Quantum Fluctuations, Pump Noise, and the Growth of Laser Radiation

R. Roy, A. W. Yu, and S. Zhu
School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332
(Received 10 July 1985)

We have measured the distribution of times for single-mode laser radiation to grow from an equilibrium spontaneous-emission background to a prescribed value of the intensity. These passage-time distributions are compared with the results of Monte Carlo simulations and demonstrate a new technique for the description of statistical fluctuations in lasers.

PACS numbers: 42.55.Mv, 05.40.+j, 42.50.+q

The growth of laser radiation from a spontaneous-emission background to a nonequilibrium steady state is an example of the decay of an unstable equilibrium state.[1] Pioneering studies of fluctuations in laser transients were performed by Arecchi and co-workers[2] and by Meltzer and Mandel.[3] Their experiments provided the first pictures of the evolution of photon number distributions in a laser (see also Pariser and Marshall[4]).

Much interest has been expressed recently in a somewhat different perspective of the transient growth of laser radiation. The intuitively simple and practically meaningful concept of first-passage times[5] (FPT) has been applied to this problem by several authors.[1] The basic question is the following: If we allow the radiation to develop from a spontaneous-emission background, what is the distribution of times for the intensity to reach a certain prescribed value? We report here the first measurements of passage-time statistics for the laser turn-on process. The technique developed provides an entirely new source of information on statistical fluctuations in lasers. Whereas photon-counting techniques are invaluable near and below threshold, the methods reported here are easily applied at much higher intensities. FPT measurements thus complement the well-established photon-counting and correlation measurements. The computations presented here examine the role of additive and multiplicative (non-Markovian) noise in determination of the qualitative and quantitative features of the passage-time distributions.

The ring dye (rhodamine 6G) laser (Fig. 1) is pumped by an argon-ion laser and is extremely stable in steady-state operation. A Pellin-Broca prism and Fabry-Perot etalon ensure single-mode operation in the steady state, and a Faraday rotator and compensator are used to obtain unidirectional operation. The cavity is 160 cm long, and the round-trip loss per pass is $\sim 12\%$.

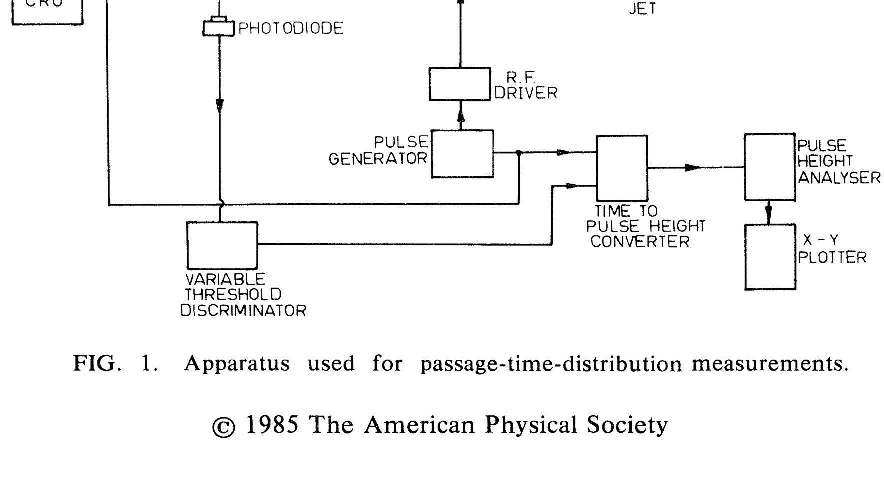

FIG. 1. Apparatus used for passage-time-distribution measurements.

An acousto-optic modulator in the laser cavity turns the laser on and off. The rise time of the modulator from maximum transmission (about a 2% insertion loss) to the maximum-loss (60% diffraction efficiency) condition requires less than 15 ns. The laser was turned on and allowed to reach steady-state intensity I_{ss} and stay on for about 600 μs. It was then turned off for 400 μs. This process was repeated several hundred thousand times.

The laser output was incident on a fast linear-response photodiode with a rise time of $\leqslant 1$ ns. As the laser is switched on with the acousto-optic modulator, a trigger pulse marks the beginning of the turn-on time. When the intensity I crosses a reference value I_{ref}, the photodiode voltage crosses a preset threshold on the discriminator, and a pulse is generated. The initial trigger pulse and this pulse are the start and stop inputs for a time–to–pulse-height converter which generates an output pulse with amplitude proportional to the separation in time between the start and stop pulses. These output pulses are fed to a pulse-height analyzer which then generates the FPT distribution for the laser radiation. The laser was operated such that its steady state was quite far above threshold, with a pump power $\sim 15\%$ in excess of the threshold pump power.

