Charge-Density-Wave Depinning: A Dynamical Critical Phenomenon?

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The dynamic coherence volume of the sliding-charge-density-wave conductor orthorhombic TaS_3 , obtained from the broad-band noise, is found to diverge as the threshold field is approached from above. In agreement with a recent theoretical treatment of the depinning process as a dynamical critical phenomenon, scaling behavior is observed for both this divergence and the charge-density-wave current. The apparent critical exponents for the coherence length and the charge-density-wave current differ substantially from the calculated mean-field values. Thermal rounding of the transition is observed at high temperatures.

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In recent years the remarkable fieldand frequency-dependent dynamical properties of slidingcharge-density-wave conductors such as NbSe₃ and TaS₃ have been a subject of considerable experimental and theoretical activity.¹ The charge-density-wave (CDW) condensate is pinned by impurities below a threshold field E_T . At higher fields it slides, leading to a strongly nonlinear dc conductivity, noise, and a variety of other phenomena. Several theoretical models have been proposed for the depinning process,¹ but no consensus has been reached. Recently, Fisher² has constructed a classical scaling theory which suggests that the depinning process at E_T is a dynamical critical phenomenon. Scaling behavior is predicted for both the velocity-velocity correlation length ξ_{n} and $I_{\rm CDW}$, the current carried by the sliding condensate:

$$\xi_{\nu} \propto (E - E_T)^{-\nu}, \tag{1}$$

 $I_{\rm CDW} \propto (E-E_T)^{\zeta}$.

In Fisher's mean-field theory, $\nu = \frac{1}{2}$ and $\zeta = \frac{3}{2}$. Experimental tests of these predictions are important in furthering our understanding of CDW conduction.

The threshold behavior of the CDW current measured in most samples is consistent with $\zeta > 1$. This contrasts sharply with the behavior predicted for any finite number of degrees of freedom,³ where $\zeta = \frac{1}{2}$, and provided one of the motivations for the theoretical approach of Fisher. However, reported values for ζ have varied considerably,¹ and the observed threshold behavior can also be explained by disorder, either static⁴ or thermal. It is thus important that the behavior of the dynamic coherence length ξ_v be studied to provide a less ambiguous test for the existence of critical behavior. A novel technique is required to measure this normally inaccessible quantity.

We have shown in a previous paper⁵ that the broadband or "1/f" noise in CDW conductors is quantitatively described by a threshold-field fluctuation model. The analysis allows the broad-band noise to be used as a direct probe of a dynamic coherence volume ξ_D^3 . The exact correspondence between ξ_D and ξ_v has not yet been established. However, we demonstrate below that ξ_D shows a critical divergence near threshold, i.e., $\xi_D \propto (E - E_T)^{-\nu}$. This divergence and the variation of the CDW current follow scaling behavior. These results provide strong evidence that the depinning transition is a dynamical critical phenomenon. Both ν and ζ differ substantially from their mean-field values.

At constant total current, the excess noise voltage is due to resistance fluctuations⁶: $\delta V^2(\omega) = I^2 \delta R^2(\omega)$. In normal Ohmic conductors, R and δR are field independent and thus $\delta V^{2\alpha} I^2$. The dependence of the noise voltage on sample volume is $\delta V^{2\alpha} (I/A)\xi_D^3$, where I is the sample length, A is the cross-sectional area, and ξ_D^3 is a dynamic coherence volume. This volume dependence implies that the noise is generated in the bulk and results from an incoherent addition of local fluctuations within regions of size ξ_D^3 . For normal conductors, ξ_D^3 represents the volume per electron $(\sim 10 \text{ Å}^3)$.

CDW conduction is intrinsically nonlinear, and $\delta R^2(\omega)$ is field dependent. For fields below E_T , there is a small amount of 1/f noise associated with Ohmic conduction by normal electrons. When the CDW depins at E_T , the magnitude of 1/f noise increases by several (~ 6) orders of magnitude, and its field dependence changes dramatically. The volume dependence of the noise is unchanged,^{5,7} implying that it is still generated in the bulk. The large increase in noise magnitude is consistent with an increase of ξ_D^3 to about 1 μ m³, which is of the same order as the static CDW phase coherence volume.

