Observation of Singly Quantized Dissipation Events Obeying the Josephson Frequency Relation in the Critical Flow of Superfluid ^{4}He through an Aperture

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We have observed highly reproducible isolated dissipation events in the flow of pure superfluid ^{4}He through an aperture of submicronic size at 10 mK, when the flow velocity just exceeds a well-defined critical threshold in agreement with Feynman's criterion for vortex emission. These events, which we interpret as phase slippages by 2π in the sense of Anderson, occur at a rate given by the Josephson frequency relation.

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This Letter deals with experimental aspects of the relation between two fundamental and largely unsettled questions in superfluid hydrodynamics, the onset of the critical regime in superfluid flow through a constriction and the rate of occurrence of quantized vorticity in such a critical flow. The ideas involved here date back in part to Onsager's and Feynman's suggestion[1,2] that, in such a flow, vortex lines or rings each carrying a quantum of circulation $K_0 = 2\pi\hbar/m_4$ are emitted when the velocity reaches a threshold depending on the orifice geometry. According to Feynman, this happens when the kinetic energy flux available in the flow becomes large enough to sustain vortex creation at the rate at which quantized circulation is generated. For a slit of width d, and for vortex lines of core radius ξ_0, this condition is fulfilled when

$$v_c = (K_0/2\pi d)\ln(d/\xi_0). \tag{1}$$

In this simple approach a number of problems are left in the shade, notably the nucleation mechanism of such a large quantum object as a vortex line and the details of its onward trajectory. These difficulties, as well as the lack of quantitative agreement between Eq. (1) and experimental observations, were noted by a number of authors.[3,4]

In far-reaching considerations on the quantum-mechanical equations of motion for the superfluid wave function. Anderson[5] pointed out that the most basic physical quantity for description of superfluid flow properties is the phase of the wave function, and he introduced the concept of phase slippage. The rate of phase slippage obeys the Josephson frequency relation:

$$\hbar\, \partial(\phi_2 - \phi_1)/\partial t = \mu_1 - \mu_2 = \Delta\mu, \tag{2}$$

$\Delta\mu$ being the chemical potential difference between quiescent regions in the two baths. Equation (2) extends the validity of the Euler-Landau equation valid for an ideal fluid,

$$\partial \mathbf{v}_s/\partial t + \nabla(\mu/m_4 + v_s^2/2) = 0, \tag{3}$$

to situations in which quantized vorticity is created. When the fluid is being accelerated without creation of vorticity, Eqs. (2) and (3) imply the usual relation between the superfluid velocity and the gradient of the phase, $\mathbf{v}_s = (\hbar/m_4)\nabla\phi$. When the critical velocity is exceeded, phase slippage takes place. For a phase slip of 2π, the total work done on the stream moving at velocity v_c in a channel of cross-sectional area s is expressed by

$$\Delta E = \rho K_0 s v_c, \tag{4}$$

which represents the stream kinetic energy loss and is independent of the details of the mechanism by which phase slippage occurs. It also corresponds, following Huggins[6] and Campbell,[7] to the energy taken from the potential flow when a vortex crosses all the streamlines, a necessary condition for a phase slip of 2π. Campbell also pointed out that in this model there is no energy available in the stream for the creation of vortices.

In this Letter, we report the observation of single phase-slip events—or vortex emission—of the type postulated above. These events are accompanied by an energy loss of the total flow related to the observed critical velocity by Eq. (4). This provides an experimental check of the Josephson frequency relation during dissipation events in superfluid ^{4}He. Previous searches for such a check, and most notably that of Richards and Anderson[8] based on the principle of synchronization between the internal events described by Eq. (2) and an external ac pressure drive, stirred a scientific controversy and failed to reach an unambiguous conclusion.[9,10] In more recent work on the subject, the synchronization method, which is plagued by difficulties of interpretation, has been abandoned to the benefit of high-Q Helmholtz resonators[11,12] which make possible the detection of very small dissipation levels.

