Naturalness for Multiscalar Models and Radiative Stability

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We show that, for a class of models in which the Higgs doublets which couple to the up quarks can communicate with those coupling to the down quarks only via mass mixing, the requirement that there should be no flavor-changing neutral processes at the tree level is stable under radiative corrections to the one-loop order. This also implies, in the context of *effective-field theories,* that we may use the low-energy Lagrangean to calculate one-loop corrections. At the two-loop level, however, heavy particles in the yet-unknown complete theory are likely to come into play.

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Many attempts have been made to solve the fermion mass and mixing puzzle. Within the "standard" model, one possibility is to trade *ad hoc* Yukawa couplings against the introduction of a nonminimal scalar structure. Such a structure, if carefully chosen, may lead to interesting relations *at the tree level* between masses and Cabibbo-Kobayashi-Maskawa matrix elements; a prototype of such a construction is provided by the Fritzsch[1] model. More often than not, however, such relations are not "natural" with respect to the issue of strangeness flavor-changing currents; namely, even if these can be tuned to be small at the tree level, uncontrollable counterterms may appear in higher orders. Obviously, demanding all the unwanted scalars to be heavy is an easy fix to this question; however, the very fact of having to tune such currents may be felt as disturbing. It is our assumption here that all scales in a low-energy effective theory should be comparable. Our query is whether a dynamical assumption can be made within this confine to understand the pattern of low-energy parameters. This approach can be quite tractable, if the answer is in the affirmative. To put it differently, we assume that the gauge-hierarchy problem, which has to do with vastly different scales and which has to do with the stability of scalar masses under radiative corrections when coupled with large scales, is a separate issue from the inner working of the low-energy sector. Its consistency is guaranteed by decoupling.

Let us briefly review the concept of naturalness. One can impose a condition on a set of input parameters in a theory. If radiative corrections do not induce infinite counterterms to violate the imposed condition, then the condition is said to be natural. There are two ways to look at "unnatural" conditions. If the Lagrangean that we are using to describe the physical situation is "the fundamental one," then infinite corrections to the imposed condition will mean that the constrained parameters should all be allowed to vary freely. By fine tuning, the condition imposed may be satisfied under perturbation to a certain order, but such a procedure is highly contrived.

The other possibility would be that the Lagrangean that we are using is an effective one, which obtains after we integrate out some heavy fields—for example, SU(3) ⊗ SU(2) ⊗ U(1) from SU(5). Then infinite counterterms for the low-energy parameters generated by use of the low-energy effective Lagrangean *may* mean that effects due to heavy particles are of order unity after certain perturbative orders. *A priori* there is nothing wrong with large radiative corrections, as highlighted by the successful calculation of the weak angle in grand-unification theory. One just has to be aware of the limitation of the tree-level results, or results to a certain perturbative order, in this situation. Mass and mixing-angle relations in the Fritzsch-type proposal may well fall into this category.

Let us turn to the problem of naturalness in relation to neutral currents. Glashow and Weinberg[2] raised the issue in the context of SU(2) ⊗ U(1) gauge theories, where they concluded that, to avoid flavor-changing neutral processes, (1) all quarks of the same charge and helicity must have the same weak isospin assignment,[3] and (2) all quarks of a given charge must receive their masses either from the vacuum expectation value of a single Higgs scalar or from a bare-mass term, in the absence of family symmetry. (We use the expression "Higgs scalars" as a shorthand to designate the scalar particles involved in the Brout-Englert-Higgs mechanism of spontaneous symmetry breaking.)

On the other hand, it is generally the case that to incorporate family symmetry, several Higgs scalars have to be introduced. In articles by Gatto *et al.*[4] and others it has been shown that family symmetry and neutral flavor conservation place severe restrictions on the symmetry and the mixing matrix. For example, if the fermion families and Higgs scalars form irreducible representations of a family symmetry, then the symmetry, up to some phases, is equivalent to a permutation symmetry, and the mixing matrix is trivial.

