Gravitomagnetic Pole and Mass Quantization

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The existence of a gravitational analog of Dirac's magnetic monopole is speculated on.

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In this note, I indulge in speculation about gravity which, if true, may have far-reaching consequences. I will first describe, and then comment critically, on the proposed notion.

The striking resemblance between Newton's law of gravity and Coulomb's law of electrostatics has been noted since the beginning of physics. It is somewhat less well-known that the post-Newtonian laws of gravity also correspond quite closely to the Maxwellian laws of electromagnetism. In the post-Newtonian approximation[1] to Einstein's theory, one expands $g_{00}=-1+g_{00}^{(2)}+g_{00}^{(4)}+\dots$, $g_{ij}=\delta_{ij}+g_{ij}^{(2)}+\dots$, $g_{i0}=g_{i0}^{(3)}+\dots$, $T^{00}=T^{00(0)}+T^{00(2)}+\dots$, $T^{i(0)}=T^{i0(1)}+\dots$, $T^{ij}=T^{ij(2)}+\dots$, and so forth. The expansion is in powers of $v\sim(GM/r)^{1/2}$, where v, M, and r denote the typical velocity, mass, and separation of the particles in the system under consideration. It is useful to name the relevant quantities: $\phi\equiv-g_{00}^{(2)}/2$, $\zeta_i\equiv g_{i0}^{(3)}$, $\rho\equiv T^{00(0)}$, $K_i\equiv T^{i0(1)}$, $\psi+\phi^2\equiv-g_{00}^{(4)}/2$, etc. Then, to the appropriate order, Einstein's equation $R_{\mu\nu}-\frac{1}{2}g_{\mu\nu}R=-8\pi GT_{\mu\nu}$ reduces to[2]

$$\nabla^2\phi=4\pi G\rho, \tag{1}$$

$$\nabla^2\zeta_i=16\pi GK_i. \tag{2}$$

The harmonic coordinate condition $g^{\mu\nu}\Gamma^{\lambda}_{\mu\nu}=0$, which reduces to

$$4\,\partial\phi/\partial t+\nabla\cdot\boldsymbol{\zeta}=0, \tag{3}$$

has been imposed.

Let us introduce the "gravitoelectric field" $\mathbf{g}\equiv-\nabla\phi$ and the "gravitomagnetic field" $\mathbf{B}\equiv\nabla\times\boldsymbol{\zeta}$. Then we can rewrite Eqs. (1) and (2) as

$$\nabla\cdot\mathbf{g}=-4\pi G\rho, \tag{4}$$

$$\nabla\times\mathbf{B}=-16\pi G\mathbf{K}+\partial\mathbf{g}/\partial t, \tag{5}$$

$$\nabla\times\mathbf{g}=0, \tag{6}$$

$$\nabla\cdot\mathbf{B}=0. \tag{7}$$

The correspondence with Maxwell's equations is exact except for Faraday's law (since we defined **g** as a pure gradient). The harmonic condition Eq. (3) appears as the Lorentz gauge condition.

Following Dirac,[3] I now speculate that pointlike "gravitipoles" exist, so that Eq. (7) is amended to read

$$\nabla\cdot\mathbf{B}=\gamma\delta^{(3)}(\mathbf{x}) \tag{7'}$$

when a gravitipole is present at the origin.

Dirac taught us that the existence of magnetic monopoles implies that electric charge is quantized. To derive the analogous quantization condition here, we must expand the action of a particle in a gravitational field:

$$\begin{aligned}\int L\,dt&=-m\int d\tau\\&=\int dt\{-m+m[\tfrac{1}{2}\mathbf{v}^2+\tfrac{1}{8}(\mathbf{v}^2)^2]\\&\quad-m(\phi+\tfrac{1}{2}\phi^2+\psi+\tfrac{3}{2}\phi\mathbf{v}^2)-m\boldsymbol{\zeta}\cdot\mathbf{v}\}.\end{aligned} \tag{8}$$

Variation of the action gives the analog of the Lorentz force law. We have arranged the terms in Eq. (8) in four groups: (1) an irrelevant additive constant, (2) the standard kinetic energy, corrected relativistically, (3) the potential energy $(-m\phi)$ of a particle in a gravitational potential ϕ, corrected relativistically, and (4) a velocity-dependent term. The last term, which can be written as a line integral

$$-m\int d\mathbf{x}\cdot\boldsymbol{\zeta}, \tag{9}$$

is the term of interest.

The argument for quantization is exactly that of Dirac. If we move a particle with mass m on a closed loop around a gravitipole, the particle's wave function acquires the phase $e^{-im\gamma/\hbar}$. We keep the mass m sufficiently far away from the gravitipole so that the post-Newtonian approximation holds to any desired accuracy. We conclude, following Dirac, that

$$m=(2\pi\hbar/\gamma)\,n,\quad n\text{ an integer}. \tag{10}$$

Mass is quantized in units of $2\pi/\gamma$.