In Fig. 2 we show three of the seven measured first-passage-time distributions for $I_{ref}/I_{ss} = 0.44$, 0.66, and 0.96, respectively. As the reference intensity is increased, the distributions move progressively to the right (i.e., the laser takes a longer time to reach that I_{ref}), grow in width, and develop a decided asymmetry. The entire time scale is 9.0 μs.

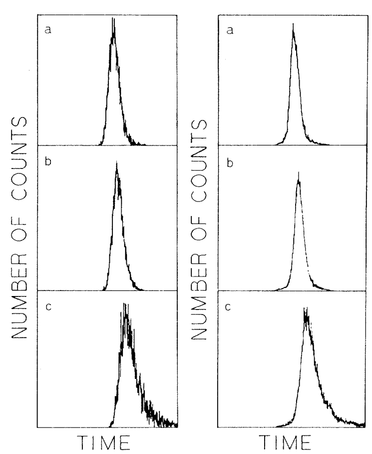

FIG. 2. Comparison of measured passage-time distributions with the results of Monte Carlo simulations of a single-mode laser model with quantum fluctuations and pump noise. (a) $I_{ref}/I_{ss} = 0.44$, (b) $I_{ref}/I_{ss} = 0.66$, and (c) $I_{ref}/I_{ss} = 0.96$. The simulation results are on the left. The other parameters are kept constant for all the simulation results: $a_0 = 2.16 \times 10^6$ s^{-1}, $A = 2.64 \times 10^6$ s^{-1}, $P = 0.0043$ s^{-1}, $P' = 3 \times 10^4$ s^{-1}, $\gamma = 2.4 \times 10^6$ s^{-1}, and integration time step $= 1.0 \times 10^{-8}$ s. The total time scale is 9.0 μs.

To interpret these measurements, we first performed Monte Carlo simulations on a standard single-mode laser model[6]:

$$dE/dt = aE - A|E|^2E + q(t), \tag{1}$$

with

$$\langle q_i(t) q_j(t') \rangle = P\,\delta_{ij}\,\delta(t-t') \quad (i,j = 1,2), \tag{2}$$

where a and A are the net-gain and self-saturation coefficients, and $q(t)$ is a δ-correlated, Gaussian, complex noise source which represents the effect of spontaneous emission on the complex dimensionless field E.[7] Simulations quickly showed that the growth in the width of the FPT distributions could not be explained adequately by this model, which is widely used.[1] We then included pump fluctuations with time scale $1/\gamma$ as follows:

$$dE/dt = a_0E - A|E|^2E + p(t)[E - (A/F_1)|E|^2E] + q(t), \tag{3}$$

$$dp/dt = -\gamma p + \gamma q'(t), \tag{4}$$

with

$$\langle q_i'(t) q_j'(t') \rangle = P'\,\delta_{ij}\,\delta(t-t'), \tag{5}$$

which implies

$$\langle p_i(t) p_j(t') \rangle = \tfrac{1}{2} P' \gamma e^{-\gamma|t-t'|}, \tag{6}$$

and Eq. (2) is still valid. a_0 is the average net gain. Here $i,j = 1,2$ for the real and imaginary parts; F_1 is the first-order factor.[7]

Thus the effects of multiplicative pump noise with a finite correlation time[8,9] and of additive white noise (quantum fluctuations) were incorporated in this model. These equations take account of noise in the inversion which will be present in both the a and A terms. If only loss fluctuations are present, the second term in the square brackets should be omitted. We find in practice, however, that this term is negligible for our simulations.

The FPT distribution is obtained by integration of Eqs. (3)–(6) step by step,[10] noting the value of the intensity I ($= |E|^2$) after each step. As soon as $I \geqslant I_{ref}$, the value of the time step is stored. This procedure was repeated 10 000 times to construct the FPT distributions shown on the left-hand side of the experimental measurements in Fig. 2. It is clearly seen that the delay, width, and asymmetry of all the distributions can be reproduced very well by the numerical simulations. A total of seven measured distributions were fitted by simulations done with the same values of the constants a_0 A, γ, P, and P', of which three are shown in Fig. 2. The constants are obtained by variation of

