# Effects of Extra Light $Z$ Bosons in Unified and Superstring Models

S. M. Barr[(a)]

*Department of Physics, University of Washington, Seattle, Washington 98195*

(Received 26 August 1985)

We discuss the low-energy effects of extra light $Z$ bosons in unified models especially in those models which might arise from the $E_8 \otimes E_8'$ superstring. We find that deviations from the standard model in neutral-current scattering data can give a very sensitive test of the presence of such bosons, of unification, and of the pattern of symmetry breaking. Such deviations have already in fact been observed. We also discuss flavor-changing effects such as $\mu \to 3e$.

PACS numbers: 14.80.Er, 11.30.Pb, 12.10.En, 12.30.Cx

In grand-unified models based on groups of rank 5 or greater the low-energy theory may contain extra Abelian gauge groups. This would imply the existence of extra $Z^0$ bosons beyond the one predicted in the standard model and discovered at CERN. In the $E_8 \otimes E_8'$ heterotic superstring theory[1] the four-dimensional effective theory is essentially a grand-unified theory. When the extra dimensions are compactified on a Calabi-Yau manifold in a manner so as to preserve a residual supersymmetry in four dimensions,[2] the gauge group of ordinary interactions is broken to $E_6$, a rank-6 group. Furthermore, the breaking at large scales is done (partly or completely) by expectation values of order parameters which are in the adjoint representation. As is well known, this favors the survival of unbroken U(1) factors in the low-energy theory. Thus, if the heterotic theory proves to be correct, it is quite likely that one or two extra $Z^0$ bosons might exist at low mass. Here we explore some of the phenomenology[3-5] associated with such particles, which henceforth we will call $Z'$ bosons. We discuss (1) deviations from the neutral-current predictions of the standard model in $\nu$-scattering and other experiments, and (2) rare processes such as $\mu^{\pm} \to e^{\pm}e^{\pm}e^{\pm}$ and other flavor-changing effects that might be mediated by such $Z'$ bosons.

Before discussing superstring unification itself let us review some earlier results[3-5] about the effects of $Z'$ bosons. In a general model with $Z'$ bosons the standard-model predictions for the neutral-current parameters, $\epsilon_L(u)$, $\epsilon_L(d)$, $\epsilon_R(u)$, $\epsilon_R(d)$, $c_{1u}$, $c_{1d}$, $c_{2u}$, $c_{2d}$, $g_V^e$, and $g_A^e$, would deviate slightly from experiment. In a model with gauge group $G_s \otimes U(1)_{Z'} \otimes \cdots$ [where $G_s \equiv SU(3)_c \otimes SU(2)_L \otimes U(1)_Y$], these deviations can be essentially arbitrary. This is not so, however, if $G_s \subset SU(5) \subset G_{\text{unified}}$. In that case the generator of $U(1)_{Z'}$ must commute with SU(5). Hence the $Z'$ bosons will have the same couplings to members of SU(5) multiplets, i.e., to the set ($u_L$, $d_L$, $\bar{u}_L$, and $e_L^+$) and also to the set ($\bar{d}_L$, $\nu_L$, and $e_L^-$). [It must be emphasized that $G_{\text{unified}}$ need not actually break to SU(5); it may break via some other sequence. All that is necessary is that $G_s$ be embedded in some SU(5) subgroup of $G_{\text{unified}}$, which is true of almost all of the unified models ever proposed.] So in grand-*unified* models in general, and the superstring theory in particular, the neutral-current predictions must satisfy certain conditions. Four such relations derived in Ref. 5 are as follows:

$$\epsilon_L(d) - \epsilon_R(d) - g_A^e = 0, \quad 2\epsilon_L(u) + 2\epsilon_R(u) + \epsilon_L(d) + \epsilon_R(d) + g_V^e = 0, \quad 2c_{1u} + c_{1d} + c_{2d} = 0,$$
$$(1 + \tfrac{2}{3}\sin^2\theta_W)\epsilon_L(u) + 2(1 - \sin^2\theta_W)\epsilon_R(u) + (1 - \tfrac{8}{3}\sin^2\theta_W)\epsilon_L(d) = 0. \qquad (1)$$

