Momentum Distribution of ^{3}He

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$S(Q,E)$ has been obtained for liquid ^{3}He for Q values between 4 and 7 $\AA^{-1}$ at temperatures of 1.2 and 0.37 K. The higher-temperature data are useful in examination of the role of final-state effects on measurements of $S(Q,E)$. Fermi-liquid properties can be determined from the low-temperature results and the data are consistent with a Fermi temperature of 1.8 $\pm$0.2 K.

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Liquid ^{4}He and ^{3}He are materials whose properties are determined to a large extent by quantum effects. Of particular importance is the fact that liquid ^{4}He is subject to Bose-Einstein statistics while ^{3}He is a Fermi liquid. Of course, interactions between atoms in the liquid insure that there is considerable modification from the statistical behavior of an ideal gas. Inelastic neutron scattering is, in principle, capable of measuring the single-particle momentum distribution of these materials which then can be directly compared to theoretical calculations. The original proposal for the scattering measurements was set forth by Hohenberg and Platzman[1] in 1966 to examine Bose-Einstein condensation in ^{4}He. In the limit of high energies the so-called impulse approximation (IA) is valid so that the helium particle momentum is determined in a straightforward way by the conservation of energy and momentum in the scattering process. However, it is becoming apparent that very high neutron energies are needed for the IA to be strictly valid, and with high neutron energies the energy resolution must be extremely good to observe the effects of interest which are on the scale of a few millivolts. It appears then that a more fruitful approach is to use moderate neutron energies and to compensate properly for the fact that the IA is not strictly valid. This point has been most recently considered by Platzman and Tzoar[2] who show that the leading correction to the IA is a small shift of the center of the scattered distribution and a nonsymmetric broadening which can be approximated by a Lorentzian width.

An important step in this approach was made by Woods and Sears[3] and Sears *et al.*[4] in that they showed that by proper analysis of a set of scattering data acquired at a series of momentum transfers Q, effects of the nonvalidity of the IA could be minimized. Indeed, previous measurements on ^{4}He made at only a single momentum transfer for momentum transfers up to 15 $\AA^{-1}$ had yielded a variety of differing results while now excellent agreement[5] has been obtained on data sets obtained at different laboratories by use of the analysis techniques of Refs. 3 and 4. In this Letter we present results for ^{3}He obtained by the technique utilized in the ^{4}He measurements and analyses. Sufficient resolution has been employed that Fermi-liquid properties of ^{3}He could be observed, and the first neutron-scattering measurement of the Fermi energy for ^{3}He is shown.

The large absorption cross section for neutrons for ^{3}He (10 000 b) makes measurements very difficult, and obviously the the quality of the data obtained is not as good as that for ^{4}He. The single-particle peak for ^{3}He has been observed by Sokol *et al.*[6] for Q values around 15 $\AA^{-1}$. Unfortunately, sufficient resolution was not available to allow the observation of the details of $n(p)$ such as the Fermi surface. The present measurements were made with the ultrasonically pulsed neutron time-of-flight spectrometer located at the High-Flux Isotope Reactor at Oak Ridge. The cross-correlation technique of data collection was used to increase the signal-to-noise ratio. The sample holder was similar to that used by Sokol *et al.*[6] with a 0.1-mm-Al front window. Gd was used to cover any surfaces that the neutron beam might strike other than the thin Al front window. The resulting spectra were very clean having a peak at energy transfer zero from the Al window and the helium single-particle peak at a finite energy. Background runs were made without the ^{3}He and showed a stronger central peak since even cadmium, placed in the sample holder to avoid scattering from the back of the sample holder, scatters more than the ^{3}He. In fact, data analysis performed with both the raw data and the raw data with the empty-cell data subtracted gave essentially identical results. This result showed that scattering from the sample cell itself was minimal.

The first series of measurements were made at a temperature of 1.2 K. This is too high a temperature to observe any significant Fermi-liquid effects but it is a good temperature to observe the effect of final-state interactions since the single-particle scattering function $S(Q,E)$ should consist of a peak with a Gaussian shape. Figure 1 shows the result from one detector bank for such a measurement where the solid line is a Gaussian fit to both the central peak and $S(Q,E)$ for ^{3}He. The fit gives a good representation of the data. Similar data were collected in thirty other detector banks at scattering angles varying from 50° to 130°.

