Subthreshold Kaon Production as a Probe of the Nuclear Equation of State

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The production of kaons at subthreshold energies from heavy-ion collisions is sensitive to the nuclear equation of state. In the Boltzmann-Uehling-Uhlenbeck model, the number of produced kaons from central collisions between heavy nuclei at incident energies around 700 MeV/nucleon can vary by a factor of ~ 3, depending on the equation of state.

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One of the main motivations for the study of heavy-ion collisions is to determine the nuclear equation of state at high densities. One suggestion is to use the flow-angle distributions[1] or a transverse-momentum analysis[2] of all the charged particles to test the nuclear equation of state. A much simpler probe such as the pions has also been suggested.[3] But further studies showed that the high-multiplicity-selected pion yields depend only weakly on the nuclear equation of state.[1] This is so because pions are strongly interacting particles and are thus sensitive only to the final stage of the reaction when the nuclear equation of state no longer plays an important role. The situation is different for kaons as their interactions with nucleons are much weaker and they cannot be reabsorbed. The study of kaon production has been carried out before at an energy of 2.1 GeV/nucleon.[4] The data are, however, consistent with the theoretical calculations based on the cascade model.[5–7] This is not surprising since one does not expect the equation of state to be important at such a high energy. In this Letter, we shall demonstrate that kaon production is sensitive to the nuclear equation of state for nuclear collisions at much lower incident energies. Using the Boltzmann-Uehling-Uhlenbeck (BUU) model for describing heavy-ion collisions, we find that, for central collisions between heavy nuclei at incident energies around 700 MeV/nucleon, the numbers of produced kaons differ by a factor of ~ 3, depending on the equation of state.

Before a microscopic calculation for kaon production from heavy-ion collisions is carried out, it is instructive to show qualitatively how the kaon-production probability depends on the nuclear equation of state. We shall see later that kaons are produced from the compression stage of the collision when both the density and the temperature are high. Assuming that only nucleons are in thermal equilibrium, one can show that the number of produced kaons is proportional to the factor $e^{-M_K/T}$ with M_K and T the kaon mass and the temperature of the system, respectively.[8] The temperature of the compressed region depends on the equation of state. A soft equation of state is expected to give rise to a higher temperature than that from a stiff equation of state, as less energy is in the form of the potential energy. To relate the temperature to the available kinetic energy, we use the familiar relation $T=\frac{2}{3}E_{\mathrm{kin}}$, which is valid at high temperatures. For the potential energy per particle, we use the following two expressions: For a stiff equation,

$$V(\rho) = -62(\rho/\rho_0) + 23.5(\rho/\rho_0)^2 \text{ MeV}, \qquad (1a)$$

and for a soft equation,

$$V(\rho) = -178(\rho/\rho_0) + 140(\rho/\rho_0)^{7/6} \text{ MeV}, \qquad (1b)$$

which give the compressibility coefficients $K \simeq 380$ and 200 MeV, respectively. If we take the density to be 2.5 times the normal nuclear-matter density ρ_0, the two equations of state give an energy difference $\Delta E \simeq 30$ MeV. For reactions between identical nuclei at an incident energy of 700 MeV/nucleon, the temperature of the compressed region is $T_1 \simeq 104$ MeV and $T_2 \simeq 85$ MeV for a soft and a stiff equation of state, respectively. The ratio of the number of produced kaons is then given by

$$R = \exp[M_K(1/T_2 - 1/T_1)] \simeq 2.9, \qquad (2)$$

which should be a measurable effect. As we shall show later, the kaon-production cross section at this energy is still appreciable with a magnitude in the region of 1 mb. From Eq. (2), we see that R approaches 1 as the temperature difference becomes smaller, which is the case in previous kaon experiments.[4] Also, the ratio will be small for pions because of the small pion mass. In our estimate we have neglected the production of pions. The inclusion of pion production will reduce the temperature substantially. Since pions are expected to reduce also the temperature difference, the ratio estimated in Eq. (2) will probably not be affected significantly.

