. Fluctuations of an impurity-roughened wall.

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[1] D. A. Huse and C. L. Henley, Phys. Rev. Lett. **54**, 2708 (1985).
[2] M. Kardar and D. R. Nelson, Phys. Rev. Lett. **55**, 1157 (1985).