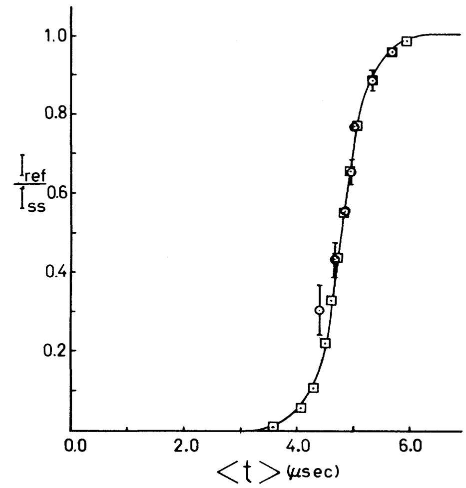

FIG. 3. Mean first-passage times from experiment (circles) and from simulations (squares) vs $I_{\mathrm{ref}}/I_{\mathrm{ss}}$. The results of the simulations with and without pump noise are indistinguishable for the mean FPT.

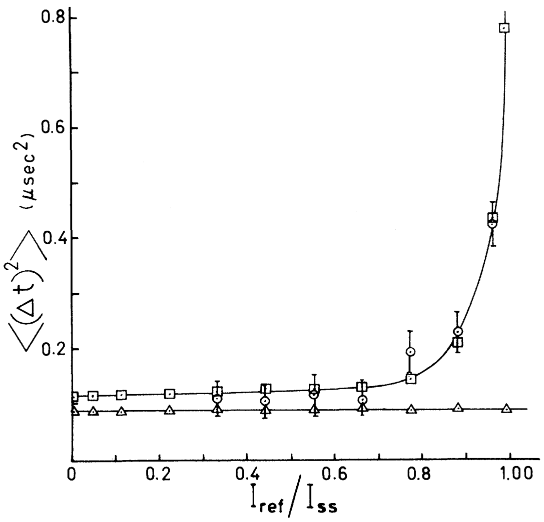

FIG. 4. Variance of the passage-time distributions from the experiment (circles), simulations without pump noise (triangles), and simulations with pump noise (squares). The bars show typical experimental uncertainties. The curves have been drawn through the simulation results.

them until the simulated distributions match the measured ones, and are quite reasonable in value.[6] The parameters are $a_0 = 2.16\times10^6\ \mathrm{s}^{-1}$, $A = 2.64\times10^6\ \mathrm{s}^{-1}$, $\gamma = 2.4\times10^6\ \mathrm{s}^{-1}$, $P = 0.0043\ \mathrm{s}^{-1}$, and $P' = 3\times10^4\ \mathrm{s}^{-1}$. A time step of 1.0×10^{-8} s was taken. These parameters imply pump fluctuations of about 2% in the net gain. The definitions of the loss, gain, and saturation coefficients are given in Ref. 7. We note that the parameters a_0 and A can be independently determined from measurements of the cavity loss, pump power above threshold, and known decay rates of the rhodamine molecule.[6,11] Thus only three parameters, P, P', and γ, are freely varied to fit a total of seven distributions; error estimates are $\sim 20\%$ for these parameters. The greater amount of "noise" in the simulated distributions reflects the fact that 10 000 stochastic trajectories were computed, whereas several hundred thousand were measured experimentally for each distribution.

Figure 3 shows a plot of $I_{\mathrm{ref}}/I_{\mathrm{ss}}$ vs $\langle t\rangle$, the mean first-passage time from the experiments and simulations. Figure 4 shows the variance of these distributions versus $I_{\mathrm{ref}}/I_{\mathrm{ss}}$.[12] Two sets of simulation results are shown, with and without pump noise. The origin of the sharp rise in the widths is clearly the pump stochasticity. However, the variances are very similiar for the smaller values of $I_{\mathrm{ref}}/I_{\mathrm{ss}}$. This indicates that the longer time scale of the pump noise (compared to the time scale of the spontaneous fluctuations) and the small values of $|E|$ do not allow it to play a dominant role in the early stages of growth of E. Quantum fluctuations dominate then, but as the field approaches steady state, it is the pump noise that becomes important, and the spontaneous emission is negligible. The time scale $(1/\gamma)$ affects the skewness of the FPT distributions; these measurements will be published later.

We comment briefly on the validity of the cubic approximation in Eq. (3); it is expected to be valid for excitations as high as 20% above threshold (see Arecchi and Degiorgio[2] and Ref. 7).