By this chain of argument, it is clear that we cannot have exact family symmetry if we are to have meaningful relations between mixing-matrix elements and masses. We should also expect the presence of uncontrollable flavor-changing neutral processes induced after some orders in perturbation, the signal of which is in the form of infinite counterterms. An answer we would like to extract is the following: Without assuming any family symmetry beforehand, at what order do we expect this to happen? As is obvious, the answer is interesting only with respect to an effective Lagrangean pertaining to a yet-unknown complete theory, because then we shall be probing at what order high-mass effects must come into play.

For the class of multi-Higgs-scalar models that we consider below, we shall find that order-unity effects due to heavy particles start coming in at the two-loop level. To put this positively with respect to those relations which connect masses and mixing-matrix elements, our results imply that we may use low-energy information to find meaningful radiative corrections to them at the one-loop level. At the same time, flavor-changing neutral processes will be under control. Starting from the two-loop level, however, we must use the complete Lagrangean which includes heavy particles to obtain corrections and to investigate flavor-changing neutral processes. To put it differently, the probable existence of new infinities at the two-loop level will mean that new flavor-changing parameters of two-loop strength may have to be added; the precise scale where this occurs is to be determined by a renormalization-group analysis of a complete theory, which we do not address in this article.

We shall carry out an analysis of a class of multi-Higgs-scalar models and investigate the consequences of the theoretical constraint of naturalness. In view of the experimental value of $\rho \equiv m_W^2/m_Z^2\cos^2\theta_W$,[5] we shall assume at the outset that the Higgs scalars are all weak isodoublets.

We define

$$q_L=\begin{pmatrix}u\\d\end{pmatrix}_L,\quad \Phi_a=\begin{pmatrix}\Phi_a^0\\ \Phi_a^-\end{pmatrix},\quad \text{and } \phi_a=\begin{pmatrix}\phi_a^+\\ \phi_a^0\end{pmatrix}.$$

Then the interaction between quarks and Higgs scalars is

$$\mathscr{L}_{\text{quarks-scalars}}=\bar{q}_L G^a\Phi_a u_R+\bar{q}_L g^b\phi_b d_R+\text{H.c.}$$

Here G^a and g^b are coupling matrices (3×3 for three families). The indices a and b label the Φ and ϕ multiplets, respectively. Note that, because of SU(2) symmetry, in principle some of the ϕ's may be the charge conjugates of some of the Φ's, but we exclude this possibility in the present consideration, which can be accomplished because of a reflective symmetry (see later).

We shall also assume that none of the Higgs particles above has a mass much higher than those of the W bosons in order to avoid the likelihood of strong couplings and to work within our assumption that disparate scales do not occur in low-energy models. We consider as neutral flavor changing the processes $(H^0)^a\to qq'$. In another approach one might deem these processes irrelevant at low energy and require flavor conservation only in $qq'\to(H^0)^a\to q''q'''$ or $qq'\to(H^0)^a\to ll'$. If this point of view is adopted, other possibilities of maintaining flavor conservation exist, e.g., by exploitation of compensations between mass-degenerate off-diagonal scalars and by use of different H's for quarks and leptons.

It is obvious that to avoid flavor-changing neutral processes at the tree level, these two sets of coupling matrices, G's and g's, must be independently, simultaneously diagonalizable with respect to the quark-mass eigenstates. We will discuss some of the conditions which yield simultaneous diagonalization later in the paper.

The mass eigenstates will be primed; we have

$$Ru_R=u_R',\quad Lu_L=u_L',\quad rd_R=d_R',\quad ld_L=d_L',$$

in which R, L, r, and l are unitary matrices. The diagonalized coupling matrices are denoted as $D^a=LG^aR^\dagger$ and $d^b=lg^br^\dagger$, and the Cabibbo-Kobayashi-Maskawa matrix is $K=lL^\dagger$. In these mass eigenstates, we have