These are identically satisfied in the standard model. If there are $Z'$ bosons, however, these will *only* be satisfied, generally, if there is unification. Thus these equations provide a precise low-energy test of grand unification *if* there are light $Z'$ bosons (perhaps, if proton decay is not seen, the only test possible). Two other very useful combinations of parameters are

$$R \equiv 2\epsilon_L(u) - \epsilon_R(u) + 2\epsilon_L(d),$$
$$S \equiv \epsilon_L(u) + 2\epsilon_R(u) + \epsilon_L(d) + 5\epsilon_R(d). \qquad (2)$$

In the standard model $R = S = 0$. If there are light $Z'$ bosons, then $R \neq 0$ and $S \neq 0$. These parameters are thus a sensitive test of the presence of such particles. Moreover, in *unified* models with light $Z'$ bosons the ratio $R/S$ can tell us a great deal about the unified symmetries and their breakings. For example, in most simple SO(10) models one finds $R/S = \frac{1}{3}$, and in most SU($N$) models (with a regular embedding of $G_s$) one finds $R/S = 2$. As we shall see, there are very definite predictions for $R/S$ in the models that one is likely to obtain as the low-energy limit of superstring theory.

We can distinguish four stages of symmetry breaking in the $E_8 \otimes E_8'$ heterotic string theory, although some of these may overlap in mass scale. (For background, see Ref. 2, Dine *et al.*,[6] and Witten,[7] where these stages of breaking are discussed in detail.) These stages are (1) the breaking[2] to $E_6 \otimes E_8'$ at the compactification scale which occurs if we require that a four-dimensional supersymmetry survive; (2) the $E_6 \rightarrow G_I$ breaking that occurs at the compactification scale if certain Wilson-loop operators[2,6–8] have group-theoretically nontrivial expectation values; (3) the further breaking to $G_s$ when certain Higgs fields develop vacuum expectation values; and (4) the weak-interaction breaking of $SU(2)_L \otimes U(1)_Y$ to $U(1)_{em}$ at a scale near $M_W$. We will refer to these, respectively, as Calabi-Yau, Wilson-loop, intermediate, and weak breaking. The Higgs fields available to do the intermediate breaking, $G_I \rightarrow G_s$, are the $G_s$ singlets in the **27** and **27*** multiplets. A **27** has two such singlets which, following Ref. 6, we denote as $S_1$ and $S_2$. (Correspondingly there are $\bar{S}_1$ and $\bar{S}_2$ in **27***). A crucial question is what scale characterizes $\langle S_1 \rangle$ and $\langle S_2 \rangle$.

In Ref. 6 the various possibilities for the intermediate group $G_I$ are enumerated and studied. There are found 27 possibilities, of which some can immediately be eliminated since the intermediate breaking cannot take them down to the standard model. The authors of Ref. 6 further eliminate a set of models, which we call "class III" models in Table I, as giving too rapid a proton decay. The point is that these models give rise at low energy to dimension-five, baryon-number–nonconserving operators coming from $g$-quark exchange. [A $g$ quark is one of the extra charge $-\frac{1}{3}$, $SU(2)_L$-singlet quarks that is contained in the **27**'s of $E_6$. The $g$ quarks get mass from the intermediate breaking.] Unless $M_g \gtrsim 10^{15}$ GeV, proton decay is too rapid. If the upper bound on the intermediate breaking scale of $10^{11}$ GeV is correct, then these class III models are not viable. However, there is another minimum (not considered in Ref. 6) to the potential for the $S_i$ at which $\langle S_i \rangle$ are $\sim M_{\rm Pl}$. It is thus premature to rule these models out. The remaining possibilities for $G_I$ we divide into class I and class II depending on whether they are rank 5 or rank 6.

