Rare Kaon Decay $K^+ \to \pi^+ l^+ l^-$

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A new treatment is proposed for the rare kaon decays $K^+ \to \pi^+ e^+ e^-$ and $K^+ \to \pi^+ \mu^+ \mu^-$. Although the individual rates seem impossible to predict accurately at present because of large cancellations of different amplitudes, it is pointed out that a measurement of certain ratios involving these decays could provide interesting information on the weak nonleptonic interaction.

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In recent years, there has been renewed interest in the physics of rare kaon decays, mainly because of their potential to indicate deviations from the standard electroweak model, in particular in the new generation of remarkably sensitive experiments currently being performed or planned (for recent reviews, see, e.g., Walter[1] and Littenberg[2]). It should be noted, however, that there are also problems within the standard model concerning the nonleptonic weak interaction that still lack a satisfactory solution. The foremost of these concerns the structure of the strangeness-changing ($\Delta S = 1$ or 2) weak Hamiltonian, in particular the origin of the $\Delta I = \frac{1}{2}$ rule.

The generic problem, e.g., the calculation of the K-π transition induced by the interplay between the strong and electroweak interaction, consists of three components in the commonly accepted Wilson expansion method: (a) identification of the relevant local field operators, (b) calculation of the quantum-chromodynamics-corrected Wilson coefficients, and (c) evaluation of the matrix elements of the operators between hadronic states. To this should be added (outside the Wilson expansion) the problem of (d) estimating the long-distance effects. In a previous paper[3] it was shown how a study of the rare kaon decays $K_L \to l^+ l^- \gamma$ ($l = e$ or μ) could shed some light on these questions, in particular the role of the so-called penguin operators first introduced by Shifman, Vainshtein, and Zakharov.[4] In this article we show that interesting information can be obtained from the decays $K^\pm \to \pi^\pm l^+ l^-$.

The diagrams contributing to this process can be divided into the two classes displayed in Figs. 1(a) and 1(b), where the box indicates the action of the $\Delta S = 1$ weak Hamiltonian (diagrams where the effective weak neutral current creates the leptons can be safely neglected). Figure 1(a) represents one of the terms where the four-quark weak-interaction Hamiltonian is acting, whereas Fig. 1(b) shows the contribution from the single-quark decay $s \to d\gamma^*$. The interesting feature of diagram 1(b) is that its contribution is sensitive to the strangeness-changing current in the vector channel. In fact, it is in processes like this that the $\Delta S = 1$ transition between vector states can be presently measured and estimated, and so far there is no experimental information on it. Whether such matrix elements are $\Delta I = \frac{1}{2}$ enhanced like the corresponding ones between pseudoscalar states [Fig. 1(a)] is of major importance for the understanding of the structure of the nonleptonic weak Hamiltonian.

To calculate the total amplitude in absolute numbers with any confidence has been found to be impossible in previous work[5-9] as a result of uncertainties in all the points (a) to (d) above and, in most cases, cancellations between large contributions of opposite signs. The main idea we want to present in this paper is that many uncertainties can be eliminated by forming the ratio of $K^+ \to \pi^+ \mu^+ \mu^-$ to $K^+ \to \pi^+ e^+ e^-$. Although suppressed by a factor of about 5 as a result of phase space, the former reaction should be easily measurable in the new rare-kaon-decay experiments; the latter decay has already been measured[10] to have a branching ratio of $(2.7 \pm 0.5) \times 10^{-7}$. As we will see,

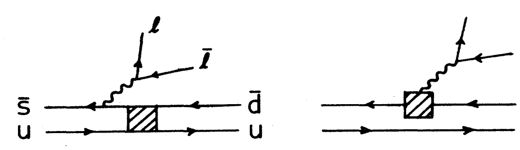

FIG. 1. Different contributions to $K^+ \to \pi^+ l^+ l^-$ (a) represents one of the diagrams where the photon transition is flavor-diagonal and is followed or preceded by a four-quark weak transition, whereas (b) involves the single-quark decays $s \to d\gamma^*$ induced by the effective flavor-changing two-quark weak Hamiltonian.

very interesting information on the QCD-corrected Wilson coefficients [point (b) above] can thus be obtained.

