Evidence for Anderson-Brinkman-Morel–Type State in a Heavy-Fermion Superconductor from Ultrasonic Attenuation

J. P. Rodriguez
Loomis Laboratory of Physics, University of Illinois at Urbana-Champaign, Urbana, Illinois 61801
(Received 4 February 1985)

The ultrasonic attenuation for a superconductor with generalized pairing is calculated via a scalar quasiparticle Boltzmann equation in the relaxation-time approximation, in the hydrodynamic limit. The attenuation follows a power law as a function of temperature T if the gap vanishes at points or lines on the Fermi surface. With an Anderson-Brinkman-Morel–type excitation spectrum, a T^2 behavior is obtained for the sound attenuation, which is consistent with the recently measured attenuation for the heavy-fermion system UPt_3.

PACS numbers: 74.30.Gn, 74.20.–z, 74.70.Rv

Ultrasonic attenuation measurements have traditionally been used in the study of superconducting metals to probe the nature of the gap. Recently, Bishop *et al.*[1] have measured the attenuation of sound in UPt_3, a heavy-fermion superconductor,[2] and have found that the normalized attenuation obeys a T^2 power law, where T is temperature. This result is inconsistent with the predictions of conventional Bordeen-Cooper-Schrieffer (BCS) theory with singlet pairing and isotropic gap. Bishop *et al.*[1] indicate that these measurements can be explained by odd-parity triplet-pairing superconductivity in UPt_3. In this Letter, we show that a p-wave triplet gap does yield a power law for the attenuation as a function of temperature. In particular, we find that at low temperature T, gap states with Anderson-Brinkman-Morel–type (ABM) excitation spectrums give attenuation proportional to T^2, while gap states with polarlike excitation spectra yield attenuation proportional to T. The latter result is in contrast to the statement made by Bishop *et al.*[1] that polarlike states give T^2 behavior in the attenuation.

There is strong evidence that the measurements on UPt_3 were performed in the hydrodynamic limit, $\omega\tau << 1$, where ω is the sound frequency and τ is the characteristic electron relaxation time. For example, the attenuation has an ω^2 frequency dependence above and below the critical temperature, and it varies roughly as the electrical conductivity on the normal-metallic side.[1] We therefore assume that propagation of sound in superconducting UPt_3 is hydrodynamic. Ultrasonic attenuation in the hydrodynamic limit is due to viscous dissipation by conduction electrons, and to obtain it we calculate the viscosity tensor for a superconductor with generalized pairing.

In principle, the near-equilibrium physics of a superconductor can be described by a matrix generalization of Landau's kinetic equation for normal Fermi liquids.[3] We assume, however, that the sound perturbation leaves Bogoliubov quasiparticles well defined, which requires that $\omega << |\Delta|$, where Δ is the gap parameter (we assume "unitary" gap, $\Delta\Delta^\dagger \propto 1$). Then the matrix equation transforms into a scalar kinetic equation for the variation of the distribution of Bogoliubov quasiparticles $\delta f_p(\mathbf{r},t)$,

$$\partial(\delta f_p)/\partial t + \nabla_p E_p \cdot \nabla_r [\delta f_p - (\partial f^0/\partial E)\delta E_p] = -\delta\tilde{f}_p/\tau_p. \quad (1)$$

In Eq. (1), we approximate the collision integral by the relaxation-time approximation. Here $\delta\tilde{f}_p$ measures the deviation from local equilibrium[4] of the quasiparticle distribution; $\epsilon_p = (p^2/2m^*) - \mu$ and $E_p = (\epsilon_p^2 + |\Delta_p|^2)^{1/2}$ are the normal and BCS quasiparticle energies; and $f^0(E) = [1+\exp(E/k_B T)]^{-1}$ is the equilibrium distribution function at temperature T. The change in quasiparticle energy is given by

$$\delta E_p = \mathbf{p}\cdot\mathbf{v}_s + (\epsilon_p/E_p)(f_0 \delta n - \delta\mu). \quad (2)$$

Here $\mathbf{v}_s$ is the superfluid velocity, δn is the change in the density from the equilibrium density, $n = k_F^3/3\pi^2$ (k_F is the Fermi wave vector), and $\delta\mu$ is the variation in the chemical potential. $F_0 = N_F f_0$ is the usual spin symmetric Landau parameter, where $N_F = k_F m^*/\pi^2\hbar^2$ is the density of states at the Fermi level (we have neglected the F_1 and higher contributions to δE_p because they do not contribute significantly to the viscosity; in fact, neither will F_0). The quasiparticle momenta $\mathbf{p}$ are measured with respect to $m\mathbf{v}_s$, where m is the band mass.