In Table I we give the results for $R/S$ for each group, $G_I$ (we put after each group, in parentheses, the designation that it received in Ref. 6). Besides the classification by group, we distinguish the various possibilities for the scales of intermediate breaking. Columns a, b, and c of Table I correspond to (a) $\langle S_1 \rangle$ and $\langle S_2 \rangle$ both "large"; (b) $\langle S_1 \rangle$ "large" and $\langle S_2 \rangle$ near the weak scale, or *vice versa;* and (c) $\langle S_1 \rangle$ and $\langle S_2 \rangle$ both near the weak scale. Clearly, in case (a) there are no *light* $Z'$ bosons and $R = S = 0$. (By "large" we mean so large that there are no measurable deviations from the standard-model neutral-current predictions.) In that case there is little interesting low-energy phenomenology beyond the standard model, and we are out of luck. We now will derive some of the results in Table I. We should note here that certain of the cases in Table I have been ruled out by various phenomenological considerations in Ref. 6, to which we refer the reader.

Let us denote the components of a **27** suggestively by

$$\begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad \bar{u}_L, \quad \bar{d}_L, \quad \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix}, \quad e_L^+, \quad g_L, \quad \bar{g}_L,$$

$$H_d = \begin{pmatrix} N_L \\ E_L \end{pmatrix}, \quad H_u = \begin{pmatrix} E_L^+ \\ N_L' \end{pmatrix}, \quad S_1, \quad S_2.$$

Here $u_L$ is that component with the same $G_s$ quantum numbers as the left-handed $u$ quark. $H_{d(u)}$ have the $G_s$ quantum numbers of the Higgs doublets that cou-

TABLE I. Values of $(R,S)$ predicted for various conceivable breaking patterns in $E_6$ superstring models. $N = 2, \ldots, 6$ stands for $SU(N)$. 1 stands for $U(1)$. The subscript $L$ means that the group contains the weak $SU(2)$ group. The classifications I,II,III, and (a),(b),(c) are explained in the text. Asterisks indicate where flavor-changing effects discussed in text are significant.

| | $G_I$ | (a) | (b) | (c) |
|---|---|---|---|---|
| I | $3_C \times 2_L \times 1_L \times 1_R$ (A1) | $R = S = 0$ | $R = S = 0$ | $R/S = 2$ |
| | $3_C \times 3_L \times 1_R$ (B1) | $R = S = 0$ | $R = S = 0$ | $R/S = 2$ |
| | $4 \times 2_L \times 1_Z$ (C1) | $R = S = 0$ | $R = S = 0$ | $R/S = 2$ |
| | $5 \times 1$ [anti-SU(5)] | $R = S = 0$ | $R = S = 0$ or $R/S = \frac{1}{3}$ | *$R/S \cong \frac{1}{3}$ or $R/S \cong -\frac{1}{2}$ |
| | $4 \times 2_L \times 2_R$ (Pati-Salam) | $R = S = 0$ | $R = S = 0$ or $R/S = \frac{1}{3}$ | *$R/S \cong \frac{1}{3}$ or $R/S \cong -\frac{1}{2}$ |
| II | (A2),(A3),(A4),(A5), (B2),(B3),(B4),(B5),(C2b) | $R = S = 0$ | $R/S = \frac{1}{3}$ | *$R/S \gtrsim \frac{1}{3}$ or $R/S \lesssim -\frac{1}{2}$ or $R/S =$ other |
| III | (C2a),(C3),(C4),(C5),(C6), (D2),(D8),(D9),(D10) | $R = S = 0$ | $R/S = \frac{1}{3}$ | |

$(A2) = 3_C \times 2_L \times 1_L \times 1_R \times 1_N$; $(A3),(A4) = 3_c \times 2_L \times 1_L \times 2 \times 1$; $(A5) = 3_C \times 2_L \times 1_L \times 3_R$;
$(B2) = 3_C \times 3_L \times 1_L \times 1_N$; $(B3),(B4) = 3_C \times 3_L \times 2 \times 1$; $(B5) = 3_C \times 3_L \times 3_R$;
$(C2) = 4_C \times 2_L \times 1 \times 1$; $(C3),(C4) = 4 \times 2_L \times 2 \times 1$; $(C5) = 5 \times 2_L \times 1$; $(C6) = 6 \times 2_L$;
$(D2) = 5 \times 1_X \times 1_Z$; $(D8) = 5 \times 2 \times 1$; $(D9) = 6 \times 2$; $(D10) = SO(10) \times 1_Z$.

