Domain Walls in Superstring Models

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The $E_8 \otimes E_8'$ superstring models in Calabi-Yau internal space have the cosmological axionic domain-wall problem, $N_{DW} \geqslant 2$, if two non-Abelian gauge groups $SU(3)_c \otimes G$ survive. Breaking G down to its Abelian subgroup removes the domain-wall problem. In general, if $SU(3)_c$ is the only surviving non-Abelian group of the superstring models, the domain-wall problem is not present, because of the model-independent invisible axion. We also point out an elegant method of solving the axionic domain-wall problem with many would-be axions.

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Recently, it has been shown[1] that $N=1$ superstring theories in ten dimensions with gauge groups O(32) or $E_8 \otimes E_8'$ are anomaly free, which has led to an enormous theoretical effort toward superstring theories and their consequences.[2-9] At present there are three known $N=1$ superstring theories: Type I O(32), heterotic $\mathrm{Spin}(32)/Z_2$, and heterotic $E_8 \otimes E_8'$. Among them, the $E_8 \otimes E_8'$ theory seems to be the most promising one in the phenomenological sense. In particular, the $E_8 \otimes E_8'$ superstring theory with Calabi-Yau internal space has attracted a great deal of attention because it leads to $d=4$ $N=1$ supersymmetry, which is useful for solution of the gauge hierarchy problem.[8]

One of the most interesting results of these anomaly-free superstring theories is the existence of the model-independent invisible axion (MI axion a_1).[2] The invisible-axion models have been extensively discussed in the literature.[10,11] At present, the allowed band[12] of the axion decay constant is $10^8 < F_a < 10^{12}$ GeV. The MI axion has the decay constant $M_{\mathrm{Pl}}/96\pi^{5/2} \approx 10^{16}$ GeV, where M_{Pl} is the Planck mass. One must fine-tune θ in the beginning or resort to the new inflationary scenario.[13] A more serious problem than this axion-decay-constant problem is the existence of domain walls in many axion models.[12,14] The boundaries of the axionic domain walls are axionic strings.[15] The domain-wall number N_{DW} in axion models must be 1.

In this paper, we mainly discuss axionic domain walls in $E_8 \otimes E_8'$ superstring theories. From the condition $N_{DW}=1$, we will conclude that E_8' must be broken so that no non-Abelian group derives from it. Namely, the domain-wall problem disfavors the supersymmetry-breaking scenario by gaugino condensates. In addition, we also conclude that all superstring models are domain-wall free (i.e., $N_{DW}=1$) if there do not exist non-Abelian interactions beyond QCD. For this observation, we use Witten's result[3] on the axionic string of the MI axion.

Our argument is unbelievably simple in the scheme of an axion effective Lagrangean.[16] Therefore, let us start by discussing the axionic domain walls in the language of the axion effective Lagrangean. The effective Lagrangean of the axion can be written as

$$\mathscr{L}[a] = \tfrac{1}{2}(\partial_\mu a)^2 + \frac{g^2}{32\pi^2}\frac{a}{F_a} F^i_{\mu\nu}\tilde{F}^{i\mu\nu}. \tag{1}$$

The axion is a periodic variable. The period in units of F_a determines the domain-wall number N_{DW}:

$$a \equiv a + 2\pi N_{DW} F_a. \tag{2}$$

The degenerate vacua can be specified by the vacuum expectation values of the axion field whose number is N_{DW},

$$\langle a \rangle = 2k\pi F_a \quad (k=0,1,\ldots,N_{DW}-1). \tag{3}$$

The axionic domain-wall number N_{DW} or the period of the axion field can be determined from the underlying theory which gives the effective Lagrangean (1) as its low-energy manifestation.[3,11,14] For the case of one axion, the form of the interaction (1) and periodicity (2) alone do not give useful information. They simply say that N_{DW} is a value satisfying $N_{DW} \geqslant 1$. On the other hand, some information may be obtained for the case of two axions *if we know the axion interactions,* even though we are blind to the underlying dynamics. Indeed, this is the case in superstring models.

For the interesting $E_8 \otimes E_8'$ superstring models with Calabi-Yau internal space,[8] the effective axion Lagrangean has been calculated[6]:

$$\mathscr{L}[a_1,a_2] = \tfrac{1}{2}(\partial_\mu a_1)^2 + \tfrac{1}{2}(\partial_\mu a_2)^2 - \frac{g^2}{32\pi^2}\left(\frac{a_1}{F_1}+\frac{a_2}{F_2}\right)F^i_{\mu\nu}\tilde{F}^{i\mu\nu} - \frac{g'^2}{32\pi^2}\left(\frac{a_1}{F_1}-\frac{a_2}{F_2}\right)F'^j_{\mu\nu}\tilde{F}'^{j\mu\nu}, \tag{4}$$

where $F^i_{\mu\nu}$ ($F'^j_{\mu\nu}$) denotes QCD (G) field strength, and $\tilde{F}^i_{\mu\nu}$ ($\tilde{F}'^j_{\mu\nu}$) is its dual. The group G is a non-Abelian subgroup of E_8' which may be useful for dynamical supersymmetry breaking. The a_1 corresponds to the MI axion defined by

$$H_{\mu\nu\rho} = M_1 \epsilon_{\mu\nu\rho\sigma} \partial^\sigma a_1, \tag{5}$$

and a_2 is a model-dependent one such that

$$H_{\mu mn} = M_2 \epsilon_{mn} \partial_\mu a_2. \tag{6}$$

Even though we do not know anything else except the interaction (4), we can say that $N_{\rm DW} \neq 1$. However, we must utilize the knowledge of the axion interactions. The argument is the following.

