# Phenomenological Aspects of Heavy Fermions

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The contrasting equilibrium and transport experimental results in the solids which exhibit heavy-fermion behavior are used to specify a phenomenological theory which puts strong constraints on a microscopic description. These constraints are not satisfied by a Fermi-liquid theory similar to that for liquid $^3He$. The heavy mass arises through the renormalization of conduction electrons by the exchange of spin fluctuations of what at high temperatures were the local moments. Expressions for the specific-heat and the magnetic-susceptibility enhancements in terms of the Landau interaction functional and predictions for some transport properties are given.

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The thermodynamic and the recent transport experiments in the solids displaying the so-called heavy-fermion behavior[1,2] can be used to put severe constraints on the nature of the many-body effects responsible for this phenomena. They can be used to formulate a phenomenological theory in terms of which different experiments may be correlated and to specify results that a microscopic theory must obtain.

The experiments I will focus on are the specific heat $C_v$, the susceptibility $\chi$, the ultrasonic attenuation $\alpha_s$, the thermal conductivity, and the local-moment relaxation rate in the normal state of $UPt_3$[3,4] or $UBe_{13}$.[5,6] Similar behavior is expected in the other heavy-fermion systems.[7] A clue to the nature of the phenomena is provided by the following two contrasting facts[8]: (1) The thermodynamic experiments, viz., the specific heat and the susceptibility, yield an enhancement of $O(10^2\text{–}10^3)$ compared to a free Fermi gas of similar electronic density. (2) Batlogg and co-workers[4] have made the remarkable observations that the ultrasonic attenuation in $UPt_3$ in the normal phase is of similar magnitude to that of an ordinary metal, say Sn or Pb. Thermal-conductivity measurements in $UBe_{13}$ and $UPt_3$ also lead to results similar to ordinary metals. Gandra *et al.*[9] have presented measurements of the local-moment relaxation rate of various rare-earth impurities in $UBe_{13}$ and again find them to be of the same order as in materials with ordinary effective mass, whereas naively a relaxation rate $\sim 10^4$ times faster would be expected.

In the usual Fermi liquid,[10] the specific-heat enhancement $C_v/V_{v0}$ is related at low temperatures to the enhancement of the quasiparticle density of states at the Fermi surface or equivalently to that of an average inverse Fermi velocity or to that of an average mass,

$$C_v/C_{v0} = m^*/m. \tag{1}$$

We shall take Eq. (1) to define $m^*/m$.

The ultrasonic attenuation for $ql \ll 1$, where $q$ is the wave vector of the sound and $l$ the electronic mean free path, can be written as[11]

$$\alpha_q = (m^* E_1^*)^2 \frac{\omega_q}{2\rho v^2 \hbar^3}(ql). \tag{2}$$

Here $\omega_q$ and $v$ are the frequency and velocity of the sound wave, $\rho$ is the density, and $E_1^*$ is the renormalized deformation potential. The ultrasonic attenuation in this regime has the temperature dependence of the conductivity and is proportional to $\omega_q^2$. This behavior is observed in $UPt_3$. Using for $l$ the value deduced from experiment, one discovers[4] that $m^*E_1^*$ is of similar magnitude as in an ordinary metal, say Sn. This means that $E_1^*$ is renormalized in the heavy-fermion system such that $(m^*E_1^*)^2$ retains its unrenormalized value. $m^*E_1^{*2}$ is proportional to $\tau_s^{*-1}$, the inverse lifetime for a sound wave to decay into particle-hole pairs. The thermal-conductivity experiment can be similarly analyzed with the conclusion that $\tau^*/m^*$ is unrenormalized, where $\tau^*$ is now the electronic momentum lifetime due to scattering from impurities.

Following the suggestion[12] that the heavy-fermion superconductors could not be ordinary phonon-induced superconductors and are likely to be anisotropic superconductors, as is liquid $^3He$, there have been arguments[13] put forth that the physics of the heavy-fermion metals is itself akin to that of liquid $^3He$. Rice *et al.*, for example, have tried to explain the mass renormalization by a Hubbard-Gutzwiller model which is quite suitable for liquid $^3He$ (where the maximum mass enhancement is about 6, before the transition to the solid state). The cancellation in the renormalizations in $\tau^*/m^*$ that we have noted above does not occur for $^3He$, however. Transport properties like ultrasonic attenuation, nuclear relaxation rate, etc., are renormalized in the usual Landau Fermi-liquid theory. It will be apparent below that the experimental results quoted above are also in disaccord with theories[14] which discard quasiparticle interactions.