$$\begin{aligned}\mathscr{L}_{\text{quarks-scalars}}={}&\bar{u}_L'D^a\Phi_a^0u_R'+\bar{d}_L'KD^a\Phi_a^-u_R'+\bar{u}_L'K^\dagger d^b\phi_b^+d_R'+\bar{d}_L'd^b\phi_b^0d_R'+\bar{u}_R'\Phi_a^{0\dagger}D^{a\dagger}u_L'\\&+\bar{u}_R'D^{a\dagger}K^\dagger\Phi_a^\dagger d_L'+\bar{d}_R'd^{b\dagger}K\phi_b^-u_L'+\bar{d}_R'd^{b\dagger}\phi_b^{0\dagger}d_L'.\end{aligned}$$

It is seen that the Lagrangean above does not give rise to any flavor-changing neutral processes at the tree level.

Our investigation of naturalness is an examination of counterterms. We shall rely heavily on power counting. In this respect we note that the nonvanishing of mixed propagators $\langle\Phi_a^0\phi_b^0\rangle$, $\langle\Phi_a^+\phi_b^-\rangle$, etc., is due to mass mixing. This makes their high-momentum behavior $\sim(1/p^2)^2$.

Note that the criteria of simultaneous diagonalizability of the G's and g's refer to properties of the bilinear combinations $G_aG_a^\dagger$ and $g_bg_b^\dagger$, which are unchanged under similarity transformations. In particular, we can make wave-function renormalizations (which can be infinite and nondiagonal) on the scalars and quarks without impeding naturalness, or the lack thereof.

At the one-loop level, up to wave-function renormalizations, we are looking for induced Yukawa couplings with infinite coefficients,

$$\mathscr{L}_{\text{induced quarks-scalars}}=\ln\Lambda\{\bar{q}_L G^a_{\text{ind}}\Phi_a u_R+\bar{q}_L g^b_{\text{ind}}\phi_b d_R+\text{H.c.}\}.$$

If the sets $G_a + (\ln\Lambda) G^a_{\rm ind}$ and $g^b + (\ln\Lambda) g^b_{\rm ind}$ can be separately, simultaneously diagonalized without fine tuning of parameters, the theory will be said to be natural to the one-loop order. This is in fact the case, as we shall show in the following.

It is convenient in this analysis to work with the quark-mass eigenstates. The only way to produce terms which may not be simultaneously diagonalizable with the tree terms is when K and $K^\dagger$ make their appearance not next to each other in a product of matrices. Now, the only interaction terms which involve K and $K^\dagger$ are those with charged scalars or charged vector bosons. Because of these remarks, we see that for quark–neutral-vector-boson vertices, charged-vector-boson exchanges have K and $K^\dagger$ standing next to each other in the amplitudes. Charged-scalar exchange [Fig. 1(a)] will give flavor off-diagonal terms, which precisely correspond to the quark wave-function renormalizations [Fig. 1(b)]. These can be absorbed by an orthogonal transformation O_L and a rescaling D_{ZL}, i.e.,

$$\bar q'_L \gamma_\mu q'_L \to \bar q'_L Z_L \gamma_\mu q'_L = \bar q'_L O^t_L (O_L Z O^t_L)\gamma_\mu O_L q'_L = \bar q'_L O^t_L D_{ZL}\gamma_\mu O_L q'_L = (\bar q'_L O^t_L D^{1/2}_{ZL})\gamma_\mu (D^{1/2}_{ZL} O_L q'_L) = \bar q''_L \gamma_\mu \bar q''_L .$$

For quark–neutral-scalar vertices, charged-scalar-exchange diagrams [Fig. 1(c)] are finite, because we need mixed scalar propagators for them to exist. Charged vector mesons [Fig. 1(d)] do not contribute, because they do not couple to the right-handed quarks. Finally, while vector-scalar exchange diagrams [Fig. 1(e)] are infinite, the K and $K^\dagger$ are positioned next to each other, so that we can use $KK^\dagger = K^\dagger K = 1$. In other words, no off-diagonal infinities are induced.

