Correlated Spontaneous-Emission Lasers: Quenching of Quantum Fluctuations in the Relative Phase Angle

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In quantum-beat and Hanle-effect experiments, spontaneous-emission events from two coherently excited states are strongly correlated. A doubly resonant laser cavity driven by such atomic configurations can have vanishing diffusion coefficient for the relative phase angle.

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In many areas of modern physics ultrasmall displacements are measured by placing a lasing medium in an optical cavity, and sensing a frequency shift associated with the change in optical path length caused by various effects, e.g., the motion of a mirror. In such experiments the frequency shift is typically determined by beating or heterodyning the light from the variable-frequency laser with that from a companion reference laser. Examples include the laser gyroscope,[1] the precision measurement of thermal-expansion coefficients,[2] and proposed variations on existing laser gravity-wave detectors.[3]

The limiting source of quantum noise in such experiments is often spontaneous-emission fluctuations[4] in the relative phase angle between the two lasers. In this Letter we show that diffusion of the relative phase angle between two such laser modes may be eliminated by preparing a laser medium consisting of "three-level" atoms, and arranging that the two transitions $|a\rangle \rightarrow |c\rangle$ and $|b\rangle \rightarrow |c\rangle$ drive a doubly resonant cavity; see Fig. 1. In this way the optical paths may be differentially affected by the external influence of interest (e.g., a gravity wave or a Sagnac frequency shift).

However, the atomic transitions driving the two optical paths are strongly correlated when the upper levels $|a\rangle$ and $|b\rangle$ are prepared in a coherent superposition as in quantum-beat[5] or Hanle-effect[6] experiments. In the quantum-beat case the coherent mixing is produced by a strong[7] external microwave signal as in Fig. 1(a). In the Hanle-effect example, the levels $|a\rangle$ and $|b\rangle$ can be taken to the "linear polarization" states formed from a single "elliptical polarization" state as shown in Fig. 1(b). The fields 1 and 2 emitted by the atoms of Fig. 1(a) will differ in frequency while fields produced by the atoms of Fig. 1(b) will differ in polarization.

In both cases discussed above the heterodyne beat note between the spontaneously emitted fields 1 and 2 shows that they are strongly correlated.[8] To see this, consider the atoms of Figs. 1 interacting with a quantized field. The state vector is given by

$$|\psi\rangle = \alpha e^{-i\phi_a}|a,0\rangle + \beta e^{-i\phi_b}|b,0\rangle + \gamma_1|c,1_1\rangle + \gamma_2|c,1_2\rangle, \tag{1}$$

where $|1_i\rangle$ is the state $\hat{a}_i^\dagger|0\rangle$, $i=1,2$, and $\hat{a}_i^\dagger$ ($\hat{a}_i$) are the creation (annihilation) operators for photons having frequency ν_i. Now the expectation value for the electric field operator $\hat{E}_1$,

$$\hat{E}_1(\mathbf{r},t) = \epsilon_1 \hat{a}_1 e^{i(\mathbf{k}_1\cdot\mathbf{r}-\nu_1 t)}, \tag{2}$$

calculated using Eq. (1) is easily seen to vanish since the states $|a\rangle$, $|b\rangle$, and $|c\rangle$ are orthogonal. Similar arguments show that $\langle \hat{E}_2\rangle$ likewise vanishes. However, the cross term[9] does not vanish:

$$\langle\psi|\hat{E}_1^\dagger E_2|\psi\rangle = \epsilon_1\epsilon_2\gamma_1^*\gamma_2\langle c|c\rangle \exp[-i(\mathbf{k}_1-\mathbf{k}_2)\cdot\mathbf{r} + i(\nu_1-\nu_2)t]. \tag{3}$$

That is, the spontaneously emitted photons at ν_1 and ν_2 are correlated.

Motivated by the preceding arguments we are led to investigate diffusion in the relative phase angle when such (three-level) lasing atoms are prepared in a coherent superposition of the upper two levels, and are placed in a doubly resonant cavity as in Fig. 1. The quantum theory of such laser configurations may be conveniently cast in terms of the following equations of motion involving Langevin quantum-noise operators[4] for the systems of Figs. 1. In both cases the operator equations of motion are found to be

$$\dot{\hat{a}}_1 = -i(\Omega_1-\nu_1)\hat{a}_1 + \alpha_1\hat{a}_1 + \alpha_{12}\hat{a}_2 e^{i\Phi(t)} - \gamma_1\hat{a}_1 + \beta_{11}(\hat{a}_1,\hat{a}_1^\dagger) + \beta_{12}(\hat{a}_1,\hat{a}_1^\dagger;\hat{a}_2,\hat{a}_2^\dagger) + \hat{F}_1(t) + \hat{G}_1(t), \tag{4a}$$

$$\dot{\hat{a}}_2 = -i(\Omega_2-\nu_2)\hat{a}_2 + \alpha_2\hat{a}_2 + \alpha_{21}\hat{a}_1 e^{-i\phi(t)} - \gamma_2\hat{a}_2 + \beta_{22}(\hat{a}_2,\hat{a}_2^\dagger) + \beta_{21}(\hat{a}_2,\hat{a}_2^\dagger;\hat{a}_1,\hat{a}_1^\dagger) + \hat{F}_2(t) + \hat{G}_2(t). \tag{4b}$$

