Inductance Effect of Runaways on Lower-Hybrid-Current Ramping

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The role of runaway electrons in current ramping by lower-hybrid waves is discussed. The back emf induced by lower-hybrid-current ramping produces runaway electrons at a rate $\dot{n}_R$ such that the additional inductance (proportional to $\dot{n}_R ec$) can be significant in opposing the current ramping. However, runaway electrons can also destabilize the oblique plasma waves through their anisotropy. The resulting turbulence can greatly reduce the runaway production rate, enabling the current-ramping rate to exceed the above limit.

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Using radio-frequency waves for the generation and sustainment of plasma current in tokamaks has been of considerable interest recently, both in theory and in experiment. Experiments with lower-hybrid (LH) waves have successfully sustained a current up to 400 kA in steady state for several seconds and created a target plasma with the plasma current raised to 100 kA.[1–5] In the case of nonstationary current (ramping and decaying), an inductive electric field is produced in the plasma to oppose the current change:

$$E=-\frac{1}{2\pi R}L_{\text{ext}}\frac{dI}{dt}\cong -L\frac{dj}{dt}, \tag{1}$$

where $L_{\text{ext}}=2\pi R/c^2[\ln(8R/a)-\frac{7}{4}]$ is the self-inductance of a torus with major radius R and minor radius a, and $L=(\pi a^2/c^2)[\ln(8R/a)-\frac{7}{4}]$ is obtained by the assumption of uniform current over the minor cross section. For E/E_0 exceeding a few percent, a significant number of runaway electrons are produced by the electric field with $v>v_R=(E_0/E)^{1/2}v_e$, where $E_0=m\nu v_e/e$ is the Dreicer field, $v_e=(T_e/m)^{1/2}$ is the electron thermal speed, and ν is the electron collision frequency. The plasma current is therefore composed of three components: $j=j_{\text{rf}}+j_b+j_R$, where j_{rf} is the part carried by the electrons in resonance with rf, i.e., $v_\parallel=\omega/k_\parallel$, $j_b\approx\sigma E$ is the current carried by the bulk electrons in response to the inductive electric field, $\sigma=ne^2/m\nu$ being the Spitzer conductivity, and j_R is the current carried by runaway electrons. Both j_b and j_R are in the opposite direction of j_{rf} in the case of current ramping. The electron distribution function in the case of rf-current ramping is shown in Fig. 1. The various components of the plasma current are then defined by $j_{\text{rf}}=e\int_{-c}^{v_{\text{ph}}}dv\,fv$, $j_b=e\int_{v_{\text{ph}}}^{v_R}dv\,fv=\sigma E$, and $j_R=e\int_{v_R}^{c}dv\,fv$, where v_{ph} is the maximum phase velocity of lower-hybrid waves. For present experiments, $j_{\text{rf}}\gtrsim j_b\gg j_R$. However, the rate of change of j_R can be significant, not only because of the runaway acceleration by the electric field, but also because runaways are being continuously produced with a rate $\dot{n}_R$, calculated from the Fokker-Planck equation as the flux crossing the surface of critical energy $mv_R^2/2$ in the velocity space, resulting in a large rate of change of high-energy current carriers. In short, this effect introduces a nonlinear plasma inductance which can significantly affect the current-ramping efficiency.

To estimate this nonlinear inductance, we use the simplified one-dimensional quasilinear, Fokker-Planck equation valid for $v\gg v_e$,

$$\frac{\partial f}{\partial t}=-\frac{eE}{m}\frac{\partial f}{\partial v}+\frac{\partial}{\partial v}D_{\text{LH}}\frac{\partial f}{\partial v}+C(f), \tag{2}$$

where $C(f)$ is the Coulomb collision operator,

$$D_{\text{LH}}(v)=\pi\left(\frac{e}{m}\right)^2\int\frac{d^3k}{(2\pi)^3}\epsilon_k\delta(\omega_k-kv\cos\theta)=\begin{cases}D_0 & \text{for } -c<\omega/k_\parallel<v_{\text{ph}},\\ 0 & \text{otherwise,}\end{cases} \tag{3}$$