We have shown the distinct roles of quantum fluctuations and pump noise in the transient growth of laser radiation, and also demonstrated the usefulness of the first-passage-time concept in obtaining a quantitative physical understanding of the laser.

The FPT distributions thus enable us to obtain consistently reasonable estimates of the parameters a_0 A, γ, P, and P', which characterize the statistical fluctuations of the single-mode laser. These techniques are applicable to any laser system that is operated in single mode, not too far above threshold, and for which the adiabatic elimination[6] of the atomic variables is possible. These are, in fact, the conditions for the validity of the third-order Lamb theory. It should also be possible to extend these methods to lasers which do not satisfy the above restrictions, and provide stringent criteria for testing the validity of theoretical models with the help of FPT measurements, which do not require elaborate photon-counting or correlation apparatus. The established counting and correlation techniques remain invaluable near and below threshold; the FPT measurements provide a new source of complementary information. Together, they assert the validity of the model given in Eqs. (3)–(6) in the description of a wide range of laser operation.

We thank S. Singh, C. H. Braden, D. C. O'Shea, P. Schulz, and E. W. Thomas for stimulating discussions and equipment loans; we are indebted to R. F. Fox for his careful reading of the paper and insightful comments. This research was supported in part by a Cottrell Research Grant from the Research Corporation.

[1]J. P. Gordon and E. W. Aslaksen, IEEE J. Quantum Electron. **6**, 428 (1970); M. Suzuki, J. Stat. Phys. **16**, 477 (1977); F. Haake, Phys. Rev. Lett. **41**, 1685 (1978); F. Haake, J. W. Haus, and R. J. Glauber, Phys. Rev. A **23**, 3255 (1981); F. T. Arecchi and A. Politi, Phys. Rev. Lett. **45**, 1219 (1980); M. R. Young and S. Singh, Phys. Rev. A **31**, 888 (1985); D. Polder, M. Schuurmans, and Q. Vrehen, Phys. Rev. A **19**, 1192 (1979); P. Goy, L. Moi, M. Gross, J. M. Raimond, C. Fabre, and S. Haroche, Phys. Rev. A **20**, 2065 (1982).

[2]F. T. Arecchi, V. Degiorgio, and B. Querzola, Phys. Rev. Lett. **19**, 1168 (1967); F. T. Arecchi and V. Degiorgio, Phys. Rev. A **3**, 1108 (1971).

[3]D. Meltzer and L. Mandel, Phys. Rev. A **3**, 1763 (1971).

[4]B. Pariser and T. C. Marshall, Appl. Phys. Lett. **6**, 232 (1965). See, also Arecchi, Degiorgio, and Querzola, Ref. 2.

[5]N. G. Van Kampen, *Stochastic Processes in Physics and Chemistry* (North-Holland, Amsterdam, 1982).

[6]M. Sargent, III, M. O. Scully, and W. E. Lamb, Jr., *Laser Physics* (Addison-Wesley, Reading, Mass., 1974); H. Haken, in *Encyclopedia of Physics,* edited by S. Flugge (Springer-Verlag, Berlin, 1970), Vol. XXV/2c; W. H. Louisell, *Quantum Statistical Properties of Radiation* (Wiley, New York, 1973); M. Lax, in *Brandeis Lectures,* edited by M. Chretien, E. P. Gross, and S. Deser (Gordon and Breach, New York, 1966), Vol. II.

[7]Sargent, Scully, and Lamb, Ref. 6.

[8]R. Graham, H. Hohnerbach, and A. Schenzle, Phys. Rev. Lett. **48**, 1396 1982); S. Dixit and P. Sahni, Phys. Rev. Lett. **50**, 1273 (1983); R. F. Fox, G. E. James, and R. Roy, Phys. Rev. A **30**, 2482 (1984); P. Jung and H. Risken, Phys. Lett. **103A**, 38 (1984); A. Hernandez-Machado, M. San Miguel, and S. Katz, Phys. Rev. A **31**, 2362 (1985).

[9]P. Hanggi, F. Marcheson, and P. Grigolini, Z. Phys. B **56**, 333 (1984); P. Lett and L. Mandel, to be published.

[10]J. M. Sancho, M. San Miguel, S. L. Katz, and J. D. Gunton, Phys. Rev. A **26**, 1589 (1982).

[11]R. B. Schaefer and C. R. Willis, Phys. Rev. A **13**, 1874 (1976).

[12]Instrumental limitations restricted us from measuring the FPT distributions for $I_{\mathrm{ref}}/I_{\mathrm{ss}} \lesssim 0.3$