Evidence against a Stable Dibaryon from Lattice QCD

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We have used standard numerical techniques of lattice quantum chromodynamics to look for evidence of the proposed doubly strange spin-zero dibaryon (the H particle), and to determine the splitting between the mass of the H and the mass of two Λ's, its lightest possible strong-decay channel. We find that the dibaryon is above the two-Λ threshold, making it unstable to strong decay.

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The reliability of lattice-gauge-theory calculations is in the process of developing from the level of quark-model-style phenomenology with little understanding of expected errors toward that of a solid calculational method. Understanding the reliability of the calculations independently of direct comparison with data involves complicated and detailed error analysis which is difficult for nonspecialists to evaluate. It would be a striking test of the emerging calculational methods to predict accurately a physical quantity before its experimental determination. Since the low-energy hadron physics which is best understood with the present methods is well explored experimentally, candidates for such a prediction are not plentiful.

A very clean candidate for such a test, if it is stable, is the doubly strange dibaryon, the H particle. This flavor-singlet, spin-zero six-quark state was shown by Jaffe[1] to have the largest possible color-hyperfine attraction among the six quarks. It was predicted by him on the basis of a bag-model calculation to have a binding energy of 80 MeV relative to the $\Lambda\Lambda$ threshold, making it stable against decay by strong interactions. The mass of a light, stable hadron can in principle be determined very accurately by experiment. In addition, the masses of the light-quark hadrons are the measurable quantities which can be calculated most accurately with the established techniques of numerical lattice gauge theory. In addition to its great intrinsic importance as a new stable light-quark hadron, its existence would shed valuable light on the possibility of the existence of stable bulk strange matter.[2] It has recently been speculated that the H may be so tightly bound that its mass is below the $N\Lambda$ threshold, making it stable against single weak decay.[3] In this case, it is a possible source for unusual cosmic-ray events recently reported from Cygnus X-3.[3] Production cross sections for doubly strange particles are not very large, and so it is conceivable that such a long-lived particle could exist and yet not have been seen so far.[4]

The H has been considered in a wide variety of phenomenological models and has always been found to be either stable or almost so. In addition to Jaffe's original estimate of 80 MeV, more detailed bag-model calculations have given binding energies of 230[5] to -10 MeV.[6] Dibaryons have recently attracted attention in chiral models, where they have an interesting interpretation as solitons associated with an SO(3) subgroup of flavor SU(3). Several estimates of the H mass have been done in chiral models, which are somewhat less successful than quark models in parametrizing the known hadron masses. These range from 1.03 to 2.10 GeV,[7] compared with the $\Lambda\Lambda$ threshold at 2.23 GeV.

In the SU(3)-symmetry limit, we have for the hyperfine splitting

$$\Delta E \propto - \sum_{\substack{i<j \\ \text{quark pairs}}} (\lambda^i \sigma^i)\cdot(\lambda^j \sigma^j), \tag{1}$$

where the λ^i and σ^i are the Gell-Mann and Pauli matrices. Jaffe showed how to compute this quantity without considering the details of the six-quark wave functions in terms of the Casimir operator of the combined color-spin SU(6) group, and obtained for the H particle a minimum eigenvalue of -24 from the **490** representation of color-spin SU(6). He obtained a binding-energy estimate of 80 MeV in the bag model. The stability of the H in quark models is simplest to see in a naive-quark-model calculation with hyperfine interactions. The hadron masses are fitted with the form

$$M = \sum_{\substack{i \\ \text{quarks}}} m_i - \sum_{\substack{i<j \\ \text{quark pairs}}} \frac{(\lambda^i\sigma^i)\cdot(\lambda^j\sigma^j)}{m_i m_j} W, \tag{2}$$

where W parametrizes the wave function at the origin and the QCD coupling constant, and m_i are the masses of the quarks. With use of the color-spin-flavor wave function of the H, which we derive later, the factor $-24/m^2$ from the eigenvalue of Eq. (1) becomes $-5/m_u^2 - 22/m_u m_s + 3/ms^2$ when flavor-SU(3)-symmetry breaking is introduced. Using the parameters of Rosner,[8] which fit the baryon spectrum quite well, and making the unjustified assumption that W is the same for baryons and dibaryons, we obtain a hyperfine splitting for the H of -348 MeV for the second term in Eq. (2). This yields a mass of 2.18 GeV and a binding energy of 50 MeV.