To study in more detail the sensitivity of kaon production in heavy-ion collisions to the nuclear equation of state, we have carried out microscopic calculations based on the BUU equation for the one-body phasespace distribution function $f(\mathbf{p},\mathbf{r})$,

$$\partial f_1/\partial t+\mathbf{v}\cdot\nabla_r f_1-\nabla U\cdot\nabla_p f_1=\int d^3p_2\int d\Omega\,\sigma(\Omega)|\mathbf{v}_1-\mathbf{v}_2|[f_1'f_2'(1-f_1)(1-f_2)-f_1f_2(1-f_1')(1-f_2')].$$

In the above, the mean-field potential $U(r)$ is related to the potential-energy density of Eq. (1) via $U=d(\rho V)/d\rho$, and the nucleon-nucleon cross section $\sigma(\Omega)$ is taken to be that in free space. Off-shell effects have been neglected in the present applications of the BUU model. In the energy region we are considering, this is probably not very serious. This model includes both the mean-field and the Pauli-blocking effects. The BUU equation is solved numerically by the method of Monte Carlo simulations and the concept of test particles. Details on the numerical method and applications of the model to heavy-ion collisions at intermediate energies are given by Bertsch and co-workers.[9] To apply the model to incident energies around 700 MeV/nucleon, it is important to include also the excitation of nucleons to delta resonances. This is done according to the method of Ref. 7.

The isospin-averaged kaon-production cross sections are taken from Ref. 5 in the form

$$\bar{\sigma}(p_{\max})\simeq(Cp_{\max}/M_K)\ \mu\text{b}, \tag{4}$$

where

$$p_{\max}=\{[s-(M_K+M_N+M_\Lambda)^2][s-(M_K-M_N-M_\Lambda)^2]/(4s)\}^{1/2}$$

is the maximum allowed kaon momentum for collisions between two particles with total center-of-mass energy $\sqrt{s}$. The constant C has values 72, 54, and 36 for collisions between two nucleons, one nucleon and one delta, and two deltas, respectively. Since we shall study collisions at energies which are much smaller than the threshold energy ~ 1.6 GeV for kaon production, we shall include final states involving lambdas, kaons, and nucleons only as they are energetically more favorable. The parametrization given by Eq. (4) describes reasonably the experimental data and works well for kaon production from high-energy heavy-ion collisions.[5-7] As Eq. (4) indicates, the kaon-production cross section is only a small fraction of the nucleon-nucleon total cross section ~ 40 mb, and we therefore treat the production of kaons perturbatively as in Ref. 5. Because of the conservation of strangeness, kaons will not be reabsorbed by the nucleons. Since we are only interested in determining the total number of kaons produced in the collision, we shall not include the weak rescattering of the kaons after their productions.

We have done a number of calculations for collisions between symmetric systems at incident energies around 700 MeV/nucleon. In Fig. 1, the total kaon-production probability P_K averaged over 100 simulations and defined by the sum of $\sigma_K/\sigma_{\text{tot}}$ from individual baryon-baryon collisions is shown as a function of the collision time for collisions of Nb+Nb at an impact parameter $b=0.5$ fm. The solid curve is obtained by use of the soft equation of state, while that from the stiff equation of state is denoted by the dashed curve. The ratio of the total kaon-production probabilities between the two is $R\sim 2.8$. This ratio is surprisingly close to the crude estimate made above with use of the thermal model. Also shown in the same figure are the values of the central density in units of the normal nuclear-matter density ρ_0 as a function of collision time. The curve with open squares is from the soft equation of state, while the curve with open circles is from the stiff equation of state. We see that the maximum density reached in the collision is higher for a soft equation of state than for a stiff equation of state. In both cases, kaons are produced when the density has reached its maximum value of $(2-3)\rho_0$. During the expansion stage of the reaction, there are essentially no kaons being produced. It is worthwhile to point out that kaons are mostly produced from deltas which result from nucleons that have already undergone a few collisions. These observations are similar to those previously reported in Refs. 5 and 6, and by Barz *et al.*[10] and Biro *et al.*[10] for heavy-ion collisions at high energies. In Table I, we show the total kaon-production probability for central collisions between various symmetric systems at an incident en-