We have already shown⁵ that the field, temperature, and volume dependences of the noise above E_T are consistent with a model based on incoherent local fluctuations of the impurity pinning force, or effective threshold field, in volumes $\sim \xi_D^3$. These fluctuations were attributed to thermally activated transitions between "metastable steady states" involving deformations of the CDW condensate over length scales $\sim \xi_D$. The model implies⁵ that

$$\delta V^2(\omega) = I^2 (\partial R / \partial V_T)^2 (l/A) \xi_D^3 E_T^2 S(\omega, T), \quad (2)$$



FIG. 1. (a) Field dependence of the noise voltage δV^2 measured at 330 Hz and of $I^2(\partial R/\partial V)^2$. The ratio ξ_D^3 is shown (b) vs V in a semilog plot, and (c) vs $(V - V_T)/V_T$ in a log-log plot.

where $V_T = lE_T$, and $S(\omega, T)$ is the spectral weight function. Recent studies⁸ of $S(\omega, T)$ are consistent with thermal activation of the transitions between metastable steady states. Moreover, S is independent of current bias for sufficiently large bias: specifically, for biases such that $\omega_0 >> \omega$, where ω_0 is the washboard or narrow-band noise frequency.¹

Equation (2) can be used to extract the field dependence of the dynamic coherence volume if the field dependence of all other quantities is known. The threshold field and sample dimensions are field independent and easily measured. For the measurement frequency used here, $\omega/2\pi = 330$ Hz, $S(\omega,T)$ is also bias independent for $(V - V_T)/V_T > 10^{-2}$. The only quantity that is not readily measured is $\partial R/\partial V_T$. In Ref. 5, R was assumed to be a function of $V - V_T$, implying $\partial R/\partial V_T = -\partial R/\partial V$, which is easily obtained from the I-V characteristic. Near V_T , this functional form for R must be valid on general grounds. The excellent tracking between δV^2 and $I^2(\partial R/\partial V)^2$ observed above $2E_T$ in Ref. 5 suggests that it is valid at higher fields also. In what follows we assume that the ratio $\delta V^2/[I^2(\partial R/\partial V)^2]$ is proportional to ξ_D^3 for all fields.

Figure 1(a) shows the field dependence of $\delta V^2(\omega = 330 \text{ Hz})$ and $I^2(\partial R/\partial V)^2$ in a log-linear plot for a 1 mm \times 20 μ m \times 40 μ m sample of orthorhombic TaS₃ (o-TaS₃) at 120 K. Beyond $V = 2V_T$, the two quantities track each other accurately.⁵ Here we have expanded the scale to emphasize the region near V_T , where the two quantities no longer track.⁹ Figure 1(b)shows a log-linear plot of their ratio, which gives the field dependence of ξ_D^3 . There is a dramatic divergence of ξ_D^3 by more than 6 orders of magnitude as V_T is approached from above. In order to determine whether this divergence obeys the scaling behavior expected for a critical phenomenon, we plot ξ_D^3 vs $\epsilon = (V - V_T)/V_T$ on a log-log scale in Fig. 1(c). For values of ϵ between 0.03 and 1, an approximate straight-line scaling behavior is obtained. The apparent slope is 3.9 which (neglecting possible anisotropy) yields v = 1.3, a substantially different value from the mean-field result, $\nu = \frac{1}{2}$. Note that the critical region extends to about $2V_T$, which is much larger than the critical region seen in typical thermodynamic critical phenomena.

Figure 2(a) shows that scaling behavior is observed for ξ_D^3 at all temperatures. The slopes of all curves are equivalent within the accuracy of our measurement. However, at high temperatures, a substantial region is evident near V_T where ξ_D^3 is saturated. The width of this region decreases as the temperature is lowered. While saturation of ξ_D^3 could occur as a result of finite-size effects, the variation of the width of the saturation region with temperature is inconsistent with this explanation. The high-field value of ξ_D decreases with increasing temperature,⁵ which would lead to less saturation at high temperatures. We therefore propose that thermal rounding is responsible for this effect. Thermal fluctuations can lead to independent motion in different regions which would not otherwise be possible in the limit $E \rightarrow E_T$, resulting in an upper limit on the divergence of ξ_D . Such fluctuations can also lead to an activated CDW current at biases below threshold.¹⁰ If this is the explanation for the saturation seen in Fig. 2, the only true critical point for the depinning transition is at T = 0.

Recent measurements of the ac conductivity¹¹⁻¹³ and noise⁸ indicate thermal activation of transitions between metastable states. Figure 2(a) illustrates another important effect of temperature: The critical behavior near V_T is suppressed. Thermal effects have been left out of most theories of CDW conduction.¹ These recent experiments point to the importance of their being included in future theoretical work.

