Energy-Level Statistics of Integrable Quantum Systems

In a recent Letter,[1] Casati, Chirikov, and Guarneri (CCG) studied statistical properties of spectra for integrable systems. They found deviations from the expected uncorrelated Poisson distribution for energy levels.[2]

In view of the far-reaching implications of this result, it is worthwhile to check the behavior of these deviations in the semiclassical limit (large quantum numbers). Therefore, I extended the calculations of CCG and computed the distribution of level spacings,

$$E_N \equiv E_{m,n} = \alpha m^2 + n^2, \quad (1)$$

for $\alpha = \pi/3$ and for consecutive sequences of 10^4 levels. Figure 1 shows the χ^2 deviation from the Poisson distribution, for the first ten bins of size $D/100$ (D is the mean spacing). The dashed line is a smoothed version which could be regarded as an estimate of $\langle\chi^2\rangle$. For $N < 2.5\times10^6$, $\langle\chi^2\rangle$ falls below 16, which represents a 10% confidence level. (If more than ten bins had been taken, the decay of the deviations would have been slower.)

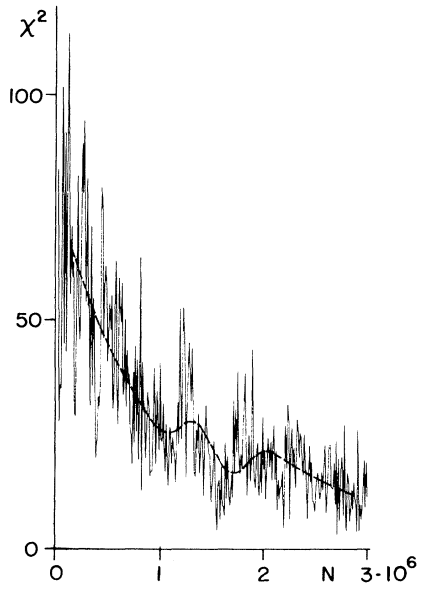

FIG. 1. χ^2 for consecutive sets of 10^4 levels. The dashed line is the value of $\langle\chi^2\rangle$ obtained by an averaging over sets of thirty consecutive data.

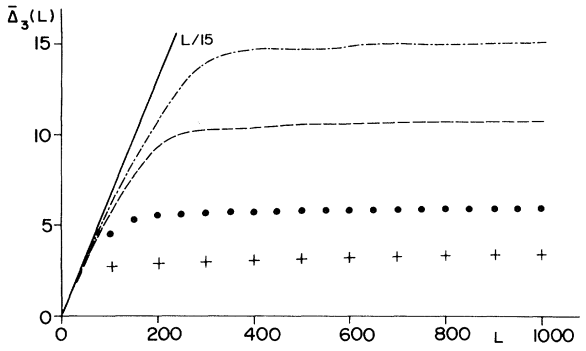

FIG. 2. The Δ_3 statistics: spectral average for (crosses) $0 < N_0 < 1850$, (solid circles) $0 < N_0 < 9000$, (dashed line) $10\,000 < N_0 < 19\,000$, and (dash-dotted line) $20\,000 < N_0 < 29\,000$. Here, N_0 is the index of the first level in the sequence for which Δ_3 is computed. The full line is the Poisson result $\bar{\Delta}_3 = L/15$.

The $\bar{\Delta}_3(L)$ rigidity measure is shown in Fig. 2. The transition from the $L/15$ behavior (which corresponds to Poisson statistics) to an almost constant value of $\bar{\Delta}_3(L)$ is shifted to larger values of L, as sequences from higher parts of the spectrum are involved. The Poisson result, $\bar{\Delta}_3 = L/15$, is obtained in the limit $N \to \infty$. This was checked for $290\,000 \to N \to 300\,000$. $\bar{\Delta}_3(L)$ was only slightly below $L/15$ and no transition was observed up to $L = 1000$.

In summary, deviations from Poisson uncorrelated statistics for energy levels in integrable systems seem to disappear in the semiclassical limit. This result was predicted by Berry and Taylor.[2]

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[1]G. Casati, B. V. Chirikov, and I. Guarneri, Phys. Rev. Lett. **54**, 1350 (1985).

[2]M. V. Berry and M. Taylor, Proc. Roy. Soc. London **356**, 375 (1977).