is the quasilinear diffusion coefficient due to lower-hybrid waves with phase velocity between $-c$ and v_{ph}, and $D_0=(e/m)^2\epsilon_k/\lambda_D^3\omega_p$. This equation suffices for both runaway current and current driven by lower-hybrid waves, as both current carriers are fast electrons with $v\gg v_e$. Equation (2) can be solved for the runaway production rate,[6]

$$\dot{n}_R=A=\frac{\sqrt{2}}{\pi}n_0\nu\left(\frac{E_0}{E}\right)^{-3/2}\exp\left(-\frac{E_0}{4E}\right), \tag{4}$$

and the time variation of the current. Separately,

$$\begin{aligned}\frac{\partial j_{\text{rf}}}{\partial t}&=-e\int_{-c}^{v_{\text{ph}}}\frac{\partial f}{\partial t}v\,dv=-e\int_{-c}^{v_{\text{ph}}}v\frac{\partial}{\partial v}D_{\text{LH}}\frac{\partial f}{\partial v}dv+\frac{e^2E}{m}\int_{-c}^{v_{\text{ph}}}v\frac{\partial f}{\partial v}dv-e\int_{-c}^{v_{\text{ph}}}vC(f)\,dv\\ &=+e\int_{-c}^{v_{\text{ph}}}D_{\text{LH}}\frac{\partial f}{\partial v}dv-\frac{e^2E}{m}n_{\text{rf}}-\nu_{\text{rf}}j_{\text{rf}}+\frac{e^2E}{m}f(v_{\text{ph}})v_{\text{ph}},\end{aligned} \tag{5}$$

since $f(v=-c)=0$ and,

$$\frac{\partial j_R}{\partial t}=-e\int_{v_R}^{\infty}dv\,v\frac{\partial f}{\partial t}=\frac{-e^2E}{m}n_R-\nu_R j_R+\frac{e^2E}{m}[f(c)c-f(v_R)v_R], \tag{6}$$

where $n_\beta \equiv \int_\beta f\,dv$, $\nu_\beta = e\int_\beta vC(f)\,dv/j_\beta$, with β being the different components, and $(eE/m)f(c)=\dot{n}_R$ is just the runaway production rate given by Eq. (4), obtained from the boundary term in the integration by parts on $(eE/m)\int^c v(\partial f/\partial v)dv$. A similar equation exists for $\partial j_b/\partial t$. In the experimental situation, the runaways have finite lifetime as a result of imperfect confinement. If the confinement time τ_c is sufficiently long that electrons can be accelerated to c, i.e., $(eE/m)\tau_c > c$, then $f(c)$ is nonvanishing and this strong inductive effect given by the last term of Eq. (6) persists. If the confinement time is so short that $f(v)\to 0$ for $v \gg v_R$, then this effect vanishes.

Summing Eqs. (5) and (6) and using Eq. (1) we obtain

$$(1+\mathscr{L})\,dj/dt=V_{\rm rf}-\nu_{\rm eff}j+\dot{n}_R ec, \tag{7}$$

where

$$V_{\rm rf}=e\int_{-c}^{v_{\rm ph}}D_{\rm LH}(\partial f/\partial v)\,dv,$$

$$\nu_{\rm eff}\simeq e\int_{-c}^{c}vC(f)\,dv/J,$$

and

$$\mathscr{L}=(a^2\omega_p^2/4c^2)[\ln(8R/a)-\tfrac{7}{4}].$$

Note that all the boundary terms dropped out except for $\dot{n}_R ec$. Equation (7) is similar to the usual circuit equation except for the last term which has the effect of an inductance and is exponentially dependent on E and thus dj/dt through Eqs. (1) and (4). When the last term of Eq. (7) becomes comparable to the left-hand side, i.e., $E_0/4E-\frac{1}{2}\ln(E_0/E)=\ln c/v_e$ or $E/E_0\approx 10\%$ for typical parameters, we expect the nonlinear inductance to be significant in opposing further current ramping by reducing the electric field.