The thermal rounding described above, and experi-



FIG. 2. Scaling of (a) ξ_D^3 vs $(V - V_T)/V_T$ and (b) I_{CDW} vs ξ_D^3 at different temperatures. Data for different temperatures are displaced arbitrarily for clarity.

mental noise, make it difficult to determine V_T accurately. This is the main source of error in our measurements of the critical exponent ν . For each temperature, V_T was determined in the following way. The quantity d^2I/dV^2 was evaluated from the *I-V* characteristic. The lowest value of *V* for which this

quantity was larger than the numerical noise was designated as V_T . We have also tested other criteria, such as choosing the first value of V where the noise was above the background. The value of V_T changes by at most 5%. Errors of this magnitude introduce uncertainties of about 5% in the value of the critical exponent. Larger ambiguities in V_T and ν may result if there is appreciable thermally activated conduction below the true V_T . The magnitude of such effects will decrease with temperature.

Within our accuracy, ν is temperature independent above 100 K: $\nu = 1.25 \pm 0.10$. This is more than twice the mean-field value $\nu = 0.5$. However, the mean-field value is not expected to apply in d = 3, and recent work¹⁴ suggests that mean-field behavior is only obtained as $d \rightarrow \infty$. Thus the discrepancy between the mean-field and measured values is not surprising.

We now turn to the critical behavior of the CDW current. There have been previous reports^{15, 16} that the critical exponent ζ is close to its mean-field value. However, both measurements found that ζ varied with temperature. In the experiment on TaS₃, ζ was found to vary from 1.3 to 2.3 as temperature decreased.¹⁵ Note that the washboard frequency ω_0 rather than I_{CDW} was measured in this experiment and that ω_0 and I_{CDW} are not always proportional.¹⁷ Also, the experimental data were fitted over a range of at most a factor of 5 in $V - V_T$.

Our measurements imply that ζ is larger than the mean-field value in TaS_3 . Figure 2(b) shows the relationship between the two scaling quantities ξ_D^3 and I_{CDW} at several temperatures. A plot of these variables against each other removes the need for precise determinations of V_T . At all temperatures, the curves show a scaling region with slope close to 1. At low temperatures $(T < 150 \text{ K}), 3\nu/\zeta = 1.0 \pm 0.1$. The value of ζ is thus $\zeta = 3.7 \pm 0.4$. Note that the relation $3\nu/\zeta = 1$ is also obeyed by the mean-field exponents. This suggests that a scaling relation may exist between the exponents. At higher temperatures the value of ζ appears to decrease while ν remains constant. However, measurements at these temperatures are less accurate because there is more thermal rounding, the change in resistance with field is small, and heating is more likely.

We have also studied I_{CDW} vs $V - V_T$. Above the field where ξ_D^3 saturates [Fig. 2(a)], the data fall on a straight line on a log-log plot with a slope consistent with the value determined from Fig. 2(b). In the region where ξ_D^3 is saturated, the apparent slope decreases. Fits in this region could give much lower values of ζ , but the saturation of ξ_D^3 indicates that thermal effects are important here. As mentioned above, the CDW current may have a thermally activated component.¹⁰ Such a component would make determination of V_T difficult and would introduce the

main source of uncertainty in our determination of the critical exponents. However, a strict lower bound on ζ can be determined. At all temperatures, plots of d^2I/dV^2 are monotonically increasing out to at least 3 times V_T . This clearly implies $\zeta > 2$. For low temperatures (T < 150 K), the increase in d^2I/dV^2 is faster than linear up to twice V_T , in agreement with our measured value for ζ .

In conclusion, we have demonstrated that the dynamic coherence length in o-TaS₃ shows a critical divergence as V approaches V_T from above. Thermal rounding of this divergence appears at high temperatures, suggesting that the only true critical point is at T = 0. The critical exponents for the dynamic coherence length and the CDW current appear to obey a scaling relation which is also obeyed by the mean-field exponents. Several important questions remain which should be addressed in future theoretical and experimental work. The scaling relation between ζ and ν should be studied and possible anisotropy in the exponents for the coherence length considered. Thermal effects appear to play an important role in these and previous measurements.^{5, 11–13} To our knowledge, no theoretical treatment of these effects has yet appeared. Finally, we note that variations in E_T with temperature may lead to complicated scaling and changes in the apparent exponents.¹⁸ This possibility should be explored and would indicate how relevant a variable temperature is.

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