To examine the H in lattice QCD, we analyzed the quark-propagator data accumulated for the calculation of hadronic coupling constants performed with Gottlieb and Weingarten.[9] For a detailed description of the definitions, methods, and results in that calculation, as well as a list of standard references for lattice gauge theory, see Ref. 9. We use a coupling constant $\beta \equiv 6/g^2 = 5.7$, which corresponds to a lattice spacing of roughly 0.9 GeV^{-1}, depending on the quantity used to set the mass scale. We worked on a $6^2 \times 12 \times 18$ lattice, with 18 taken as the Euclidean time direction. The transverse size of 6 is roughly the size of a single hadron. A total of twenty gauge configurations was analyzed. Each configuration was separated by 500 Metropolis sweeps, after equilibration for 1000 sweeps. Quark propagators were calculated for hopping parameters $K = 0.325$, 0.34, and 0.355, which correspond to pions of mass around 900, 750, and 600 MeV, respectively. The hadron masses obtained for these quark masses must be extrapolated to the correct physical limit of $m_\pi = 138$ MeV. We will analyze a scaled mass splitting for the H which is not very sensitive to the extrapolation. The valence approximation was used, with the effects of internal quark loops ignored.

Hadron masses are obtained from the two-point functions of multiquark operators having the same quantum numbers as the hadron in question. The long-distance exponential falloff is determined by the energy of the lowest-energy hadron state to which the operator couples. Since hadron masses were not the primary goal of the calculations of Ref. 9, detailed analyses of mass splittings were not presented. For comparison with the results of the H calculation, we show in Fig. 1 the scaled mass splitting $[(M_\Delta - M_{\mathrm{nuc}})/M_{\mathrm{nuc}}] \times (938\ \mathrm{MeV})$, as a function of the Euclidean time. The relative splitting rescaled to megaelectronvolt units is relatively insensitive to the effects of extrapolation in the quark mass. The effective mass at a given Euclidean time is defined as the logarithm of the ratio of the values of the two-point function at adjacent time slices. This should approach the mass of the lowest lying hadron state asymptotically in the limit of large time. At very short times, the propagators fall off very rapidly. The falloff is dominated by the spreading of the almost free quarks, so that little splitting between hadrons with the same quark content is observed at short times. At larger times, dynamical effects become important and the splitting rises to an asymptotic value. Statistical errors are estimated from

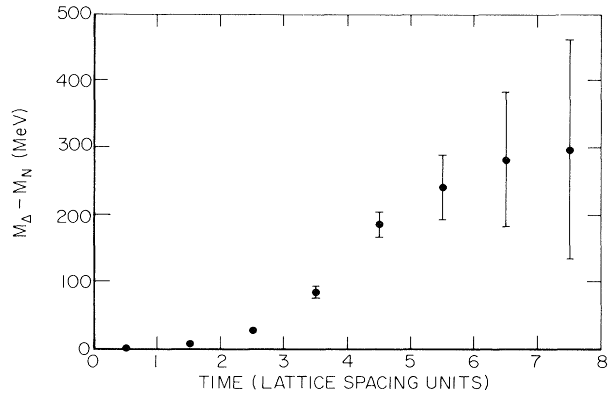

FIG. 1. The Δ-nucleon mass splitting as a function of the Euclidean time, extrapolated to the physical quark masses. Vertical lines are statistical errors.

the fluctuations of analyses performed on data sets with one lattice at a time removed (the "jackknife method"). Removal of small sets of contiguous lattices allows testing for the presence of correlations between gauge configurations, as described in Ref. 9. Negligible correlation was found for masses with 500 sweeps separating the configurations. The mass splittings for the six baryons not used as inputs agreed with experiment to well within the statistical errors of (20–30)%. The $\pi\rho$ splitting was too low by 30%; the KK^* splitting was too low by over a factor of 2. The $N\rho$ mass ratio was too large by 25%. These results will have to serve as a rough guide to the reliability of the calculations in the absence of a solid analysis of all sources of error in the lattice calculations.