As an example, let us consider a case when high enough rf power is applied for current rampup. With the assumption that the electron distribution is close to thermal initially, $\dot{n}_R$ will take some time (typically $\gtrsim 100\nu^{-1}$) to approach the value given by Eq. (4). Equations (7) and (1) then predict an initial jump in $E=-V_{\rm rf}L/(1+\mathscr{L})$ followed by temporal decay due to dissipative and inductive effects. For $j_{\rm rf}\sim j_b \gg j_R$, $\nu_{\rm eff}\simeq\nu_b$. If we multiply Eq. (4) by a factor $(1-e^{-t/\tau_R})$, where $\tau_R=100\nu^{-1}$ is the runaway setup time, and treat $V_{\rm rf}$ as a given initial parameter, we find from Eqs. (7) and (1) that E experiences a much faster decay than in the absence of the runaway effect (Fig. 2). Since the current-rampup rate is proportional to E, this tends to saturate or even decrease the rampup efficiency.

However, for E/E_0 exceeding a few percent, depending on the ratio of plasma to cyclotron frequencies ω_p/Ω, the runaways can drive oblique plasma waves with $\omega=\omega_p k_\parallel/k$ unstable via anomalous Doppler resonance $\Omega+\omega=k_\parallel v_\parallel$ because of their anisotropy in energy distribution ($\epsilon_\parallel \gg \epsilon_\perp$), where $\epsilon_\parallel$ and $\epsilon_\perp$ are energy parallel and perpendicular to the magnetic field. The minimum parallel phase velocity of the unstable waves is about v_R.[7] This instability can effectively halt the runaway production by enhanced pitch-angle scattering of the resonant particles with $v_\parallel \gtrsim v_D = v_R\Omega/\omega_p$, therefore reducing the nonlinear

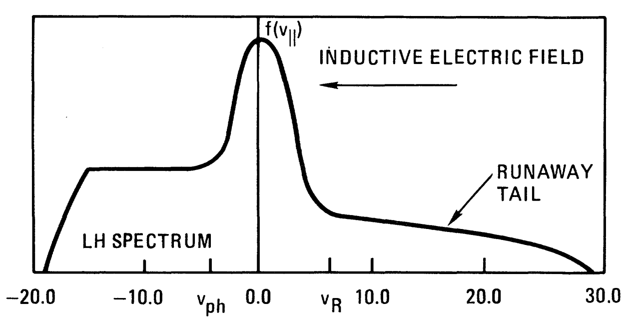

FIG. 1. Schematic of electron velocity distribution in LH-current ramping.

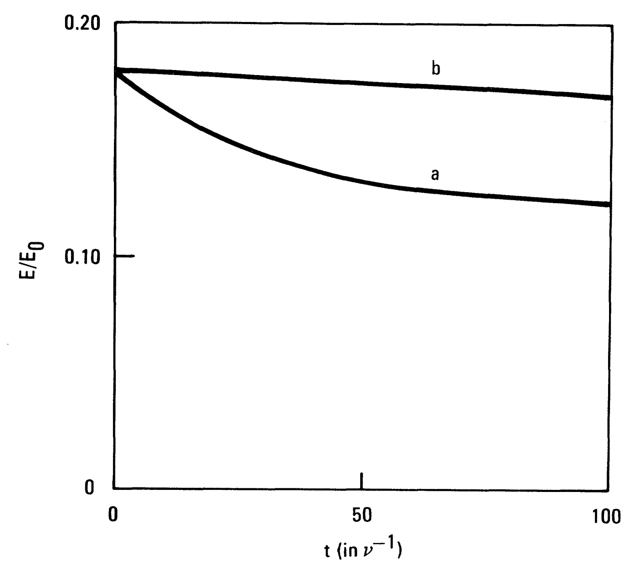

FIG. 2. Temporal evolution of inductive electric field with runaway correction (curve a) and without runaway correction (curve b) for $T_e=500$ eV, $n_0=2\times10^{12}$ cm^{-3}, $R=100$ cm, $a=20$ cm, $v_{\rm ph}-c/8$, $v_m=c/2$, and runaway setup time of $100\nu^{-1}$.

inductance and allowing current ramping at higher efficiencies. Thus, depending on ω_p/Ω, there may be a window in E/E_0 for which the plasma is stable and there is significantly reduced current ramping due to the large nonlinear plasma inductance.