ple to the $d$ ($u$) quarks, and similarly for the others. Let us decompose the **27** of $E_6$ under the maximal subgroup $SU(3)_c \otimes SU(3)_L \otimes SU(3)_R$: $\mathbf{27} \to (3, 3^*, 1) + (3^*, 1, 3) + (1, 3, 3^*)$. Under the further decomposition $SU(3)_L \to SU(2)_L \otimes U(1)_L$ we have

$$(3,3^*,1) = (3,2,1)^{(-1)} + (3,1,1)^{(2)} = \begin{pmatrix} u_L \\ d_L \end{pmatrix}^{(-1)} + g_L^{(2)};$$

$$(3^*,1,3) = (3^*,1,3)^{(0)} = (\bar{u}_L, \bar{d}_L, \bar{g}_L)^{(0)};$$

$$(1,3,3^*) = (1,2,3^*)^{(1)} + (1,1,3^*)^{(-2)} = \left[ \begin{pmatrix} E_L^+ \\ N_L' \end{pmatrix}, \begin{pmatrix} N_L \\ E_L^- \end{pmatrix}, \begin{pmatrix} \nu_L \\ e_L^- \end{pmatrix} \right]^{(1)} + (e_L^+, S_1, S_2)^{(-2)}.$$

Here the superscripts are the $U(1)_L$ charges, $T_L$. Denote by $T_R$ and $T_N$ the $SU(3)_R$ generators that are $\mathrm{diag}(-\frac{2}{3}, +\frac{1}{3}, +\frac{1}{3})$ and $\mathrm{diag}(0, \frac{1}{2}, -\frac{1}{2})$, respectively, in the bases $(\bar{u}_L, \bar{d}_L, \bar{g}_L)$. The weak hypercharge is given by $Y/2 = -\frac{1}{6}T_L + T_R$.

Now consider a group $G_I$ of rank 6. If $\langle S_2 \rangle$ is large and $\langle S_1 \rangle$ is near the weak scale, $G_I$ will break at the large scale to a rank-5 group that contains $SU(3)_C \otimes SU(2)_L \otimes U(1)_L \otimes U(1)_{Y''} = G_s \otimes U(1)_{Y''}$. $Y''$ is the linear combination of $T_L$, $T_R$, and $T_N$ which is zero for $S_2$ and is orthogonal to $Y/2$. That is, $Y'' = T_L + \frac{3}{2}T_R + 5T_N$. There is only one light $Z'$, and it couples to $Y''$. Since $Y''(\bar{u}_L) = Y''(u_L) = Y''(d_L) = Y''(e_L^+) = -\frac{1}{3}Y''(\nu_L) = -\frac{1}{3}Y''(e_L^-) = -\frac{1}{3}Y''(\bar{d}_L)$, we have that $R/S = +\frac{1}{3}$. If $\langle S_1 \rangle$ is large and $\langle S_2 \rangle$ is near the weak scale, we get a situation essentially equivalent. [We could relabel the fields so as to interchange $S_1 \leftrightarrow S_2$, $H_d \leftrightarrow (^{\nu}_{e^-})$, $\bar{g}_L \leftrightarrow \bar{d}_L$.] Again, therefore, $R/S = \frac{1}{3}$. If $\langle S_1 \rangle$ and $\langle S_2 \rangle$ are both near the weak scale [case (c)], then there are two light $Z'$ bosons of similar mass, and so there is no sharp prediction for $R/S$. However, as we shall see later on, in these breaking schemes [$\mathrm{rank}(G_1) = 6$, $\langle S_1 \rangle$ and $\langle S_2 \rangle$ near the weak scale] the absence of flavor-changing processes tells us that there is little mixing between $\bar{d}_L^{(0)}$ and $\bar{g}_L^{(0)}$ and between $e_L^{-(0)}$ and $E_L^{-(0)}$. That is $\bar{d}_L^0$ is either mostly $\bar{d}_L^{(0)}$ with only a *small* admixture of $\bar{g}_L^{(0)}$, or mostly $\bar{g}_L^{(0)}$ with only a *small* admixture of $\bar{d}_L^{(0)}$. In the former case $R/S = \frac{1}{3} + O(\langle S_1 \rangle^2 / \langle S_2 \rangle^2)$, and in the latter case $R/S = -\frac{1}{2} + O(\langle S_1 \rangle^2 / \langle S_2 \rangle^2)$. (Of course, we get the same results if $\langle S_1 \rangle > \langle S_2 \rangle$ as this case is equivalent under a relabeling of the fields as noted above.)