Our experimental observations have been performed in such a miniaturized low-frequency Helmholtz resonator of about 7 mm^3 internal volume immersed in the main superfluid bath at zero applied pressure and cooled to about 10 mK by a dilution refrigerator. A drawing of the resonator is shown in Fig. 1. Its two main parts are the disk supporting the microaperture through which the superfluid flows and develops inertia, and the flexible diaphragm which provides the (very weak) restoring force and monitors the flow. This system resonates at a frequency $\Omega_m/2\pi$ of about 2.81 Hz in ^{4}He. The microaperture is milled by means of a focused ion beam[13] into a thin film of nickel of thickness $l = 0.2$ μm. It has the shape of a rectangular slit of width $d = 0.3$ μm and large dimension (breadth) $b = 5$ μm. Its edges are somewhat irregular. The flexible diaphragm is made out of a 7.5-μm-thick Kapton foil, coated with a 60-nm-thick aluminum film on both sides. This diaphragm is mounted in the cell unstretched at liquid-nitrogen temperature so as to be as flexible as possible at liquid-helium temperature. Its position is read by two separate means. The first readout device uses a high–slew-rate rf SQUID[14] as sensing element. Its principle of operation is analogous to that of the superconducting accelerometer devised by Paik for gravity-wave measurements.[15] A flat coil ($L_c \sim 2$ μH) faces at close distance (50 μm) the aluminized Kapton diaphragm. This coil is connected to the SQUID input and to a large tank coil ($L_t \sim 100$ μH). A current I_0 of up to 1 A is trapped in the loop L_c-L_t. When the diaphragm is moving, the Al film, which behaves as a type-II superconductor, modulates L_c and induces a current in the signal coil of the SQUID. The ultimate sensitivity of this displacement gauge is of the order of 10^{-13} m in a bandwidth extending from dc to 1 kHz. The corresponding differential pressure sensitivity is about 6×10^{-12} bar. The second readout is provided by a conventional bridge measurement of the capacitor formed by the diaphragm and a flat plate of known geometry separated by a spacer of specified thickness at room temperature (50 μm). This capacitor serves two purposes. The first is to drive the resonance by a well-controlled electrostatic force. The second is to calibrate the ultrasensitive SQUID displacement readout. The accuracy of the calibration depends on the knowledge of the spacer thickness at low temperature. With this knowledge, we can infer from standard voltage and capacitance measurements the values of the applied electrical force, the volume flow rate, and the membrane elastic constant.[16]

The system is driven on resonance by a sinusoidal voltage synchronized electronically to the SQUID output. A dc bias voltage is applied to linearize the excitation. Starting from the idle state, the peak amplitude of the oscillation A grows linearly with the time as shown in Fig. 2. At a critical level of oscillation A_c, this amplitude changes in a sudden manner by a quite reproducible fraction $\Delta A/A_c = (0.0187 \pm 5)\%$. A close-up observation of the wave form reveals that this sudden jump by which the system dissipates energy occurs at the maximum membrane velocity, i.e., when the critical velocity is exceeded in the aperture. The time over which this event takes place is shorter than our instrumental time resolution of a few milliseconds. A drive power of the order of 4×10^{-19} W is necessary to maintain the oscillation amplitude close to A_c; the corresponding quality factor is $Q = 14\,000$. The energy dissipated per event is quite accurately known[16]:

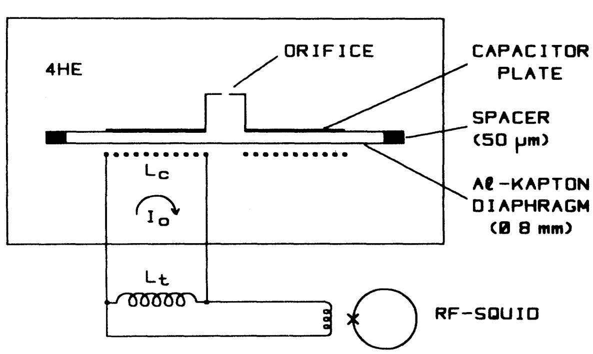

FIG. 1. Schematic drawing of the experimental cell.

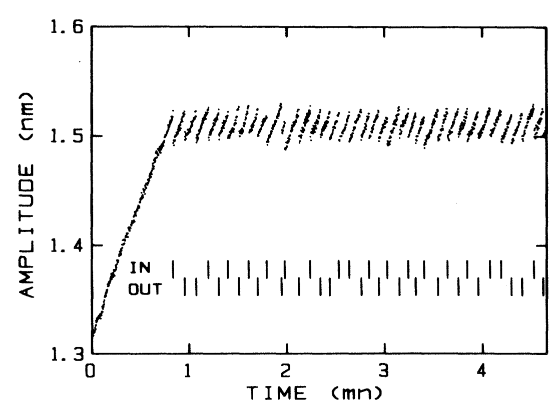

FIG. 2. Peak amplitude of the membrane oscillation vs time for a constant drive power of 2.4×10^{-18} W. The high correlation between inward and outward jumps can be seen from the ticks drawn below each jump.

$\Delta E = 1.21 \times 10^{-17}$ J.

When the drive level is increased, the jumps occur more frequently until several jumps occur during the same half-period. The phenomenon can then be traced by measurement of the change in peak amplitude ΔA from one half-period to the next versus drive level as the latter is gradually ramped up. For each drive level, the energy driven into the Helmholtz oscillator is dissipated by emission of a given number of quanta per half-period. This number varies by plus or minus one unit, depending on the leftover of quantization. As the drive level increases, the resulting behavior of ΔA follows the fishbone pattern represented in Fig. 3(a). A more direct illustration of the energy loss per half-period is obtained in Fig. 3(b) by subtraction from the fishbone pattern of the change in amplitude that the system would have experienced in the absence of jumps, i.e., the linear extrapolation of the first sloping segment in Fig. 3(a). It is seen quite clearly that the energy loss per half-period is lumped into steps. Up to at least fifty such steps can be resolved. The step amplitude is equal to that of isolated events. Steps fully overlap two by two as required by energy conservation. At high drive levels, the oscillatory motion is far from sinusoidal in shape but tends toward a sawtooth shape with a limiting slope fixed by the critical velocity.