The cancellation in $\tau^*/m^*$ is reminiscent of renormalizations in electronic properties due to electron-phonon interactions.[15] What is relevant here is not the electron-phonon interactions but only that an enhancement of $m^*/m$ without a renormalization in $\tau^*/m$ follows if the momentum dependence of the self-energy $\Sigma(\mathbf{k},\omega)$ is negligible compared to the energy dependence. Let the single-particle Green's function $G(\mathbf{k},\omega)$ be given by

$$G_\sigma^{-1}(\mathbf{k},\omega)=\{[\omega-\epsilon(\mathbf{k})]-\Sigma_\sigma(\mathbf{k},\omega)\}, \tag{3}$$

where near the Fermi surface the band-structure energy $\epsilon(\mathbf{k})=\mathbf{v}_0\cdot(\mathbf{k}-\mathbf{k}_\mathrm{F})$. For

$$\begin{aligned}&\partial\Sigma/\partial\omega >> v_0^{-1}\partial\Sigma/\partial k,\\ &G_\sigma^{-1}(\mathbf{k},\omega)=Z^{-1}[\omega-Z^{-1}\mathbf{v}_0\cdot(\mathbf{k}-\mathbf{k}_\mathrm{F})],\end{aligned} \tag{4}$$

where $Z$ is the quasiparticle amplitude or wave-function renormalization,

$$Z=(1+\partial\Sigma/\partial\omega)^{-1}. \tag{5}$$

The specific-heat enhancement or $m^*/m$ is then given by

$$m^*/m=Z^{-1}. \tag{6}$$

In calculating any transport property, like the ultrasonic attenuation by a process as in Fig. 1, the matrix element (vertex) renormalization cancels against the self-energy under the condition (4) for long-wavelength perturbations. This follows from a Ward identity as shown by Prange and Kadanoff.[15] Physically the cancellation in $\tau^*/m^*$ can be understood from the condition (4), which amounts to saying that the self-energy rides with the chemical potential so that in any perturbation where the local chemical potential is altered and relaxes towards equilibrium the renormalization is not felt.

Prange and Kadanoff explicitly demonstrate that $dn/d\mu$, where $n$ is the electronic density and $\mu$ the chemical potential, is unaffected by the renormalizations if condiction (4) is satisfied. In the usual Fermi-liquid theory (as applied to liquid $^3$He or ordinary metals) in which the frequency and momentum dependence of the self-energy are on a similar scale, $dn/d\mu$ is renormalized by the $s$-wave part of the Landau scattering function $F_0^s$.

One can follow Prange and Kadanoff and obtain that

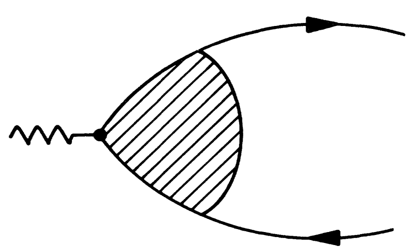

FIG. 1. The ultrasonic-attenuation process. The particle-hole lines are fully renormalized as is the vertex by many-body effects as well as by impurity scattering.

the effective-mass enhancement for heavy fermions is given for spherical bands by

$$\begin{aligned}m^*/m&=Z^{-1}\\&=1+\sum_{\sigma'}\int d^2S'\,\phi(\mathbf{k}\sigma,\mathbf{k}'\sigma')/v_k\\&=1+F_0^s,\end{aligned} \tag{7}$$

where $\phi$ is the Landau quasiparticle scattering function and integration is over the Fermi surface. Thus $m^*/m$ depends on the $s$-wave scattering amplitude of the fermions, not the $p$-wave scattering amplitude as in the usual Fermi-liquid theory.[9] This is at first surprising, since the usual result follows from Galilean invariance. It is natural, however, in situations[15] where the momentum is shared by more than one type of excitation, but only one of them gives a linear specific heat and the effective mass is defined only in terms of it.

There is one important difference between the physics of electron-phonon renormalizations and the phenomena in the heavy fermions. Magnetic susceptibility is not renormalized by electron-phonon interactions. This is due to the fact the Landau interaction function $\phi(\mathbf{k}\sigma,\mathbf{k}'\sigma')$ in this case is diagonal in spin indices. We obviously need, beside the condition (4), an interaction function with off-diagonal spin components. This means that the bosons that are exchanged by the conduction electrons must carry spin.

An expression for the magnetic susceptibility for the case that $\phi$ includes spin-flip components can be derived by a simple extension of the Prange and Kadanoff procedure.[15] The answer is

$$\frac{\chi}{\chi_0}=\frac{m^*/m}{1+F_0^a}, \tag{8}$$

where $F_0^a$ is obtained from the antisymmetric spin component of $\phi_0$. The experimental results[16] on the heavy fermions imply that $m^*/m>10^2$ but $\chi/\chi_0\lesssim 2m^*/m$. This means that $\phi_0(\uparrow\uparrow)>10^2$, but $\phi_0(\uparrow\uparrow)-\phi_0(\uparrow\downarrow)\lesssim\frac{1}{2}$; there is very little exchange interaction among the heavy fermions compared to the direct interaction.