It has been known for some time that QCD corrections may have a drastic effect on the $sd\gamma$ vertex which, in particular, enters the calculation of diagram 1(b). Remarkably, the sign is reversed for the charge-radius term by these corrections,[7,11–13] and the magnetic term (not active in our process) is dramatically enhanced through a screening of the Glashow-Iliopoulos-Maiani cancellation mechanism.[12,14] The key to our separation of the charge-radius term from the radiative amplitude of Fig. 1(a) is the observation that the form-factor dependence (on the invariant mass of the lepton pair) is different for the two types of diagrams. For the low values of Q^2 relevant here, diagrams of type 1(a) give rise to ρ- and ω-dominated form factors, whereas, as shown later, diagram 1(b) contains a K^* pole in addition. Since the e^+e^- and $\mu^+\mu^-$ pairs partly populate different regions of phase space, their ratio is sensitive to form-factor effects and thus to the relative strengths (including the sign) of the two types of contributions.

The matrix element for $K^+ \to \pi^+ l^+ l^-$ can be written $M = L^\mu l_\mu$, where

$$L^\mu = i\int d^4x \exp[i(p_K - p_\pi)\cdot x]\langle \pi | T(H_{\rm wk}(x) J^\mu_{\rm em}(0)) | K\rangle = \tilde{f}_+(s)(p_K + p_\pi)^\mu + \tilde{f}_-(s)(p_K - p_\pi)^\mu \qquad (1)$$

and $l_\mu = -i(e^2/s)\bar{u}(p_-)\gamma_\mu v(p_+)$. Here p_K, p_π, p_-, and p_+ are the four-momenta of the particles, $s = (p_- + p_+)^2 = (p_K - p_\pi)^2$. As a result of current conservation, only $\tilde{f}_+(s)$ contributes to this process and furthermore $\tilde{f}_+(0) = 0$ (reflecting the fact that the process $K^+ \to \pi^+\gamma_{\rm real}$ is forbidden). Defining then $f_+(s)$ by $\tilde{f}_+(s) = s f_+(s)$ one obtains the invariant-mass distribution

$$d\Gamma/ds = (\alpha^2/12\pi m_K^3)|f_+(s)|^2 \lambda^{3/2}(s)(1 - 4m_l^2/s)^{1/2}(1 + 2m_l^2/s), \qquad (2)$$

where $\lambda(s) = m_K^4 + m_\pi^4 + s^2 - 2m_K^2 m_\pi^2 - 2m_K^2 s - 2m_\pi^2 s$ and where for our purpose it is essential not to neglect the lepton mass and the s dependence of f_+, where the dynamics resides. To extract the various pieces of f_+ corresponding to the two classes of diagrams, we first write f_+ in a form consistent with gauge invariance and vector meson dominance:

$$f_+(s) = (G_F/\sqrt{2}) s_1 c_1 c_3 a(\xi)/4\pi^2[\sin\xi(1 - s/m_\rho^2)^{-1} + \cos\xi(1 - s/m_\rho^2)^{-1}(1 - s/m_{K^*}^2)^{-1}], \qquad (3)$$

where $s_1 = \sin\theta_1$ etc. (θ_1 and θ_3 are Kobayashi-Maskawa angles) and ξ is a parameter such that $\tan\xi$ describes the relative weight of the two terms ($|\xi| \leq \pi/2$); $a(\xi)$ is a normalization factor that can be determined from the experimental rate for $K^+ \to \pi^+ e^+ e^-$. Since we will mostly be concerned with ratios, such as

$$R(\xi) = \Gamma(K^+ \to \pi^+\mu^+\mu^-)/\Gamma(K^+ \to \pi^+ e^+ e^-), \qquad (4)$$

$a(\xi)$ will not enter our results. The inclusion of the form-factor dependence in Eq. (3), not considered in previous treatments, will have interesting measurable consequences as shown below. The form factor should really include a sum over different vector meson resonances ($\rho, \omega, \phi, \ldots$) and take into account their infinite widths. In our numerical calculations this has been done without affecting significantly the results obtained when we keep only the lowest-lying poles (see Ref. 3 for a more complete expression for the form factor).