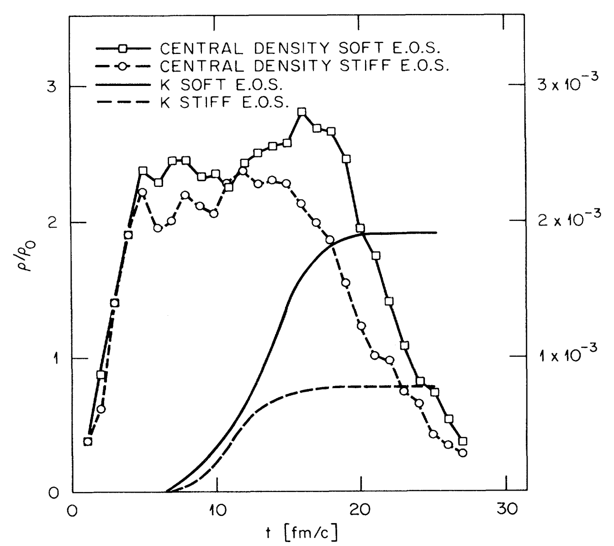

FIG. 1. Central density ρ/ρ_0 and total kaon-production probability P_K as functions of the collision time for reactions between Nb nuclei at an incident energy 700*A* MeV and at an impact parameter $b=0.5$ fm.

TABLE I. Kaon-production probability P_K for collisions between nuclei with mass number A at an incident energy 700 MeV/nucleon and at an impact parameter $b = 0.5$ fm. Results from both a soft and a stiff equation of state are shown. The ratio between the two is given by R.

A	P_K(soft)	P_K(stiff)	R
20	8.8×10^{-5}	5.2×10^{-5}	1.7
40	3.2×10^{-4}	1.3×10^{-4}	2.5
93	2.0×10^{-3}	7.0×10^{-4}	2.8

TABLE II. Kaon-production probability P_K as a function of beam energies for collisions between Nb nuclei at an impact parameter $b = 0.5$ fm. Results are from both a soft and a stiff equation of state. The ratio between the two is given by R.

E/nucleon (GeV)	P_K(soft)	P_K(stiff)	R
0.65	9.0×10^{-4}	3.3×10^{-4}	2.8
0.70	2.0×10^{-3}	7.0×10^{-4}	2.8
0.80	4.0×10^{-3}	2.6×10^{-4}	1.5
1.00	1.7×10^{-2}	1.1×10^{-2}	1.5
1.20	5.1×10^{-2}	3.3×10^{-2}	1.5
1.40	1.2×10^{-1}	7.8×10^{-2}	1.5

ergy of 700 MeV/nucleon averaged over several hundred simulations. For both equations of state, the kaon-production probability P_K increases with the mass number A of the colliding nuclei. The mass dependence of the kaon-production probability is consistent with that of Refs. 5, 6, and 10. The ratio R also increases as the system becomes heavier. This allows the determination of the nuclear equation of state by measurement of the ratio of cross sections. We find for collisions of Nb+Nb at 700 MeV/nucleon an impact-parameter-averaged cross section of 0.89 ± 0.04 and 0.46 ± 0.03 mb for the soft and the stiff equations of state, respectively. Calculations at a lower beam energy of 650 MeV/nucleon show a reduction of the cross section by a factor of 2 but leave the ratio of the kaon-production probability using different equations of state essentially unchanged. This ratio becomes smaller for calculations at higher beam energies as shown in Table II. In this table we see a sudden decrease of the sensitivity of kaon production to the nuclear equation of state for heavy-ion collisions at energies above ~ 800 MeV/nucleon. This is due to the influence of the Fermi momentum which has a value ~ 270 MeV/c and makes kaon production from initial nucleon-nucleon collisions energetically allowed once the incident energy exceeds ~ 800 MeV/nucleon.

In our calculations, we have not included kaon production from the reactions of secondary pions with nucleons or deltas. Previous studies at high energies[7,10] indicated that this mechanism contributes only $\sim 25\%$ to the total kaon production. We therefore do not expect significant changes in our results when the $\pi N \to \Lambda K$ reaction channel is included.

In summary, we have carried out microscopic calculations of kaon production at subthreshold energies using the Boltzmann-Uehling-Uhlenbeck equation. We find that kaon production is sensitive to the nuclear equation of state. For central collisions between heavy systems, the number of produced kaons can differ by a factor of ~ 3, depending on the equation of state. All the kaons are created at the stage of maximum compressions and are therefore sensitive to the equation of state at high densities. The sensitivity diminishes as the colliding systems become smaller or the incident energies get higher. Since lambda particles are produced in association with kaons, one expects that lambda production at subthreshold energies will also be sensitive to the nuclear equation of state. It is therefore extremely interesting to carry out measurements of kaons or lambdas for collisions between heavy nuclei at incident energies around 700 MeV/nucleon.

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