Let us now examine the rank-5 groups. The easiest to analyze are the groups $SU(3)_C \otimes SU(2)_L \otimes U(1)_L \otimes U(1)_R$, (A1) and $SU(3)_C \otimes SU(3)_L \otimes U(1)_R$, (B1). As before, $Y/2 = -\frac{1}{6}T_L + T_R$. But now the $Z'$ couples to that charge, $Y'$, which is a linear combination of $T_L$ and $T_R$ only, and which is orthogonal to $Y/2$; namely, $Y' = 2T_L + 3T_R$. If either $\langle S_1 \rangle$ or $\langle S_2 \rangle$ is large, then $Z'$ becomes heavy and $R = S = 0$. But if both $\langle S_1 \rangle$ and $\langle S_2 \rangle$ are near the weak scale, then $R \neq 0$, $S \neq 0$. It is easy to show that $Y'(\bar{u}_L) = Y'(u_L) = Y'(d_L) = Y'(e_L^+) = -2Y'(\nu_L) = -2Y'(e_L^-) = -2Y'(\bar{d}_L)$, so that $R/S = 2$.

The case of model C1 is analyzed in a similar manner and gives the same results for $R/S$ as A1 and B1, as shown in Table I (even though group theoretically it is somewhat different).

There are two more rank-5 groups to consider, namely, "anti-SU(5)"[9] and the Pati-Salam group $SU(4)_L \otimes SU(2)_L \otimes SU(2)_R$. We do not give the details here. The results (given in Table I) are quite similar to the class II models. However, the predictions for case (c) are sharper as there is here only one light $Z'$ and hence no uncertainty depending on the relative size of $\langle S_1 \rangle$ and $\langle S_2 \rangle$.

Now, there is one further prediction of *all* of the breakings that give $R/S = \frac{1}{3}$, namely, that $S \leq 0$. This can be shown by examination of the expressions for $S$ given in Ref. 3. One might worry that, if the $Z'$ is light enough to give interesting results on the $(R,S)$ plot, it would mix excessively with the $Z^0$ and give $Z^0$ mass inconsistent with experiment. Recent results[10] from CERN give $M_Z = 92.4 \pm 1.1 \pm 1.4$ GeV, about a 2.7% uncertainty. For comparison let us take the case of a rank-6 type-(b) model in which $R/S = \frac{1}{3}$. If we assume that $\langle S_1 \rangle > \langle H_u \rangle \sim \langle H_d \rangle > \langle v \rangle$, then one can easily show that $\Delta M_Z / M_Z \cong (2/15\sqrt{10})S$. If we take the experimental value of $S = -0.479 \pm 0.25$, then we find $\Delta M_Z / M_Z \cong (2 \pm 1)\%$. So the slight deviation that already exists between the standard model and experiment could conceivably be an effect of extra $Z'$ boson(s); however, it would require a careful analysis to see whether there is any evidence presently for such particles.

An interesting feature of some of the breaking patterns that we have been considering is that they allow certain flavor-changing processes to occur, such as $K^0$-$\bar{K}^0$ mixing, $K_L^0 \to \mu^+\mu^-$, $\mu^\pm \to e^\pm e^\pm e^\mp$, and $\mu \to e\gamma$. This is due to the presence of extra leptons $(E^+, E_-, N, N')$ and quarks $(g, \bar{g})$ with which the ordinary leptons and quarks can mix.