We now write the distribution at local equilibrium as

$$\delta f_p^{\mathrm{le}} = (\partial f^0/\partial E)(\delta E_p - \mathbf{p}\cdot\mathbf{v}_n), \quad (3)$$

where $\mathbf{v}_n$ is the velocity of the "normal" flow, which in a metal is equal to the velocity of the lattice. Note that we have neglected temperature fluctuations and have taken $\delta|\Delta_p| = 0$, which is a good approximation within corrections of $|\Delta|^2/\epsilon_F^2$ in the attenuation. We are dealing with nonmagnetic phenomena and so have dropped spin indices. Now, in the hydrodynamic limit we let $\delta f_p = \delta f_p^{\mathrm{le}}$ and $\delta\mu = \delta\mu^{\mathrm{le}} = \delta n(\partial\mu/\partial n)_{\mathrm{le}} = \delta n(1+F_0)/N_F$ on the left side of Eq. (1). Then with the help of the linearized continuity equation we obtain

$$\delta\tilde{f}_p = \tau_p v_F p_F \left(\frac{\epsilon_p}{E_p}\right)\left(\frac{\partial f^0}{\partial E}\right)\left(\hat{p}_i\hat{p}_j - \frac{\delta_{ij}}{3}\right)\partial_i v_j^n, \tag{4}$$

where p_F and v_F are respectively the Fermi momentum and the Fermi velocity and $\hat{\mathbf{p}}$ is the unit vector along $\mathbf{p}$. In the above expression for $\delta\tilde{f}_p$ we have dropped terms that yield superfluid viscosities.[5] We now write the stress tensor in the form $\pi_{ij} = \pi_{ij}^{\rm le} - \sigma_{ij}$, where $\pi_{ij}^{\rm le}$ is the local equilibrium part and where σ_{ij} is the dissipative part given by[6]

$$\sigma_{ij} = -(\partial\pi_{ij}/\partial\mu)_{\rm le}\delta\tilde{\mu} - \sum\nolimits_{ps}(p_i p_j m^{*-1} + \delta_{ij} f_0 n)(\epsilon_p/E_p)\delta\tilde{f}_p. \tag{5}$$

Here $\delta\tilde{\mu}$ measures the deviation of the chemical potential from its local equilibrium value. Since the local values of $\delta\mu$ and δf_p are chosen so that they give the correct local density and current, we require that the contribution which $\delta\tilde{f}_p$ and $\delta\tilde{\mu}$ make to δn vanishes; i.e.,[7]

$$(\partial n/\partial\mu)_{\rm le}\delta\tilde{\mu} + \sum\nolimits_{ps}(\epsilon_p/E_p)\delta\tilde{f}_p = 0, \tag{6}$$

from which we obtain $\delta\tilde{\mu}$. We further write $\pi_{ij}^{\rm le} = \delta_{ij}P$, where P is the pressure. Then by the Gibbs-Duhem relation $(\partial\pi_{ij}/\partial\mu)_{\rm le} = n\delta_{ij}$. After solving for $\delta\tilde{\mu}$ from Eq. (6), substituting it, along with Eq. (4), into Eq. (5), and replacing $\sum_{ps}$ by $\int(d\Omega/4\pi)d\epsilon N_f$ in Eq. (5), we obtain for the viscosity tensor

$$\eta_{ijkl} = -3np_F\int\frac{d\Omega}{4\pi}d\epsilon\, l_p\frac{\partial f^0}{\partial E}\frac{|\epsilon|}{E}(\hat{p}_i\hat{p}_j - \tfrac{1}{3}\delta_{ij})(\hat{p}_k\hat{p}_l - \tfrac{1}{3}\delta_{kl}), \tag{7}$$

where $\sigma_{ij} = \eta_{ijkl}\partial_k v_l^n$. Here $l_p \equiv (|\epsilon_p|/E_p)v_F\tau_p$ is the effective mean free path associated with the degradation of strain in the Fermi liquid of a quasiparticle of momentum $\mathbf{p}$. Note that expression (7) for the viscosity tensor is independent of F_0.