To illustrate the calculation of ξ, we first consider the free-quark model (without QCD gluonic corrections). Removing the W and the heavy quarks t, b, and c from the low-energy effective theory, one obtains[5,7,9]

$$H_{\rm eff} = -4(G_F/\sqrt{2}) s_1 c_1 c_3 (\bar{s}_i \gamma_\mu u_i)_L (\bar{u}_j \gamma^\mu d_j)_L - G_F/\sqrt{2}(8\alpha/9\pi)[\lambda_c \ln(m_c/\mu) + \lambda_t \ln(m_t/m_c)](\bar{s}_i\gamma_\mu d_i)(\bar{l}\gamma^\mu l), \qquad (5)$$

where $\lambda_k = V_{kd}V_{ks}^*$ ($k = c$ or t) (V is the Kobayashi-Maskawa mixing matrix), i and j are color indices, and L means left-handed projection; μ is a typical hadronic mass scale 0.5–1 GeV. The transition $K \to \pi l \bar{l}$ is then induced by matrix elements of the first (four-quark) operator in (5) evaluated between pseudoscalar states to order e^2 and matrix elements of the second (two-quark) operator $(\bar{s}_i\gamma_\mu d_i)(\bar{l}\gamma^\mu l)$ (Q_7 in the terminology of Ref. 7) evaluated between vector states to order e^0. The latter can be related by a quark isospin rotation to the matrix element that enters the decay $K^+ \to \pi^0 e^+ \nu_e$, which has been shown experimentally[15] to exhibit a K^* dominance behavior as claimed above. The evaluation of the contribution from the first term in (5) is somewhat elaborate but has been performed in detail in Ref. 5. Taking their reuslt we find this to correspond to a value of $\tan\xi \approx 1.0$ ($\xi = 0.67$). Calculating QCD corrections to (5) in the leading logarithmic approximation[5,7,11–13] we find, again using values from Ref. 5, $\tan\xi \approx -0.3$ ($\xi = -0.29$). We caution at this point, however, that although the authors of Refs. 5, 7, and 11–13 agree on the change of sign of the $sd\gamma$ charge-radius term resulting from the QCD corrections, the various procedures give differences by factors up to ≈ 1.5. As mentioned before, the QCD corrections change both the relative sign and magnitude of the two terms.

As a last example, we have also calculated these processes in the old Sakurai model[16] where we find a form factor proportional to

$$\left(1-\frac{s}{m_\rho^2}\right)^{-1}\left[\frac{-f_\pi f_K}{m_\rho^2\sqrt{2}}+\frac{4}{3f_{K^*}f_\rho(1-s/m_{K^*}^2)}\right]. \quad (6)$$

We take this model as representative of the phenomenological models which have a universally built-in $\Delta I=\frac{1}{2}$ enhancement, unlike the penguin mechanism[4] which gives large matrix elements between pseudoscalar states but is inoperative between vector states.[3] Inserting in (6) $f_\pi=131$ MeV, $f_K=148$ MeV, $f_{K^*}=f_\rho\approx 6.0$, we thus obtain $\tan\xi\approx -0.63$ ($\xi = -0.54$). It is interesting to note that this pure long-distance model gives the same sign for ξ as the QCD-corrected short-distance model.

In Fig. 2 we show the predicted ratio $R=\Gamma(K^+\to\pi^+\mu^+\mu^-)/\Gamma(K^+\to\pi^+e^+e^-)$ as a function of ξ. As can be seen, the difficult model predictions calculated above should be distinguishable in an experiment with an accuracy better than around 15%. For the three models discussed above, we find, respectively, R(free-quark model) = 0.23, R(QCD) = 0.25, and R(Sakurai) = 0.28. In any case, the predicted deviations from the pure phase-space prediction $R=0.196$ (using a constant form factor) should be easily measurable in the new generation of rare-kaon-decay experiments.[1,2] For $\xi=\pm\pi/2$, which corresponds to no K^* contribution, we find $R(\pm\pi/2)\approx 0.22$. For $\xi\approx 0$, where the single-quark decay dominates, $R(0)\approx 0.25$. It is interesting to note in Fig. 2 that for most values of ξ, $0.20 < R(\xi) < 0.25$. Other values only appear when $\xi\approx -\pi/4$ ($\tan\xi\approx -1$), because of the cancellation between the two terms in (3). For ξ somewhat larger than this value, this cancellation is most effective for small s and thus affects more the $K^+\to\pi^+e^+e^-$ decay where the kinematic threshold in s is lower than for the muonic decay. For ξ somewhat smaller than $-\pi/4$ the cancellation is more effective for large s and thus diminishes the muonic decay relatively more than the electronic one. This is the reason for the rapid variation of R near $\xi=-\pi/4$.