If a_1 and a_2 are periodic variables, they must be identified as

$$a_1 \equiv a_1 + 2\pi N_1 F_1, \tag{7}$$

$$a_2 \equiv a_2 + 2\pi N_2 F_2, \tag{8}$$

where N_1 and N_2 are positive integers, and F_1 and F_2 are axion scales. The topological charge is quantized. Therefore, the effective interaction (4) implies

$$\langle a_1 \rangle / F_1 + \langle a_2 \rangle / F_2 = 2\pi \times \text{integer}, \tag{9}$$

$$\langle a_1 \rangle / F_1 - \langle a_2 \rangle / F_2 = 2\pi \times \text{integer}. \tag{10}$$

The number of vacuum states is counted by the number of sets of the axion vacuum expectation values $\{\langle a_1 \rangle / F_1, \langle a_2 \rangle / F_2\}$ satisfying the quantization condition [Eqs. (9) and (10)] within the periods [Eqs. (7) and (8)].[17] The resulting disconnected degenerate vacua are $\{2n\pi, 2m\pi\}$ and $\{(2n+1)\pi, (2m+1)\pi\}$, where $n = 0, 1, 2, \ldots, N_1 - 1$, and $m = 0, 1, 2, \ldots, N_2 - 1$. Therefore, the number of domain walls is

$$N_{\rm DW} = 2 N_1 N_2. \tag{11}$$

The minimum number for $N_{\rm DW}$ is 2, and the model is cosmologically harmful.[14] The mechanism of Lazarides and Shafi[18] cannot be realized in superstring theories. Two superstring axions with Calabi-Yau internal space are not consistent with the assumption of two non-Abelian forces.

To cure the cosmological domain-wall problem for the given axion Lagrangean (4), we must break the extra Yang-Mills group G descending from E_8'. Then the G gauge fields $A_\mu^{\prime i}$ cannot develop any surface term, and we can safely put $F_{\mu\nu}^{\prime i} = 0$ in the axion Lagrangean (4). In theories with the broken-G gauge interaction, one combination of a_1 and a_2 is the invisible axion, and the other combination is a true Goldstone boson. In this case, the domain wall number is the number of disconnected vacuum manifolds where each connected component has the topology of the circle. The circle topology of a vacuum manifold means the existence of a massless Goldstone boson which corresponds to the aforementioned orthogonal component $(-F_1 a_1 + F_2 a_2)/(F_1^2 + F_2^2)^{1/2}$. The axion is $(F_2 a_1 + F_1 a_2)/(F_1^2 + F_2^2)^{1/2}$.

Because the G interaction is broken, only Eq. (9) gives the condition for a ground state. If one defines $\theta_1 = 2\pi a_1 / F_1$ and $\theta_2 = 2\pi a_2 / F_2$, then the vacuum manifold corresponds to $\theta_1 + \theta_2 =$ integer on the torus of $\theta_1 \equiv \theta_1 + N_1$, $\theta_2 \equiv \theta_2 + N_2$. We illustrate the vacuum manifold in Fig. 1 for the cases of $N_1 = 2$, $N_2 = 5$ and $N_1 = 2$, $N_2 = 4$. From these examples, one can see that $N_{\rm DW}$, that is, the number of disconnected circles, is the maximal common divisor of the period numbers N_1 and N_2.

The domain-wall problem does not exist, i.e., $N_{\rm DW} = 1$, if N_1 or N_2 takes the value 1 or if N_1 and N_2 are relatively prime. We again point out that this solution exists only when there do not exist extra unbroken non-Abelian gauge interactions descending from E_8'. Our next step is to determine N_1 and N_2, which requires information on the underlying theory. Fortunately enough, Witten[3] determined $N_1 = 1$, and hence we obtain $N_{\rm DW} = 1$ irrespective of N_2 if the G interaction is broken at least to its Abelian subgroup.