The attenuation of sound with wave vector $\mathbf{q}$ and polarization $\hat{\mathbf{u}}$ (a unit vector) is given by $\alpha = (q^2/\rho c_s)\bar{\eta}$,[8] with $\bar{\eta} = \eta_{ijkl}\hat{u}_i\hat{q}_j\hat{q}_k\hat{u}_l$; ρ is the mass density and c_s is the speed of sound related to F_0 by[6] $c_s^2 = (m^*n/3\rho)v_F^2(1+F_0)$. Consider the case in which l_p is given by a constant l. Then, for longitudinal polarizations, we obtain for the attenuation, normalized to its value in the normal state,

$$\frac{\alpha}{\alpha_n} = \frac{5}{2}\frac{l}{l_n}\int\frac{d\Omega}{4\pi}f^0(|\Delta_p|)[1 - 3(\hat{\mathbf{p}}\cdot\hat{\mathbf{q}})^2]^2. \tag{8}$$

Expressions similar to Eq. (8) in form have been obtained previously by Combescot in the context of superfluid ^{3}He, but in the large F_0 limit.[9] Above, $\alpha_n = \frac{4}{15}(q^2/\rho c_s)p_f l_n n$ is the attenuation of longitudinal sound in a normal metal. The assumption of a constant mean free path is clearly valid for an isotropic gap. However, though in general the mean free path l_p will vary as a function of the quasiparticle momentum if the gap is anisotropic, Eq. (8) remains valid at low temperatures compared to the critical temperature T_c, for gaps with nodes on the Fermi surface. In that case, the major contributions to the viscosity in expression (7) come from the regions of momentum in the vicinity of the nodes, and l is then the mean free path of those low-lying excitations. Of course, we have implicitly assumed that the mean free path is finite at these nodes, which is required in the limit of hydrodynamic sound propagation, where in fact $ql \ll 1$. Evaluating (8) for an isotropic gap, an ABM-like gap, and a polar-like gap, we obtain the following results:

$$|\Delta_p| = \Delta,\quad l = l_n:\quad \alpha/\alpha_n = 2f^0(\Delta),\quad \text{for all } T. \tag{9a}$$

$$|\Delta_p| = \Delta(1-\hat{p}_z^2)^{1/2}:\quad \begin{aligned}\alpha_\parallel/\alpha_n &= (5\pi^2/6)(l/l_n)(k_BT/\Delta_0)^2 + O(T^4),\\ \alpha_\perp/\alpha_n &= (5\pi^2/24)(l/l_n)(k_BT/\Delta_0)^2 + O(T^4),\quad \text{for } T\to 0.\end{aligned} \tag{9b}$$

$$|\Delta_p| = \Delta|\hat{p}_z|:\quad \begin{aligned}\alpha_\parallel/\alpha_n &= \tfrac{5}{2}(\ln 2)(l/l_n)(k_BT/\Delta_0) + O(T^3),\\ \alpha_\perp/\alpha_n &= \tfrac{55}{16}(\ln 2)(l/l_n)(k_BT/\Delta_0) + O(T^3),\quad \text{for } T\to 0.\end{aligned} \tag{9c}$$

The subscripts $\parallel$ and $\perp$ denote, respectively, $\hat{\mathbf{q}}$ along and perpendicular to $\hat{\mathbf{z}}$. Notice that expression (8) gives an infinite slope for $(\alpha/\alpha_n)(T)$ at T_c, as long as $|\Delta_p| \sim T_c(1-T/T_c)^{1/2}$ for T near T_c.

We see that the anisotropic gap states give power laws for the normalized attenuation as a function of temperature. In particular, the ABM-like gap displayed in Eq. (9b) is consistent with the low temperature T^2 dependence of α/α_n in UPt_3, as long as the ratio l/l_n is finite and nonzero at $T=0$. The inclusion of momentum dependence in the mean free path l_p only produces higher-order corrections, in powers of T/T_c, to results (9a)–(9c). A fit of the re-

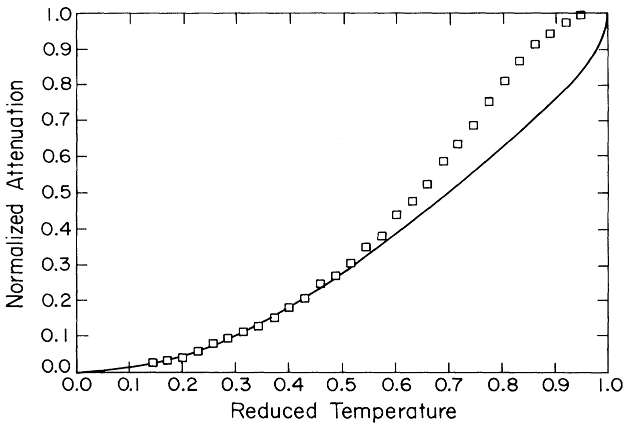

FIG. 1. The theoretical normalized attenuation vs normalized temperature for ABM-like gap with compression along axis of azimuthal symmetry. Also plotted are data for attenuation in UPt_3 at 508 MHz obtained from Bishop *et al.*, Ref. 1 (squares).