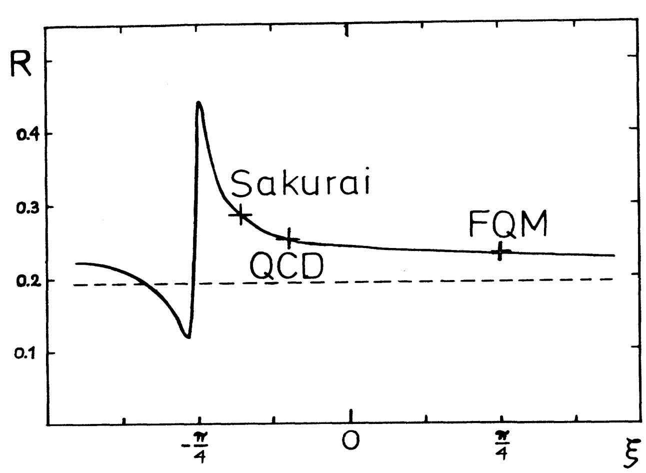

FIG. 2. $R=\Gamma(K^+\to\pi^+\mu^+\mu^-)/\Gamma(K^+\to\pi^+e^+e^-)$ as a function of the parameter ξ, which determines the relative strength of the contributions shown in Fig. 1(a) and Fig. 1(b) [see Eq. (3)]. Shown are the predictions from the Sakurai model, QCD, and the free-quark model (FQM). The dashed horizontal line $R=0.196$ is the prediction that would follow if form-factor effects were neglected.

Another, in this region more efficient, way to extract information on ξ is then to look at the invariant-mass distribution of the lepton pairs for each of the decays separately. An experimentally easily measurable quantity is, e.g., the fraction of events with invariant mass greater than some fixed value, say $s > 0.5(m_K-m_\pi)^2$. This particular ratio displays a behavior as a function of ξ which is very similar to that of R. By measuring both this ratio and R it should be possible to tell whether a deviation of R from the phase space prediction is attributable to the form-factor effects discussed here or if there are other, unknown effects (e.g., nonuniversality for e and μ as is the case for a light Higgs boson[17]).

For experimental reasons (to avoid the e^+e^- background from $K^+\to\pi^+\pi^0$ followed by π^0 Dalitz decay) a cut in invariant mass, $s > m_\pi^2$, is often imposed for the $K^+\to\pi^+e^+e^-$ decay. This reduces the number of events by typically 30% corresponding roughly to an overall rescaling of R without affecting much the shape displayed in Fig. 2.

It should finally be noted that the results obtained in this paper could be of relevance also for other rare-kaon-decay experiments currently being considered. By use of Eqs. (2) and (3), the background from $K\to\pi l\bar{l}$ to processes like $K\to\pi\mu e$ and $K^+\to\pi^+\pi^0\to\pi^+e^+e^-$ can be estimated. In particular, one should observe that because of the vector nature of Eq. (1) and the s-dependent form factor in Eq. (3), the invariant-mass distribution differs significantly from that obtained from phase space only.

To conclude, we want to emphasize the importance of experimentally measuring the kaon decays $K\to\pi l\bar{l}$ since they may give interesting and unexpected information on the strangeness-changing weak interaction. An accurate measurement of R could throw light on several interesting features of the nonleptonic interaction: (a) the relative size of the terms describing the two-quark and four-quark processes; (b) the relevance of the QCD corrections which lead to a change of sign for the two-quark transition term; (c) the size of the weak matrix element between vector states; (d) the existence or nonexistence of light Higgs bosons contributing to these decays.

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