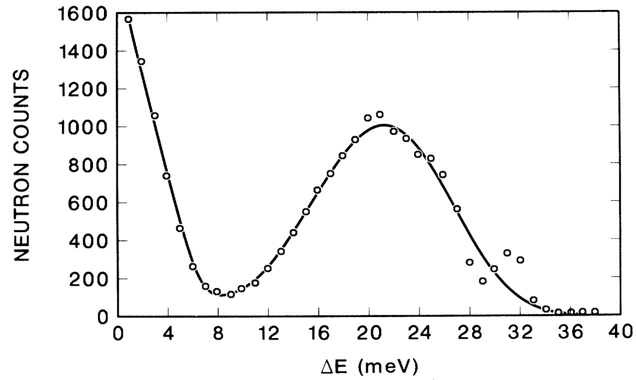

FIG. 1. Measurement of $S(Q,E)$ for ^{3}He at 1.2 K for the detector bank at a scattering angle of 81.6°. The solid line is a least-squares fit with a Gaussian peak at $\Delta E = 0$ and a Gaussian form for $S(Q,E)$.

$S(Q,E)$ was obtained from the raw data by the technique suggested by Sears[7] for highly absorbing samples after subtraction of the background counting rate obtained from the neutron energy-gain time channels which contain only room background counts.

It has been observed from earlier measurements[8] on ^{4}He that the positions and widths of the single-particle peaks of $S(Q,E)$ varied as a function of Q. It took some effort to understand the origin of the oscillations in the position and width of $S(Q,E)$ with Q, but the situation appeared to be settled by Martel *et al.*[9] who suggested that the oscillations had the same origin as oscillations in the ^{4}He-^{4}He scattering cross section. Indeed, the first strong peak in the curve of the widths of $S(Q,E)$ vs Q for ^{4}He occurs at about 4 Å^{-1} where there is a strong peak in the ^{4}He-^{4}He cross section.[10] The Q position of the second peak in the width of $S(Q,E)$ does not correspond so well to the ^{4}He-^{4}He cross section; however, it appears that the ^{4}He-^{4}He cross section is an important ingredient in the understanding of the width and position oscillations.

The ^{3}He-^{3}He scattering cross section can be expected to have oscillations that are out of phase[11] with the oscillations for ^{4}He since different statistics are obeyed. One might in turn expect that the width and position oscillations in $S(Q,E)$ for ^{3}He will occur at Q values that are different than for ^{4}He. The top of Fig. 2 shows a graph of the constant-Q width of the Gaussian distribution for $S(Q,E)$ divided by Q and plotted versus Q. This should be a straight line if the IA is obeyed exactly but clearly it is not in the region of our measurement for Q between 4 and 7 Å^{-1}. The widths in constant Q were obtained from the widths in constant angle by techniques similar to that used by Hilleke *et al.*[12] The bottom portion of Fig. 2 shows the difference between the observed peaks in $S(Q,E)$ and the free-particle scattering position $\hbar^2Q^2/2M_{\rm He}$. Although the Q range of the measurements is limited, it is apparent that variations occur in the peak positions and the widths of $S(Q,E)$ for ^{3}He, but at different positions than for ^{4}He. Indeed, the deviation of the position of $S(Q,E)$ from $\hbar^2Q^2/2M_{\rm He}$ is a maximum at about 4.5 Å^{-1} for ^{4}He but appears to be a minimum at this Q for ^{3}He. For ^{4}He a minimum in the width oscillations occurs at about 4.5 Å^{-1} while the minimum seems closer to 5.5 Å^{-1} for ^{3}He. Because of the difficulty of the experiment, the width and position results have rather large statistical uncertainties. However, it is clear that variations occur in the width and positions of $S(Q,E)$ and that these variations can be traced at least in part to variations in the He-He scattering cross section for each isotope. The problem is obviously more complicated than this, and a more complete picture of the final-state interactions is needed than can be given by two-particle scattering. The value of the average kinetic energy per particle is directly related to the quantity $\Delta E/Q$ shown in Fig. 2. This quantity obviously varies for different Q values but from the average of all data a value of 10.7 K is calculated in good agreement with previous results.[6]