cent data on UPt_3 using this gap state in Eq. (8), with $\mathbf{q}$ along $\hat{\mathbf{z}}$, over the entire temperature range appears in Fig. 1. The temperature dependence of the gap has been approximated by the following interpolation:

$$\Delta(T) = \Delta_0 \tanh\left\{\pi\left(\frac{k_B T_c}{\Delta_0}\right)\left(\frac{\Delta C}{C_n}\right)^{1/2}\left[\left(\frac{T_c}{T}\right)-1\right]^{1/2}\right\}.$$

Here we have taken $\Delta_0/k_B T_c = 2.9$, the specific heat jump as $\Delta C/C_n = 0.5$, $l/l_n = 1$, and have evaluated the expression in Eq. (8) numerically. The fit is quite good, except near T_c. As mentioned before, our expression for the normalized attenuation as a function of temperature does not yield the finite slope at T_c that is observed experimentally. It is possible that this excess attenuation near T_c could be a result of collective oscillations of the superconducting order parameter[10] which we have not accounted for in this calculation.

In conclusion, we find good agreement with the recent ultrasonic attenuation measurements on UPt_3 if we assume an ABM-type quasiparticle excitation spectrum. Here, we disagree with the statement made by Bishop *et al.*[1] that a T^2 law for the attenuation below T_c is typical for a polarlike state. It should be borne in mind that the ABM pairing is not the only generalized gap state that has an ABM-type excitation spectrum. In fact the spin-orbit and crystal-field energies in UPt_3 are strong, and therefore the gap parameter $\Delta_{\alpha\beta}(\mathbf{k})$ must transform as an irreducible representation of the crystal symmetry, where real space and spin space are rotated simultaneously because of the $\mathbf{J}$ symmetry.[11] What this calculation does suggest is that superconducting UPt_3 has a gap that vanishes at *points* on the Fermi surface in a manner similar to the ABM state, though the pairing may not be precisely ABM.

I would like to thank Professor Gordon Baym for suggesting this calculation and for providing helpful guidance through to its completion. I would also like to thank Professor A. J. Leggett, Professor C. J. Pethick, and Dr. C. Yu for illuminating conversations on this material. This work has been supported in part by the National Science Foundation under Grant No. DMR 84-15063.

[1]D. J. Bishop, C. M. Varma, B. Batlogg, E. Bucher, Z. Fisk, and J. L. Smith, Phys. Rev. Lett. **53**, 1009 (1984).

[2]G. R. Stewart, Rev. Mod. Phys. **56**, 755 (1984).

[3]O. Betbeder-Matibet and P. Nozières, Ann. Phys. (N.Y.) **51**, 392 (1969).

[4]For a definition of the concept of local equilibrium, see G. Baym and C. J. Pethick, in *The Physics of Liquid and Solid Helium Part II,* edited by K. H. Bennemann and J. B. Ketterson (Wiley, New York, 1978), p. 1.

[5]I. M. Khalatnikov, *An Introduction to the Theory of Superfluidity* (Benjamin, New York, 1965).

[6]Baym and Pethick, in Ref. 4.

[7]This is a consequence of the fact that

$$\delta n = \sum_{ps}\left[\frac{\epsilon_p}{E_p}\delta f_p - \tanh\left(\frac{E_p}{2k_B T}\right)\frac{|\Delta_p|^2}{2E_p^3}\delta\epsilon_p\right]$$

for $\delta|\Delta_p| = 0$, with a unitary equilibrium gap. Here $\delta\epsilon_p$ measures the change in the normal quasiparticle energy with respect to the chemical potential. See P. Wölfle, Prog. Low Temp. Phys. **7a**, 191 (1978).

[8]We have neglected to include an additional term in the sound attenuation arising from thermal conductivity. It is small compared to the viscosity term for typical values of the relevant parameters in heavy-fermion systems. See L. Landau and E. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1984).

[9]R. Combescot, Phys. Rev. B **12**, 4839 (1975).

[10]Wölfle, Ref. 7.

[11]P. W. Anderson, Phys. Rev. B **30**, 4000 (1984).