Irregular coarse-grained dissipation associated with the appearance of vorticity has already been reported in film flow by Adie and Armitage.[17] We have also observed sudden jumps of erratic amplitude in the present cell filled with impure ^{4}He containing about 0.2% of ^{3}He. In the case of pure ^{4}He, when the temperature is raised above a few hundred millikelvins,

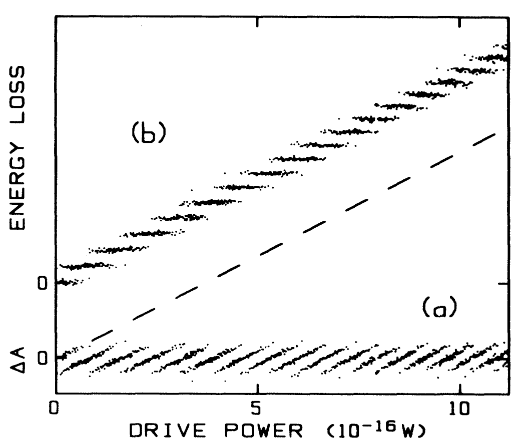

FIG. 3. (a) Peak amplitude change from one half-period to another when the drive level is ramped up linearly (sweep time ~ 450 s). (b) Obtained from (a) by subtraction of the dashed line, this step pattern represents the energy dissipated in half a period.

damping appears, and the peak amplitude records become very noisy. At about 800 mK, the Q drops to about 1100, but steps can still be resolved. A most remarkable feature of the low-temperature data presented in Figs. 2 and 3 is the reproducibility of the dissipation event. We are led to infer from this feature that these events correspond to phase slippage of precisely 2π between the inside and the outside of the cell in the experimental conditions prevailing in the present cooldown. As such a jump by 2π corresponds to a velocity change $\Delta v/v_c = 0.0187$, the total phase difference across the orifice at the critical velocity is about $53 \times 2\pi$, i.e., the phase coherence remains quite well-established over a fairly large number of periods.

We have studied the probability of event occurrence when the fluid flows into or out of the cell and discovered that jumps on inward and outward flows are very highly correlated, as shown in Fig. 2. No sequence of more than three successive jumps in the same direction was ever seen in these experiments, the most probable sequence being an alternating succession of inward and outward jumps. We interpret the high inward-outward correlation as an indication that circulation or vorticity is somehow trapped in the system. It seems to us possible to trap vorticity in the orifice either because of its fairly irregular shape or because of accidental partitioning by air or water microcrystallites. Also, we cannot rule out the possible existence of a parallel leak in the form of a long and narrow path at some place in the cell[18] which would turn aside a small fraction of the flow and still have little influence on the Helmholtz frequency. If remnant vorticity is present, the critical velocity should depend on the direction of the trapped circulation, which in turn depends on the sign of the previous jumps. A detailed statistical analysis yields indications that such is the case with a quantum change in v_c of the order of 0.3%. This provides the mechanism by which the system balances itself in total volume flow.

The critical volume flow rate is $\Omega_m A_c S_H$, S_H being the hydraulic area of the membrane, i.e., the cross-sectional area of the rigid piston giving the equivalent displacement.[19] The value of this quantity is such that in Eq. (4) $\Delta E = 1.01 \times 10^{-17}$ J. The combined inaccuracies on this figure due to the calibration procedure, the correction on S_H, and the possible existence of a parallel derivation of $\sim 15\%$ of the flow may add up to $\pm 50\%$. Thus the relation between the energy jump induced by the dissipation event and the critical velocity expressed by Eq. (4) is well obeyed. This provides, admittedly not to a great accuracy as yet, an experimental verification of the Josephson frequency relation in a superfluid.

If we assume that the orifice area is equal to its geometrical value, we find $v_c = 55$ cm/s. This value is in good agreement with Feynman's criterion [Eq. (1)]

and its variants[3] with $d = 0.3$ μm and $\xi_0 = 1.3$ Å.[20] In this model, the emitted vortex ends up moving freely in bulk fluid. Then its energy corresponds to that of a ring of radius 1.8 μm moving with a velocity of 5 cm/s. As Eq. (1) is satisfied, it is not surprising that the ring perimeter is equal to that of the orifice. The ring impulse is about 10 times larger than the kinetic momentum lost by the Helmholtz resonator, $I = \Delta E/v_c = \rho K_0 s$. This indicates that the ring must gather its momentum by interaction with the wall, which is still an open problem.[21] These experimental results obtained at very low temperatures place precise constraints on critical-velocity theories involving vortex creation and motion for which a detailed model is, as it seems to us, still lacking.[7] It is nonetheless gratifying that single phase-slippage events whose existence was envisioned long ago have finally been observed.

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[18]The 0.2-μm-thick Ni foil has been inspected under a scanning electron microscope and a transmission electron microscope and is believed to be free of pinholes.

[19]S_H is calculated from the known geometrical area S_G with the assumption that the membrane takes the shape of a paraboloid: $S_H = S_G/1.63$.

[20]Such an agreement relies on the knowledge of s which may be lower, because of various contaminations, than the geometrical value. In particular, we have observed that the resonance frequency varies from one run to another and is always sizably lower than estimated from a simple mechanical behavior of the membrane.

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