Let us consider the mass matrix for the charge $-\frac{1}{3}$ quarks. It is of the form

$$(\bar{d}_i^{(0)} \;\; \bar{g}_i^{(0)}) \begin{pmatrix} f_n^{ij}\langle N \rangle & f_1^{ij}\langle S_1 \rangle \\ f_\nu^{ij}\langle \nu \rangle & f_2^{ij}\langle S_2 \rangle \end{pmatrix} \begin{pmatrix} d_j^{(0)} \\ g_j^{(0)} \end{pmatrix}. \qquad (3)$$

Here we denote Higgs fields by the symbols for the fermions with the same $G_s$ quantum numbers in the same notation explained above. $i,j$ are family labels, so that, e.g., $d_2 = s$. The (0) superscripts refer to the weak eigenstates. Were $\langle \nu \rangle$ and $\langle S_1 \rangle$ to vanish, there would be no mixing between $d$ and $g$. However, $\langle S_1 \rangle$

must be nonzero and somewhat larger than about 300 GeV or else the lightest $Z'$ will have a large mixing with the $Z^0$. So at least the $\bar{d}_{iL}$ and $\bar{g}_{jL}$ should have some mixing. Since these have the same $SU(2)_L \otimes U(1)_Y$ charges, the $Z^0$ coupling is still flavor diagonal in the charge $-\frac{1}{3}$ right-handed quarks (at tree level); not necessarily so, however, with the $Z'$ boson(s).

For breakings A1, B1, and C1, the $Z'$ coupling is flavor diagonal for the $\bar{d}$ and $\bar{g}$, but it is not for the remaining breakings. Thus there are tree-level diagrams involving $Z'$ exchange, which contribute to strangeness-changing neutral processes. For these to be significant, the $Z'$ mass must be near the weak scale, and the mixing between the $\bar{d}$ and $\bar{g}$ (which goes like $\langle S_1\rangle/\langle S_2\rangle$) must not be supersmall. Hence only if $\langle S_1\rangle$ and $\langle S_2\rangle$ are near the weak scale will we get substantial flavor-changing effects (see Table I). If we call the weak eigenstates $(\bar{d}_{iL}^{(0)},\bar{g}_{iL}^{(0)}) \equiv \bar{D}_{iL}^{(0)}$ and the mass eigenstates $(\bar{d}_{iL},\bar{g}_{iL}) \equiv \bar{D}_{iL}$, then there is a unitary matrix $U$ such that $\bar{D}_{iL} = U_{ij}\bar{D}_{iL}^{(0)}$. One can estimate (using the vacuum-insertion approximation) the contribution of $Z'$ exchange to the $K_L$-$K_S$ mass difference,

$$\delta\left(\frac{\Delta M_K}{M_K}\right) = \frac{5}{16}\left(\frac{4\pi\alpha}{\sin^2\theta_W}\right)\left(\frac{g_{Z'}}{g_2}\right)^2\left|\sum_{i=1,2,3} U_{\bar{d}\bar{g}_i}U^*_{\bar{s}\bar{g}_i}\right|^2 (F_K/M_{Z'})^2. \tag{4}$$

$g_{Z'}$ and $g_2$ are the coupling constants of the extra $U(1)_{Z'}$ and the $SU(2)_L$ (where the generators are normalized consistently in $E_6$). Using $F_K \simeq 1.14 m_K$ and comparing to the experimental value of $\Delta M_K/M_K \cong 0.7\times 10^{-14}$, we have

$$(g_{Z'}/g_2)(M_Z/M_{Z'})\left|\sum_{i=1}^3 U_{\bar{d}\bar{g}_i}U^*_{\bar{s}\bar{g}_i}\right| \leq 1.7\times 10^{-4}. \tag{5}$$

If $R$ and $S$ are to be nonvanishing, then we see that $M_Z/M_{Z'}$ cannot be too small. Thus this tells us that the $\bar{d}_{iL}$ and $\bar{g}_{iL}$ mixing is small, a result we used in the previous section.