An explicit expression for the quark-model wave function of the H is required for the lattice calculation. This is much more complicated than the wave functions for the two- and three-quark hadrons. The color and spin part of the H wave function is the color-singlet, spin-singlet part of the **490** representation of the combined color-spin SU(6) group.[1] The **490** is represented by the Young tableau with two rows of three boxes each. The color and spin wave function may be obtained by symmetrization of the color and spin indices of two trios of quarks, yielding spin-$\frac{1}{2}$ color octets and spin-$\frac{3}{2}$ color decuplets. Only the two octets may be combined to make an overall color singlet. When the color and spin indices of the two spin-$\frac{1}{2}$ color octets are combined into a color singlet and spin singlet, three pairs of indices from the two octets are antisymmetrized as required by the **490** symmetry. Flavor indices are then arranged to obtain Fermi symmetry, yielding a flavor singlet. This results in a huge number of explicit terms. The correctness of this expression was checked by our applying the color-spin operator of Eq. (1) in explicit form to it to obtain the correct eigenvalue, -24. The number of terms in the wave function is squared in the two-point-function calculation, making the analysis program with this expression much too time consuming. A more tractable form of the wave function may be obtained by the reexpression of it in terms of quarks of the same flavor. The quarks in each pair must be antisymmetric in overall color and spin, and so must be in either spin-0 color sextets or spin-1 color $\mathbf{3}^*$'s. The allowed combinations are three sextets, three $\mathbf{3}^*$'s, or two $\mathbf{3}^*$'s and a sextet. There are 138 nonzero terms in all (out of a possible 15^3). The 15×15 flavor-pair propagators may be constructed relatively quickly at each lattice site from the quark propagators. These are combined with the flavor-pair wave function into the full wave function. The calculation of the H two-point function from the quark propagators performed in this way required two weeks of VAX 11/780 central-processing-unit time, compared with a few days for all the rest of the spectrum and coupling-constant analysis combined. We may also obtain the flavor-pair wave function directly by starting from an arbitrary combination of trios of flavor pairs in color and spin singlets and using isospin-raising operators to find the combination which gives a flavor singlet.[10] The **490** symmetry then follows from Fermi statistics. Because of the complexity of the wave function, the correctness of the derivation and programming was checked by our deriving and programming the wave function along the two completely separate routes: the flavor-pair basis and the quark basis. The results were checked and agreed at an intermediate step and in the output of the analysis programs.

If the H exists as a stable particle, it is thought to be a tightly bound six-quark state, with a radius possibly not much larger than the radii of the ordinary hadrons. On the lattice in the infinite-volume limit, we should find the pole for the H in its proper place. In a finite volume, if the H really is tightly bound, finite-volume errors should be comparable to those for the ordinary hadrons. If the H is unstable, the dominant singularity in the H two-point function will be that of the lightest physical state to which it couples, at $2M_\Lambda$. If the H prefers to exist as a pair of independent Λ's, finite-volume effects may be very large when the two Λ's are squeezed into a lattice barely big enough to fit a single hadron, making the dominant singularity in the H propagator appear to be above $2M_\Lambda$.

Our results are shown in Figs. 2 and 3. Figure 2 shows the H two-point function as a function of the Euclidean time, with $K=0.355$ for the light quarks and 0.34 for the strange quark. For comparison, the square of the Λ two-point function is also shown. At short times, the two plots fall rapidly and almost identically. The falloff is dominated by the effects of the almost free quarks spreading out from the local operator which creates the state. In Fig. 3, we show the relative mass splitting $[(M_H-2M_\Lambda)/M_\Lambda]\times(1115\text{ MeV})$ obtained from adjacent time slices, extrapolated to the physical-quark-mass limit. At short times, the splitting is very small, but always positive. A large positive splitting develops asymptotically. The qualitative effect is very insensitive to quark mass. Very similar graphs are obtained for all quark masses used in the calculation, as well as for the extrapolated results. As a further check, we extrapolated all three flavors of quarks to the chiral-symmetry limit and obtained almost identical results for the splitting. This is in contrast to the calculations in chiral models which are extremely sensitive to the details of chiral-symmetry breaking.[11]