When the coherent mixing of levels $|a\rangle$ and $|b\rangle$ is produced via a microwave signal having frequency ω_0, the phase angle Φ is given by

$$\Phi(t)=(\nu_1-\nu_2-\omega_0)t-\phi, \tag{5a}$$

where ϕ is the (microwave determined) atomic phase difference $\phi_a-\phi_b$. In the case of polarization "labeling" of fields 1 and 2 as per Fig. 1(b) the phase angle is

$$\Phi(t)=(\nu_1-\nu_2)t-\phi, \tag{5b}$$

where ϕ is again the relative phase between levels $|a\rangle$ and $|b\rangle$ but determined this time by the state of elliptical polarization of the pump light used to excite the atoms.

The frequencies Ω_i, $i=1,2$, are the empty-cavity frequencies while ν_i denotes the actual lasing frequencies. The detailed form of the linear gain constants (α_1 and α_2) and the cross-coupling coefficients (α_{12} and α_{21}) need not concern us here; however, we note that they are determined respectively by the diagonal and off-diagonal elements of the atomic density matrix.

The nonlinear self-saturation and cross-coupling terms are denoted by β_{11} and β_{12} and will not be necessary for the purposes of this paper. The linear loss rate is γ_1. Finally, the quantum Langevin-noise operators $\hat{F}_1$ and $\hat{G}_1$ are associated with the gain, cross coupling, and loss for mode 1. Identical definitions apply to mode 2 with the interchange of the indices 1 and 2. The noise operators are defined by their diffusion coefficients

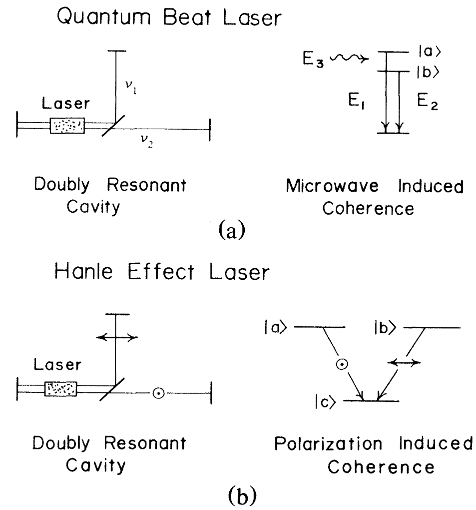

FIG. 1. (a) Active medium consisting of coherently excited three-level atoms drives doubly resonant cavity. Similar atomic configurations are prepared in quantum-beat experiments. Coherence is generated by, e.g., an external microwave field. (b) Active medium is prepared in coherent excitation of states $|a\rangle$ and $|b\rangle$, which decay to state $|c\rangle$ via emission of radiation of differing polarization states as in Hanle-effect experiments. A polarization-sensitive mirror is used to couple a doubly resonant cavity.

$$\langle \hat{G}_i^\dagger(t)\hat{G}_j(t')\rangle = 2\gamma_i\bar{n}_i\delta_{ij}\delta(t-t'), \tag{6a}$$

where $\bar{n}_i$ is the thermal photon number for the ith cavity.[10] For the active medium

$$\langle \hat{F}_i^\dagger(t)\hat{F}_j(t')\rangle = 2D_{ij}\delta(t-t'), \tag{6b}$$

and the diffusion constants are determined from the generalized Einstein relation

$$2D_{ij}=\left\langle \frac{\partial}{\partial t}(\hat{a}_i^\dagger a_j)\right\rangle - \left\langle\left(\frac{\partial \hat{a}_i^\dagger}{\partial t}\right)\hat{a}_j\right\rangle - \left\langle \hat{a}_i^\dagger\left(\frac{\partial \hat{a}_j}{\partial t}\right)\right\rangle. \tag{6c}$$