In the charged-lepton sector the mass matrix is of a form analogous to Eq. (3). The mixing of order $\langle S_1\rangle/\langle S_2\rangle$ for the leptons is among the weak doublets, $e^-$ and $E^-$. There are several diagrams that need to be considered in the calculation of $\mu \to 3e$. Let us neglect all but the $Z'$ exchange contribution. We find[11]

$$\frac{\Gamma(\mu^\pm \to e^\pm e^\pm e^\mp)}{\Gamma(\mu^\pm \to e^\pm \nu\bar{\nu})} \cong \frac{19}{1024}\left(\frac{g_{Z'}}{g_2}\right)^2\left(\frac{M_W}{M_{Z'}}\right)^2\left|\sum_{i=1}^3 U_{\mu^- E_i}U_{\bar{e}E_i}\right|^2 \leq 2.4\times 10^{-12};$$

therefore

$$(g_{Z'}/g_2)(M_W/M_{Z'})\left|\sum_{i=1}^3 U_{\mu^- E_i^-}U^*_{e^- E_i^-}\right| \leq 1.1\times 10^{-5}. \tag{6}$$

---

[(a)]Present address: Brookhaven National Laboratory, Upton, N.Y. 11973.

[1]D. J. Gross, J. Harvey, E. Martinec, and R. Rohm, Phys. Rev. Lett. **54**, 502 (1985), and references therein. For a review of superstrings, see, e.g., J. H. Schwarz, Phys. Rev. **89**, 223 (1982). See also M. Green and J. Schwarz, Phys. Lett. **149B**, 117 (1984).

[2]P. Candelas, G. T. Horowitz, A. Strominger, and E. Witten, "Vacuum Configurations for Superstrings" (to be published). The first suggestion of $E_6$ as a unifying group was by F. Gürsey, P. Ramond, and P. Sikivie, Phys. Lett. **60B**, 177 (1976).

[3]S. M. Barr and A. Zee, Phys. Lett. **92B**, 297 (1980), and **110B**, 501(E) (1982).

[4]For a thorough review and analysis of neutral-current phenomenology, see J. E, Kim, P. Langacker, M. Levine, and H. H. Williams, Rev. Mod. Phys. **53**, 211 (1981). See also W. Marciano and A. Sirlin, Phys. Rev. D **29**, 945 (1984). For discussions of extra $Z$ bosons, see N. Deshpande, in *High Energy Physics—1980,* edited by L. Durand and L. G. Pondrom, AIP Conference Proceedings No. 68 (American Institute of Physics, New York, 1981), p. 431; A. Davidson, Phys. Rev. D **20**, 776 (1979); R. N. Mohapatra and R. E. Marshak, Phys. Rev. Lett. **44**, 1316, 1643(E) (1980); P. Fayet, Phys. Lett. **96B**, 83 (1980); E. H. deGroot, G. J. Gounaris, and D. Schildknecht, Phys. Lett. **90B**, 427 (1980); V. Barger, E. Ma, and K. Whisnant, Phys. Rev. D **26**, 2378 (1982); R. W. Robinett and J. L. Rosner, Phys. Rev. D **26**, 2396 (1982), and **25**, 3036 (1982). For a recent study, see P. Langacker, R. W. Robinett, and J. L. Rosner, Phys. Rev. D **30**, 1470 (1984), and references therein.

[5]S. M. Barr, Phys. Lett. **128B**, 400 (1983).

[6]M. Dine, V. Kaplunovsky, M. Mangano, C. Nappi, and N. Seiberg, to be published.

[7]E. Witten, "Symmetry Breaking Patterns in Superstring Models" (to be published).

[8]J. Breit, B. Ovrut, and G. Segré, University of Pennsylvania Report No. UPR-0279T, 1985 (to be published).

[9]A. De Rújula, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **45**, 413 (1980); S. M. Barr, Phys. Lett. **112B**, 219 (1982).

[10]L. Mapelli, CERN Report No. CERN-EP/85-84, 1985, in Proceedings of the Fifth Topical Workshop on Proton Antiproton Collider Physics, Saint-Vincent, Aosta Valley, 25 February–1 March 1985 (to be published).

[11]For a recent dramatic improvement in the experimental bound, see W. Bertl *et al.* (SINDRUM collaboration), Swiss Institute for Nuclear Research Report No. PR-85-06, 1985 (to be published).