The fact that the singularity in the H two-point function appears above rather than at $2M_\Lambda$ is a finite-volume effect, and so there are clearly large finite-volume errors in this calculation. On the other hand,

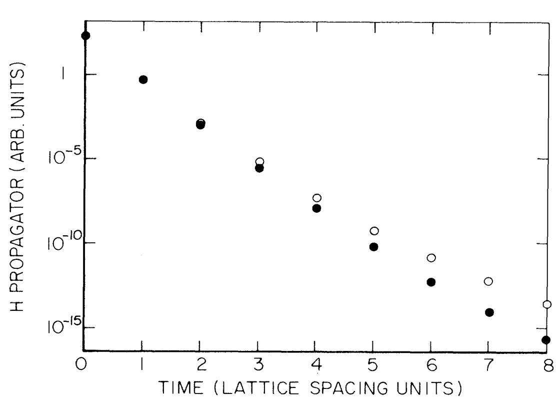

FIG. 2. The H two-point function (black dots) and the square of the Λ two-point function (white dots) as a function of Euclidean time. $K=0.355$ for the light quarks and 0.34 for the strange quark.

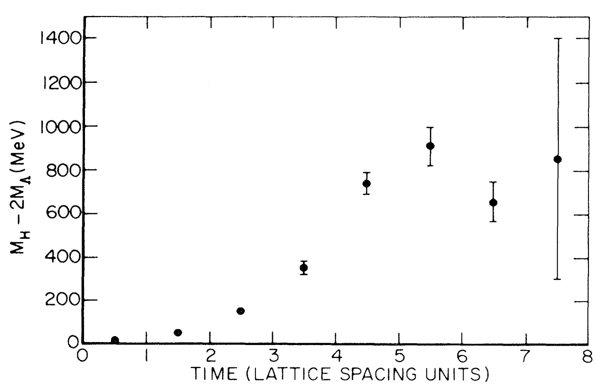

FIG. 3. The mass splitting between the H and two Λ's extrapolated to the physical quark masses. Vertical lines are statistical errors. The splitting is positive for all time separations and for all quark masses used in the calculation.

finite-volume errors on a tightly bound H would not necessarily act to decrease its splitting. In a study of finite-volume effects in the sine-Gordon model with periodic boundary conditions,[12] the binding was increased as the volume decreased and the particles were pushed deeper into their potential well. Furthermore, the same sign of the splitting is observed in our data at short times before the quarks have spread out enough to feel the effects of the finite volume. Another source of uncertainty is the question of whether the lattice is long enough in the time direction that we are seeing the asymptotic form of the splitting. The last four data points in Fig. 3 are consistent with being flat, but because of the large statistical error of the last point the last three are also consistent with a falling splitting as time increases. This raises the question of whether the very-large-time behavior might be different from the short- and intermediate-time behavior. This type of behavior might be expected if the H existed as a stable, lightly bound deuteronlike object. (This is not a very likely possibility, on the basis of meson-exchange potential-model calculations.[13]) A potentially serious way that this calculation could go wrong is in an underestimation of the splitting between the H and the center of the dibaryon multiplets. The H is stable in quark-model estimates precisely because of a very large spin splitting of this sort (about -350 MeV). In the spectroscopy data described above, most of the spin splittings for the known hadrons agreed well with experiment, but two of them were low by up to a factor of 2. A worst-case scenario for this calculation to go wrong might be that the H is somewhat bound in real life, a bad misestimate of the spin splitting makes it somewhat unbound on the lattice with our approximations, and finite-volume errors magnify that effect into the very large splitting seen in our data. We do not consider this likely, but it cannot be excluded.

To sum up, we do not see a negative splitting between the H and two Λ's at any combination of quark masses or at any separation of the hadron operators, although as we have pointed out, there are ways that this calculation could go wrong. Much better lattice calculations for the H will be possible in the near future. Although the present calculation lends no support to the attractive possibility of a stable H, more work, both on the lattice and in experiment, is clearly desirable.

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[10]We thank W. A. Bardeen for showing us this method.

[11]We thank Chiara Nappi for a conversation on this point.

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