The growth rate due to anomalous Doppler resonance must exceed the damping due to Landau and collisional damping $\nu/2$ for the mode to be unstable:

$$\gamma=\frac{\omega_k\pi^2}{2}\sum_{n=1,0}\frac{\omega_p^2}{k^2k_\parallel}\int_0^\infty d^2v_\perp j_n^2\left(\frac{k_\perp v_\perp}{\Omega}\right)\left(-\frac{n\Omega}{v_\perp}\frac{\partial}{\partial v_\perp}+k_\parallel\frac{\partial}{\partial v_\parallel}\right)f_0-\frac{\nu}{2}=\gamma_0+\gamma_1-\frac{\nu}{2}. \tag{8}$$

The stability boundary in E/E_0 and ω_p/Ω space has been reported by Liu and Mok[8] for a steady-state runaway distribution with a cutoff velocity (due to loss) $v_0\approx 12v_e$. For a given E/E_0, the instability is sensitively dependent on ω_p/Ω; for $\Omega/\omega_p=2$, instability can occur at $E/E_0\simeq 3\%$, and for $\Omega/\omega_p\simeq 3$, instability requires $E/E_0\gtrsim 10\%$. The instability boundary has also recently been verified by use of the two-dimensional Fokker-Planck equation; the detailed results will be published subsequently.[9]

The unstable plasma waves scatter the Doppler-resonant electrons with $v_\parallel=(\Omega+\omega)/k_\parallel$ in pitch angle, enhancing their perpendicular energy while reducing their parallel energy, resulting in a nearly isotropic distribution centered around $v_D=(\Omega/\omega_p)v_R$. This isotropization, when projected in the $v_\parallel$ direction, appears as a backflow against the electric field acceleration leading to a nonlinear instability of parallel-propagating plasma wave and effectively stopping the runaway production. To describe this quantitatively, we have to use a two-dimensional quasilinear Fokker-Planck equation,

$$\frac{\partial f}{\partial t}=-\frac{e}{m}E\frac{\partial f}{\partial v}+\frac{\partial}{\partial v_\parallel}D_{\rm LH}\frac{\partial}{\partial v_\parallel}f+\frac{\partial}{\partial v_\parallel}D_0\frac{\partial f}{\partial v_\parallel}+\left(\frac{\partial}{\partial v_\parallel}-\frac{v_\parallel}{v_\perp}\frac{\partial}{\partial v_\perp}\right)D_1\left(\frac{\partial}{\partial v_\parallel}-\frac{v_\parallel}{v_\perp}\frac{\partial}{\partial v_\perp}\right)f$$
$$+\frac{\partial}{\partial v_\parallel}\nu\left(v_\parallel f+v_e^2\frac{\partial f}{\partial v_\parallel}\right), \tag{9}$$

where

$$D_0=2\pi\int[d^3k/(2\pi)^3](k_\parallel/k)^2(W_k+W_{k_\parallel})\delta(\omega-k_\parallel v_\parallel),$$
$$D_1=(\pi/2)\int[d^3k/(2\pi)^3](k_\parallel/k)^2(k_\perp v_\perp/\Omega)^2W_k\delta(\Omega-k_\parallel v_\parallel),$$

are the quasilinear diffusion coefficients for Landau resonant particles with unstable waves (including nonlinearly unstable $k_\perp=0$ modes with energy $W_{k_\parallel}$ and linearly unstable $k_\perp\neq 0$ modes with energy W_k, and for Doppler resonant electrons, respectively.

By integration over $v_\perp$, Eq. (9) becomes

$$\frac{\partial F}{\partial t}=-\frac{e}{m}E\frac{\partial F}{\partial v_\parallel}+\frac{\partial}{\partial v_\parallel}(D_{\rm LH}+D_0)\frac{\partial F}{\partial v_\parallel}+\frac{\partial}{\partial v_\parallel}\bar{D}_1\left[\frac{\partial}{\partial v_\parallel}(TF)+v_\parallel F\right]+\frac{\partial}{\partial v_\parallel}\nu\left(v_\parallel f+v_e^2\frac{\partial f}{\partial v_\parallel}\right), \tag{10}$$

where $F\equiv 2\pi\int v_\perp d^2v_\perp f$, $(TF)\equiv 2\pi\int d^2v_\perp(v_\perp^3/2)f$, and $\bar{D}_1\equiv D_1(v_\perp^2/2)^{-1}$. In a steady state

$$\frac{e}{m}E\frac{\partial F}{\partial v_\parallel}-\frac{\partial}{\partial v_\parallel}\bar{D}_1\left[\frac{\partial}{\partial v_\parallel}(TF)+v_\parallel F\right]=0,\quad \text{for } c\geqslant v>v_D. \tag{11}$$