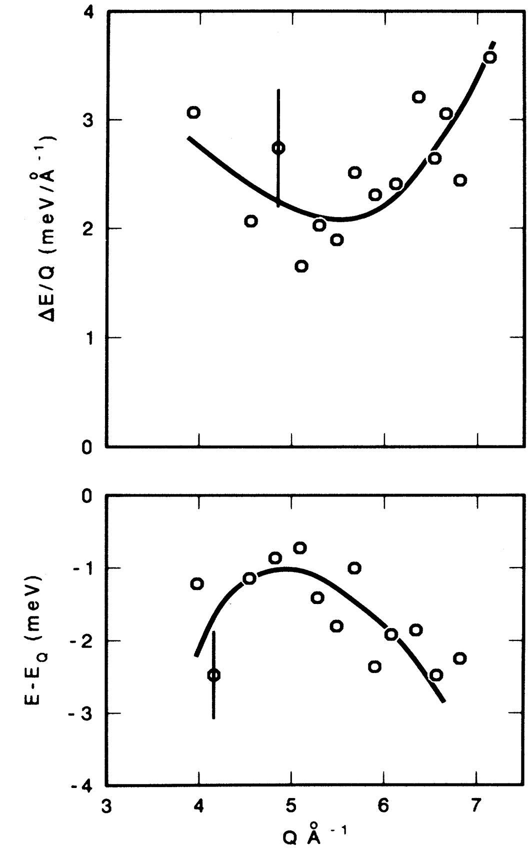

FIG. 2. Top: Widths in constant Q of $S(Q,E)/Q$ for various Q values. Bottom: The calculated recoil position $\hbar^2Q^2/2M_{\rm He}$ subtracted from the measured position of the Gaussian distribution.

As ^{3}He is cooled to lower temperatures, it becomes a quantum liquid and $S(Q,E)$ is no longer Gaussian as for a classical system. The main objective of the experiment was to observe the Fermi-liquid effects and in particular to observe the position of the Fermi surface. The lowest temperature that could be obtained in the cryostat used for the experiment was 0.37 K and so measurements were made at this temperature. The ^{3}He sample was under its own vapor pressure for this temperature. The momentum distribution $n(p)$ was obtained in the same manner as was used earlier for ^{4}He.[5] A great advantage of time-of-flight measurements for these materials is that data can be obtained for many Q's simultaneously. Thus while the statistical accuracy at one Q may not be satisfactory, the sum of the $n(p)$ distributions from perhaps twenty Q values gives an accurate result. The experimental resolution for $n(p)$, which is about 0.3 Å^{-1} FWHM, is a compromise between obtaining sufficient neutron intensity and having sufficient resolution to observe the Fermi surface. This resolution is somewhat coarser than the broadening caused by final-state effects as calculated by Platzman and Tzoar[2] so that this broadening can be neglected to first order in the present experiment. Finally, the resolution of the experiment sufficiently broadens the Fermi-surface effects in $S(Q,E)$ that temperatures lower than that used are unnecessary. This means that comparison of the data to calculations of $n(p)$ for low temperatures[13] is not appropriate and comparison with the ideal-gas function is sufficient for the purposes of this experiment. Figure 3 shows the result of the measured $n(p)$ compared to the Fermi distribution

$$n(p)=\frac{1}{\exp[(E-E_\text{F})/k_\text{B}T]+1}. \tag{1}$$

A least-squares fit of Eq. (1) convoluted with the experimental resolution is shown by the solid line in Fig. 3. The Fermi temperature E_F/k_B was the adjustable parameter and the fitting gave $T_\text{F}=1.8\pm0.2$ K with use of an effective mass of $3.08M_\text{He}$. The error reflects the statistical analysis only and assumes that the procedure used to obtain $n(p)$ from the measured result is exact. The Fermi temperature obtained is in good agreement with generally accepted values.[14]

The above result shows that information about the Fermi-liquid parameters of ^{3}He can be obtained by state-of-the-art neutron-scattering techniques. The next step would be to achieve sufficient resolution so that ground-state calculations of $n(p)$ could be checked in detail. This would require lower temperatures which would be straightforward, and a more proper account of the final-state effects which would be possible by utilization of the formalism developed in Ref. 2. Sufficient neutron intensity to make higher resolution possible is a considerable problem and will have to await the development of a higher source flux.

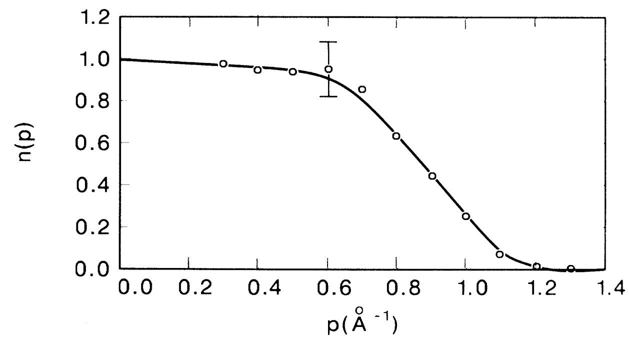

FIG. 3. $n(p)$ for ^{3}He at 0.37 K. The solid line is a least-squares fit by the ideal-gas Fermi distribution for $T_\text{F}=1.8$ K.

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