The Einstein relation[11] (6c) is evaluated by use of the equation of motion for the density matrix describing the laser radiation field $\hat{\rho}(\hat{a}_1,\hat{a}_1^\dagger;\hat{a}_2,\hat{a}_2^\dagger)$, that is,

$$\frac{d\hat{\rho}}{dt}=\sum\nolimits_{ij}\hat{\mathscr{L}}_{ij}\hat{\rho}, \tag{7}$$

where the linear gain and cross-coupling Liouville operators are given by[12]

$$\hat{\mathscr{L}}_{ii}\hat{\rho}=-\tfrac{1}{2}[\alpha_i\hat{\rho}\hat{a}_i\hat{a}_i^\dagger+\alpha_i^*\hat{a}_i\hat{a}_i^\dagger\hat{\rho}-(\alpha_i+\alpha_i^*)\hat{a}_i^\dagger\hat{\rho}\hat{a}_i], \tag{8a}$$

$$\hat{\mathscr{L}}_{12}\hat{\rho}=-\tfrac{1}{2}[\alpha_{12}\hat{\rho}\hat{a}_2\hat{a}_1^\dagger+\alpha_{21}^*\hat{a}_2\hat{a}_1^\dagger\hat{\rho}-(\alpha_{12}+\alpha_{21}^*)\hat{a}_1^\dagger\hat{\rho}\hat{a}_2]e^{i\Phi}, \tag{8b}$$

$$\hat{\mathscr{L}}_{21}\hat{\rho}=-\tfrac{1}{2}[\alpha_{21}\hat{\rho}\hat{a}_1\hat{a}_2^\dagger+\alpha_{12}^*\hat{a}_1\hat{a}_2^\dagger\hat{\rho}-(\alpha_{21}+\alpha_{12}^*)\hat{a}_2^\dagger\hat{\rho}\hat{a}_1]e^{-i\Phi}. \tag{8c}$$

With use of Eqs. (6), (7), and (8) we find the diffusion coefficients as they appear in Eq. (6b) to be

$$D_{ii}=\tfrac{1}{4}(\alpha_i+\alpha_i^*),\quad i=1,2, \tag{9a}$$

and

$$D_{12}=\tfrac{1}{4}(\alpha_{21}+\alpha_{12}^*)e^{-i\Phi},\quad D_{21}=D_{12}^*. \tag{9b}$$

We may now proceed to calculate the phase diffusion time for our heterodyne average $\langle a_1^\dagger a_2 \rangle$ with use of Eqs. (4)–(9) in the usual way (see, e.g., SSL[4] Sect. 20-3). We associate $\hat{a}_i(t)$ with the complex signal $\rho_i \exp[-i\theta_i(t)]$ and write

$$\langle \hat{a}_1^\dagger(t)\hat{a}_2(t)\rangle = \rho_1\rho_2 \exp\{-\tfrac{1}{2}\langle[\theta_1(t)-\theta_2(t)]^2\rangle\}. \qquad (10)$$

The phases in Eq. (10) are determined from Eqs. (4) to be

$$\theta_i(t) = \int_{t_0}^{t} dt' \frac{i}{2\rho_i}[\hat{F}_i(t)e^{i\theta_i} - \hat{F}_i^\dagger(t)e^{-i\theta_i}]. \qquad (11)$$

Inserting Eq. (11) into Eq. (10) and using Eqs. (9), we find

$$\langle \hat{a}_1^\dagger(t)\hat{a}_2(t)\rangle = \rho_1\rho_2 \exp(-Dt), \qquad (12a)$$

where

$$D = \frac{1}{16}\left\{\left(\frac{\alpha_1}{\rho_1^2} + \frac{\alpha_2}{\rho_2^2}\right) - \frac{(\alpha_{12}+\alpha_{21}^*)e^{-i\psi}}{\rho_1\rho_2}\right\} + \text{c.c.} \qquad (12b)$$

and $\psi = \Phi + \theta_1 - \theta_2$ with θ_i being the phase of the ith field.

Now in deriving Eqs. (12) we have made use of the fact that the phase angle ψ will (under normal operating conditions) lock to a constant value. This constant phase angle is determined by the locking equation of motion for $\psi(t)$ as obtained from Eqs. (4), namely[13]

$$\dot{\psi}(t) = a - b\sin\psi(t), \qquad (13)$$

where the locking parameter b is determined (in part) by α_{12} and α_{21}, and a is a small frequency difference which can be made to vanish by a proper choice of $\Omega_1 - \Omega_2$. In that case ψ locks[14] to zero and the rate of phase diffusion as given by Eq. (12b) vanishes; the terms in curly brackets cancel. Detailed analysis, to be published elsewhere, shows that this can be satisfied for a variety of laser parameters. For example, when all α's were equal and $\rho_1 = \rho_2$ then D as defined by Eq. (12b) would obviously vanish when $\psi = 0$. Further discussion of this point and experimental arrangements designed to maximize signal-to-noise ratio will be given in a future paper.

In addition to its intrinsic interest within the field of quantum optics, the possibility of noise suppression[15] via correlated-emission lasers holds promise for application in several areas of research. The analysis of such potential applications will be presented elsewhere.