Including wave-function renormalizations, we have, e.g.,

$$\bar u'_L G^a u'_R \to \bar u'_L Z^{1/2}_L G^a Z^{1/2}_R u'_R = \bar u'_L O^t_L (O_L Z^{1/2}_L O^t_L) O_L G^a O^t_R (O_R Z^{1/2} O^t_R) O_R u'_R$$
$$= (\bar u'_L O^t_L D^{1/2}_{ZL}) O_L G^a O^t_R (D^{1/2}_{ZR} O_R u'_R) = \bar u''_L O_L G^a O^t_R u''_R .$$

As remarked before, if the G^a's are simultaneously diagonalizable among themselves, so will $O_L G^a O^t_R$ be.

Then, to the one-loop order, the requirement of simultaneous diagonalizability of the coupling matrices is stable under radiative corrections.

To discuss naturalness at the two-loop order, we need the self-interaction of the scalars. We shall assume that the overall interaction is invariant under the reflections

$$\Phi \to -\Phi, \quad u_R \to -u_R,$$

and/or

$$\phi \to -\phi, \quad d_R \to -d_R .$$

The Lagrangean density is

$$\mathscr{L}_{\rm scalars} = \lambda^{(1)}_{abcd}(\Phi^\dagger_a\Phi_b)(\Phi^\dagger_c\Phi_d) + \lambda^{(2)}_{abcd}(\phi^\dagger_a\phi_b)(\phi^\dagger_c\phi_d) + \lambda^{(3)}_{abcd}(\phi^c_a\Phi_b)(\Phi^\dagger_c\phi^{c\dagger}_d) + \lambda^{(4)}_{abcd}(\Phi^\dagger_a\Phi_b)(\phi^\dagger_c\phi_d)$$
$$+ [\lambda^{(5)}_{abcd}(\phi_{a_c}\Phi_b)(\phi^c_c\Phi_d) + \text{H.c.}] + [\lambda^{(6)}_{abcd}(\Phi^c_a\Phi_b)(\phi^c_c\phi_d) + \text{H.c.}],$$

$$\lambda^{(1)}_{abcd} = \lambda^{(1)}_{cdab} = \lambda^{(1)*}_{badc}, \quad \lambda^{(2)}_{abcd} = \lambda^{(2)}_{cdab} = \lambda^{(2)*}_{badc}, \quad \lambda^{(3)}_{abcd} = \lambda^{(3)*}_{dcba}, \quad \lambda^{(4)}_{abcd} = \lambda^{(4)*}_{badc},$$

in which

$$\phi^c = (i\tau_2\phi)^t, \quad \Phi^c = (i\tau_2\Phi)^t .$$

Again, the question is whether we shall induce infinite Yukawa counterterms, such that, without fine tuning, the simultaneous-diagonalizability requirement is violated. This is in fact the case, as we shall demonstrate in the following.

A diagram which generates off-diagonal Yukawa couplings is, for example, Fig. 1(f), which produces

$$\sim \ln\Lambda(-\lambda^{(3)}_{cabd} + \lambda^{(4)}_{badc})\bar d'_R d^\dagger_c K D_b D^\dagger_a K^\dagger d'_L \phi^{0\dagger}_d .$$

One possibility for this not to violate naturalness is that the coefficient vanishes because of some symmetry, such as $\lambda^{(3)}_{cabd} = \lambda^{(4)}_{badc}$. However, we can easily verify that this is not consistent with the counterterm structure of $\lambda^{(3)}$ and $\lambda^{(4)}$ at the one-loop level, in particular, due to quarks. The other alternative is that $d^\dagger_c K D_b D^\dagger_a K^\dagger$ are diagonal. If we assume that some of the d's are nonsingular, as they are related to down-family masses which are nonvanishing, it follows that $K D_b D^\dagger_a K^\dagger$ are diagonal. Again, upon stipulating that some of the D's are nondegenerate and nonsingular, we can easily show that the Cabibbo-Kobayashi-Maskawa matrix K can have one and only one nonvanishing element in each row and in each column. This is hardly what is known in reality.