Just as in the case of electron-phonon mass enhancement,[15] most transport experiments—ultrasonic attenuation, thermal conductivity, resistivity, spin-lattice relaxation, etc.—if dominated by electron scattering, are unaffected by the renormalizations, while some like cyclotron resonance frequency are affected. It is interesting though that the $\tau^*/m^*$ measured by the de Haas–van Alphen effect is also unrenormalized. Photoemission measures the spectral function of the single-hole Green's function. This means that the narrow feature near the Fermi level in such experiments will have a spectral weight proportional to $Z$. The cancellations in renormalizations in transport properties will survive into the superconducting or antiferromagnetic state of the heavy fermions.

Recent results of inelastic neutron scattering[17] from $UPt_3$ are consistent with the form of the self-energy deduced here. $\chi(q,\omega)$, within the experimental uncertainty, is observed not to depend on $q$ and as a function of $\omega$ at low $\omega$ is that of a Fermi liquid characterized by $(m^*/m)(1+F_0^a)^{-1}$. $\chi(q,\omega)$ satisfies sum rules implying that the total magnetic response, which at high temperatures is that of free spins, appears at low temperatures in the form of a Fermi liquid.

I will now give a simple argument for the conditions under which a very heavy fermion mass of $O(10^2-10^3)$ is mandated. Consider the contributions to the entropy of an ordinary rare-earth metal or compound. There is a small linear contribution $\gamma T$ from the fermions corresponding to a mass of $O(1)$, and a contribution from the local moments which at room temperature is nearly temperature independent and two to three orders of magnitude larger than the $\gamma T$ contribution. The spin contribution to the entropy decreases as temperature is decreased, and as illustrated in Fig. 2 has a change in slope at the magnetic ordering temperature $T_N$ which is typically of $O(10\text{ K})$. Below $T_N$, one gets the spin-wave contribution to the entropy. The heavy fermions are described by the same Hamiltonian as the typical rare-earth solid albeit with somewhat different parameters. Suppose that we demand that there be no magnetic phase transition[18] and that the low-energy excitations in these solids be purely fermionic, i.e., there is a gap of $\sim T_F$ which is the same order of magnitude as $T_N$ for spin-flucutation excitations, then as illustrated in Fig. 2, a heavy-fermion mass of $O(10^2-10^3)$ is necessary, since total entropy in the present problem is the same as for the ordinary rare earths and the high-temperature entropies must be identical.

The discussion above may be summarized by the statement that heavy fermions arise from ordinary conduction electrons by exchange of spin fluctuations with total spectral weight similar to that carried by the local moments and that if these spin fluctuations must have a gap of $O(T_F)$, the specific heat of the fermions will be $O(T/T_F)$. One really observes only the spin-fluctuation entropy but in a fermionic form, i.e., $\sim T$. The effective induced interaction between the fermions is highly retarded in time but local in space. This ensured the disappearance of the renormalization in most transport experiments.

A look at the resistivity[1] of the heavy-fermion metals allows one to make another useful statement. In $CeCu_6$ and $PuBe_{13}$, there is a resistivity minimum as a function of temperature at about 100 K, reminiscent of the Kondo effect, and a sharp drop in the resistivity below 10 K. In $CeAl_3$, $UBe_{13}$, $CeCu_2Si_2$, etc., resistivity decreases (approximately logarithmically with a characteristic scale of a few hundred degrees) above about 10 K to about room temperature, where present

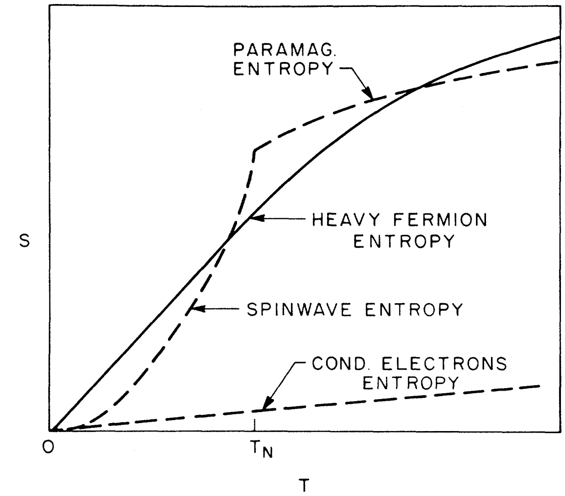


FIG. 2. A schematic diagram useful for arguing the condition under which a heavy-fermion mass is obtained—see text. The dashed lines refer to the entropy contribution in a typical rare-earth metal or metallic compound, and the solid line to the entropy in a heavy-fermion material. The phonon contribution to the entropy has been ignored.

measurements stop. Below about 10 K there is a precipitous drop. The suggestion is that there are at least two characteristic temperature scales in the problem, one of the isolated-moment Kondo temperature and a lower one, where coherence develops between different sites and the resistivity drops. This is also implied by a recent microscopic theory.[19]

It is amusing to note the phenomenological connection of the heavy-Fermi problem to that of quark confinement. Asymptotic freedom in our case means existence of free spins at high energies. The transition from this regime without any magnetic-symmetry breaking to a regime where only fermionic excitations exist corresponds to the low-energy regime of a color-singlet state of the quarks with only the baryons free. Magnetic fluctuations or bosons appear massive.

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