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[1]See, for example, the recent review by W. W. Chou *et al.*, Rev. Mod. Phys. **57**, 61 (1981).

[2]J. Berthold and S. Jacobs, Appl. Opt. **15**, 2344 (1976).

[3]We have found the review by K. Thorne, Rev. Mod. Phys. **52**, 285 (1980), to be very helpful. For a review of recent optical work see the excellent review by C. Borde and co-workers, Ann. Phys. (Paris) **10**, 201 (1985); see also *Quantum Optics, Experimental Gravitation, and Measurement Theory,* edited by P. Meystre and M. O. Scully, NATO Advanced Study Institute Series B, Vol. 94 (Plenum, New York, 1983).

[4]M. Lax, in *Statistical Physics, Phase Transitions, and Superfluidity,* edited by M. Chretien *et al.* (Gordon and Breach, New York, 1966), Vol. 2; H. Haken, *Handbuch der Physik* (Springer-Verlag, New York, 1970); M. Sargent, III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, Mass., 1974) (hereafter referred to as SSL).

[5]The pioneering work in quantum-beat experiments was carried out by G. Series and co-workers. The author thanks W. Sandle for stimulating discussions in this regard. The atomic-beam experiment of S. Baskin provides a dramatic example of this effect. Papers especially relevant to the present work are W. Chow, M. Scully, and J. Stoner, Phys. Rev. A **11**, 1380 (1975); R. Herman, H. Grotch, R. Kornblith, and J. Eberly, Phys. Rev. A **11**, 1389 (1975); I. Senitzky, Phys. Rev. Lett. **35**, 1755 (1975).

[6]W. Hanle, Z. Phys. **30**, 93 (1924). An account of this problem oriented along the present lines is given by M. Scully, in *Atomic Physics 1,* edited by B. Bederson, V. W. Cohen, and F. M. Pichanick (Plenum, New York, 1969), p. 81.

[7]The microwave field must be a strong field since states $|a\rangle$ and $|b\rangle$ cannot be coupled by a dipole transition, since the transitions $|a\rangle$ to $|c\rangle$ and $|b\rangle$ to $|c\rangle$ obey dipole selection rules. One would therefore anticipate a weaker coupling between the upper two levels and would compensate for this by, for example, driving the transition with a strong external field. Alternatively one could use, e.g., a molecular system in which parity is not a good quantum number.

[8]The notion of correlated spontaneous emission is discussed in Ref. 7 and also more recently in M. Scully and K. Druhl, Phys. Rev. A **25**, 2208 (1982). See also the interesting recent paper by I. Senitzky, J. Opt. Soc. Am. B **1**, 879 (1984).

[9]For a discussion of optical correlation theory see R. Glauber, in *Quantum Optics and Electronics,* edited by B. DeWitt *et al.* (Gordon and Breach, New York, 1964). We are here using the Onsager regression theorem as applied in Chap. 18 of SSL, Ref. 4.

[10]We may take the number of thermal photons $\bar{n}_i$ to be zero for the optical transitions here considered.

[11]The partial derivatives in Eq. (6) remind us that the Einstein relation is found from mean drift only (ignoring diffusion). The Einstein relation measures the extent to which the usual product rule of differentiation does not apply to stochastic processes.

[12]The exact form of the α's need not concern us here and will be given in the quantum-beat laser case in a forthcoming paper by M. Scully and S. Zubairy. The Hanle-laser parameters will be published in a paper by J. Gea-Banacloche and M. Scully.

[13]A discussion of mode locking as it appears in the present problem can be found in SSL (Ref. 4), or in Ref. 1.

[14]Although the phase locks, information associated with path length differences can still be extracted by various strategies. For example, the relative phase between the two beams will be shifted by the frequency differential of interest according to the expression $\delta\psi = a/b$ for the small frequency shifts we wish to measure. Other measurement strategies using correlated spontaneous-emission lasers will be discussed elsewhere.

[15]In conversations with my colleagues it has been necessary to point out that this work is unrelated to the beautiful "two-photon-squeezed state" studies of C. Caves, J. Shapiro, D. Walls, H. Yuen, and co-workers. However, upon completion of this paper it was pointed out to the author that there is an interesting connection between the present work and the cross-correlation studies of B. Dalton and P. Knight, J. Phys. B **15**, 3997 (1982), and Opt. Commun. **42**, 411 (1982); and of T. Kennedy and S. Swain, J. Phys. D **17**, 1751 (1984). In their studies they consider the noise reduction effects of *nonlinear* (third-order) cross coupling whereas our coupling is first order. They find that the linewidth can be reduced by a factor of one-half through this nonlinear coupling effect whereas our results lead to a vanishing linewidth. The author is indebted to S. Zubairy for pointing out these references.