The electric field acceleration is balanced by the back flow due to pitch-angle scattering:

$$\frac{\partial}{\partial v_\parallel}D_0\frac{\partial F}{\partial v_\parallel}-\frac{e}{m}E\frac{\partial F}{\partial v_\parallel}+\frac{\partial}{\partial v_\parallel}\nu\left(v_\parallel F+v_e^2\frac{\partial F}{\partial v_\parallel}\right)=0,\quad \text{for } v_D>v>v_R. \tag{12}$$

Note that $E\,\partial f/\partial v_\parallel$ ensured that $\partial f/\partial v_\parallel\gtrsim 0$ for $v_D\geqslant v\geqslant v_R$ and there can be no Parail-Pogutse nonlinear oscillation. Integrating Eq. (11) over $v_\parallel$ when solving for F introduces the runaway production rate in the form of an integration constant which we set equal to zero. The reason which we have yet to justify is that runaways are negligible. With the assumption that $D_0\to\infty$ is appropriate for a high level of $k_\perp=0$ turbulence due to the nonlinear instability, the marginal stability condition $\gamma_1=-\gamma_0+\nu/2$ becomes

$$\partial(TF)/\partial v_\parallel+v_\parallel F=\alpha, \tag{13}$$

with $\alpha=2n_0\nu\Omega^2/\pi\omega_p^3\sin^2\theta_0\cos\theta_0$, as $\gamma_0\approx 0$. Here θ_0 is some propagation angle of the magnetized plasma waves, and $\bar{D}_1=FE/\alpha$ is determined by substitution of Eq. (13) into Eq. (11).

Equation (13) is sometimes solved by the assumption of an *Ansatz* for f in $v_\perp$ with guidance from numerical solutions of the Fokker-Planck equation. Here our purpose is just to demonstrate the possibility of clamping of runaway production by the instability and so instead we use the approximate solution obtained by Parail and Pogutse[7] which depends logarithmically on $v_\parallel$. Then

$$\bar{D}_1(v_\parallel)=\frac{FE}{\alpha}\simeq\left(\frac{\omega_p^3}{\Omega^2}\right)\left(\frac{E}{E_0}\right)e^{-E_0/4E}\ln\left[\frac{v_D(1-\omega_p^2/\Omega^2)}{(v_e^2+v_\parallel^2)^{1/2}-v_R}\right], \tag{14}$$

also decays logarithmically with $v_\parallel$ and is very sensitive to E_0/E. The slower than $1/v_\parallel$ decay of $\bar{D}_1$ implies that the wave-enhanced pitch-angle scattering can be very effective in reducing runaway production when E/E_0 exceeds a few percent. We have used Eq. (14) in the Fokker-Planck equation and numerically verified that this is the case.[9]

In conclusion, the large inductance due to runaway production and the corresponding increase in current carriers can effectively clamp the current-ramping efficiency by lower-hybrid waves if the runaways are well confined. The plasma instability due to the anisotropic energy distribution ($\epsilon_\parallel >> \epsilon_\perp$) in runaway electrons, however, helps to stop the runaway production and ensures the efficient current ramp. Depending on the value of ω_p/Ω, it is possible in present experiments for E/E_0 to exceed the threshold for large runaway production with no instability, resulting in ineffective current ramping. A refined estimate of the electric field threshold can be obtained from a two-dimensional Fokker-Planck solution of the runaway production rate.[10]

In the case of current decay, the LH waves and the induced electric field accelerate electrons in the same direction, greatly enhancing the runaway production rate and destabilizing plasma waves.

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