Parenthetically, we cannot completely exclude the possibility that the tree terms and these divergent terms would be diagonalized at the same time. For instance, one may imagine, for the sake of argument, a

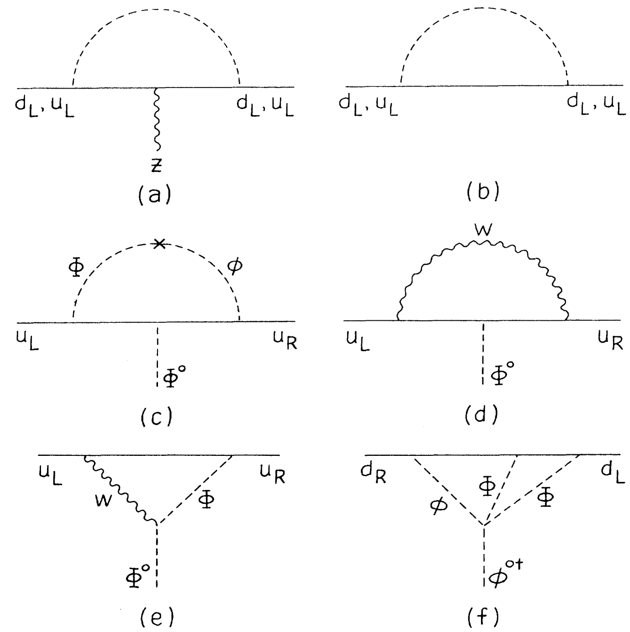

FIG. 1. (a)–(f) Some diagrams which must be considered for an analysis of one- and two-loop naturalness of flavor-changing neutral processes.

toy model with only two fields ϕ_1 and ϕ_2, which have *identical* couplings to the quarks. If they have furthermore identical "ϕ^4" couplings and masses, they are just as good as being one field, and no flavor-changing neutral current can arise here. The possibility of this simultaneous diagonalization seems, however, somewhat remote in general and can be discussed only on a case-by-case basis.

Necessary and sufficient conditions for the simultaneous diagonalizability of a set of coupling matrices have been stated in the literature.[4] Their connection with physical requirements is, however, not immediate. For instance, we can formulate the following theorem: Let A_i be a set of *nonsingular* matrices such that at least one product, say $A_1^\dagger A_1$, is nondegenerate. Then $A_i = UD_iV$ for all i, where U and V are unitary and D_i's are diagonal, if and only if $[A_1^\dagger A_1, A_l^\dagger A_k] = 0$ for $l \leq k$. (This formulation is due to P. Federbush.)

The conditions on nondegeneracy and nonsingularity of the individual matrices are essential, and unfortunately cannot be directly related to the nondegeneracy and the nonvanishing of the quark masses themselves, which only provide information on $\sum_a G^a \langle \Phi_a \rangle$ and $\sum_b g^b \langle \phi_b \rangle$.

Notice that the question of singular matrices is not rhetorical, since many models are expected to use coupling matrices with only a few entries. Also, it is an easy matter to check that some popular couplings [e.g., for two generations,

$$\begin{pmatrix} 0 & \epsilon \\ \epsilon & 0 \end{pmatrix} \phi_1 + \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} \phi_2$$

in Fritzsch's model] do not obey the criterion of simultaneous diagonalization. Obviously, other sets in some similar scenario can be used [such as

$$\begin{pmatrix} 0 & \epsilon \\ \epsilon & 1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & \epsilon \\ \epsilon & 0 \end{pmatrix}$$

in the example above, but the physical motivation here is far from transparent].

To conclude, there remains the problem of the building of a model which satisfies our criteria to explain the correct mass pattern. We are working on it, and we hope that we can report positively on it in the future. On the other hand, we have emphasized in this article that one should pay special attention to the compatibility between obtaining a correct mass pattern and suppressing flavor-changing neutral processes. In many models proposed, such as the original model of Fritzsch, an uncontrollable flavor-changing problem will occur.

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