Witten observed[3] that the superstring itself in heterotic string theories is the axionic string for the MI axion a_1. He also notes that this axion string is the boundary of a single axionic domain wall. This result can be obtained by the counting of the color charge carriers which are trapped in the string.[19] Witten's result means in our effective axion Lagrangean language that a_1 is the periodic field variable with

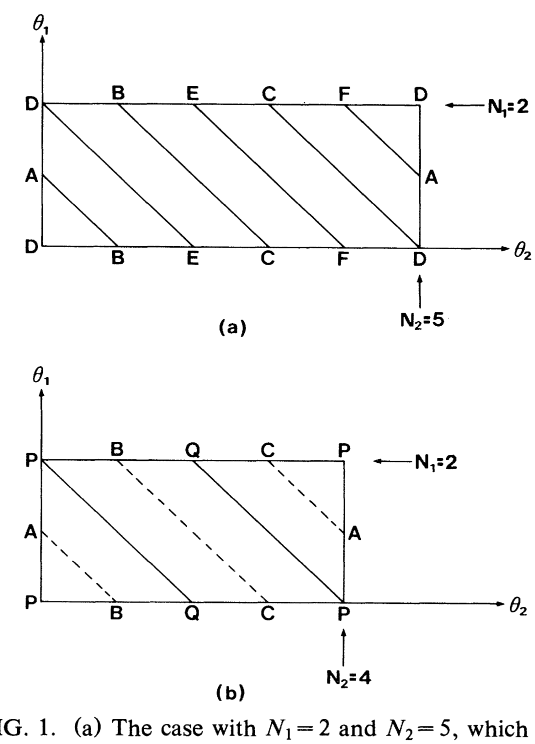

FIG. 1. (a) The case with $N_1 = 2$ and $N_2 = 5$, which gives $N_{\rm DW} = 1$. The single circle is *ABCDEFA*. (b) The case with $N_1 = 2$ and $N_2 = 4$, which gives $N_{\rm DW} = 2$. The two disconnected circles are *ABCA* and *PQP*.

period $2\pi F_1$. Therefore, $N_1=1$.

So far we have considered the domain-wall problem in the standard "big bang" cosmology. It is well known that the domain-wall problem disappears if there was an epoch of new inflation at a temperature below the axion scale. Then G need not be broken. In any case, this new inflation may be necessary to solve the notorious monopole problem. It has been already pointed out that the Hosotani mechanism breaking E_6 predicts monopoles with nonconventional magnetic charges.[20] The same thing can happen for the breakdown of G into its Abelian subgroups by the Hosotani mechanism.

The existence of the MI axion a_1 with period $2\pi F_1$ has a far-reaching consequence for other realizations of superstring theories. It was noticed[5] that a_1 generates the cosmological energy-density problem as a result of its large decay constant $F_1 \sim 10^{16}$ GeV. To remove this problem, Barr[7] compactified in non-Calabi-Yau space and obtained the Peccei-Quinn symmetry $U(1)_P$, which is not the quinticity of SU(5). Therefore, the Peccei-Quinn symmetry $U(1)_P$ can be broken at the desired scale $10^8 < v_{PQ} < 10^{12}$ GeV. Inevitably, this method gives a large value of $N_P = \mathrm{Tr}P[SU(3)]^2$ because of the repetition of generations. But as has been seen in the preceding discussion, $N_P > 1$ does not generate the domain-wall problem because of the a_1 with period $2\pi F_1$.

This picture can be readily generalized to the case with many would-be axions. The axion Lagrangean is

$$\mathscr{L} = \sum_{k=1}^{n} \left\{ \tfrac{1}{2}(\partial_\mu a_k)^2 + \frac{g^2}{32\pi^2}\frac{a_k}{F_k} F^i_{\mu\nu}\tilde{F}^{i\mu\nu} \right\}, \qquad (12)$$

where $F^i_{\mu\nu}$ is the gluon field strength. Note that the color gauge interaction is the only non-Abelian interaction. With the would-be axion a_1 with period $2\pi F_1$, the effective domain-wall number is $N_{DW}=1$, irrespective of the periods of the other a_k's. This mechanism reveals an elegant way of solving the domain-wall problem in axion models. If the theory contains a would-be axion with $N_1=1$ like the MI axion a_1 in superstring theories, the domain-wall problem is automatically cured without a further extension of the theory. With several U(1)'s, this has been studied by Barr, Gao, and Reiss.[21]

We summarize our results. We conclude that the $E_8 \otimes E_8'$ model with Calabi-Yau internal space has the domain-wall problem if the hidden non-Abelian group G descending from E_8' survives for the dynamical supersymmetry breaking. Therefore, the consideration of axionic domain walls in the standard cosmology disfavors the supersymmetry-breaking scenario by the gaugino bilinear condensates $\langle \bar{\chi}\Gamma^{mnp}\chi \rangle$. If G is broken down to its Abelian subgroup, the domain-wall problem disappears because of the MI axion a_1. By the same reasoning, we also conclude that for any low-energy realization of superstring theories with QCD as its only non-Abelian gauge interactions, the axionic domain-wall problem is absent irrespective of the periods of the other would-be axions. Nevertheless, a new inflation may be necessary to dilute the monopoles arising through the Hosotani mechanism.

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