An obvious and almost tautological objection is that the Einstein-Hilbert action must be modified to accommodate gravitipoles, just as the Maxwell action must be modified to accommodate magnetic monopoles. I imagine that the history of the magnetic monopole may be repeated. After Dirac's paper, many authors tried to modify electromagnetic theory by including path-dependent quantities.[4] There was considerable debate whether a field theory of magnetic monopoles may be formulated. Eventually, 't Hooft and Polyakov showed that the magnetic monopole exists as an extended solution[5] in certain non-Abelian gauge theories. Nowadays, most theorists believe that electromagnetism is a piece of a grand unified theory and that magnetic monopoles exist. Might it not turn out that Einstein's theory is but a bigger piece of a bigger theory and that gravitipoles exist?

My speculation raises a number of physical issues, some of which I cannot fully resolve at this point. Nevertheless, I feel that it may be useful to air this speculation and to discuss its phenomenological consequences.

(1) The notion that mass is quantized has long been one of the wilder speculations in circulation. I must emphasize that there is certainly no experimental evidence whatsoever of mass quantization. If mass is quantized, the unit $m^* = 2\pi/\gamma$ would have to be tiny. We will address the question of how tiny m^* has to be.

(2) Mass quantization, if true, may have profound consequences in physics. Imagine moving a nucleus, say, around a gravitipole. Mass quantization implies that the binding energy of every level in every possible nucleus is quantized. This appears to impose some conditions on the fundamental couplings of Nature.

(3) Level splittings in atoms and molecules set stringent bounds on the mass quantization unit m^*. Molecular level splittings are of order 10^{-14} eV. But most remarkably, in the University of Washington experiment on the electric dipole moment of ^{129}Xe, an energy splitting of 10^{-23} eV has been observed.[6] However, it must be emphasized that in all these experiments one measures the change in precession frequencies of a large number of atoms (or molecules) in external electric and magnetic fields. Mass quantization, if true, would surely require revision of our understanding of energy and mass, and it is far from clear whether Planck's relation $E = \hbar\omega$ relating frequency to energy would remain intact. Thus I am not sure whether m^* has to be as small as 10^{-23} eV.

(4) The preceding raises the question of whether photon energy would be quantized. The post-Newtonian approximation certainly does not hold for a photon (and it is not clear what it means to move a photon adiabatically around a gravitipole). However, it seems to me that one can consider a *Gedankenexperiment* in which one constructs a hollow sphere with a mirrored interior and a hole through which one can inject a photon. As far as the external gravitational field is concerned, the mass of the sphere, after a photon is trapped inside, would increase by an amount equal to the photon energy.

(5) What about the one-phonon excitation energy of a macroscopic object? This may be absorbed into the object's gravitational energy. For instance, a shift in size of δr in a 10^3-kg object with size $\sim 10^3$ cm shifts its gravitational energy by $(GM^2/r)(\delta r/r) \sim (10^2$ eV$)(\delta r/r)$.

(6) Regarding the quantization of an atom's mass, one may wonder whether a small difference may be absorbed in the gravitational binding energy between the atom and the gravitipole. This corresponds to the issue of whether the mass being quantized is only the rest mass. One might think that, to the post-Newtonian order to which we are working, the action in Eq. (8) may also be written as

$$m\left(-1+\left\langle \tfrac{1}{2}v^2+\ldots-\phi+\ldots\right\rangle\right) \times \int dt\,(1+\boldsymbol{\zeta}\cdot\mathbf{v}). \qquad (8')$$

The bracket denotes some sort of vaguely defined average. One might then argue that it is the quantity in front of the integral sign which is to be quantized. However, I believe that this argument is without merit since conceptually one can take the atom as far away from the gravitipole as one likes, so that the correction to the rest mass is negligible. Also, formally, it is not clear what Eq. (8′) means as an action.

(7) There is no reason why the gravitipole has to be extremely massive. (Indeed, Dirac's magnetic monopole carries no electric charge.) Experimentally, gravitipoles may be produced with tiny cross sections in pairs in hadronic collisions, and thus there is probably no significant lower bound on the gravitipole mass. On the other hand, one might speculate that the Planck mass provides a natural mass scale for the gravitipole.

(8) It is amusing to work out what would happen were there a gravitipole in our vicinity, say inside the sun. The acceleration of a planet is given by

$$\frac{d\mathbf{v}}{dt} = -\frac{GM_\odot}{r^2}\hat{\mathbf{r}} + \mathbf{v}\times\frac{\gamma}{4\pi r^2}\hat{\mathbf{r}}. \qquad (11)$$

Let us consider a circular planetary orbit for simplicity. A moment's thought reveals that the orbit is lifted away from the sun (see Fig. 1): The $\mathbf{v}\times\mathbf{B}$ force acts perpendicular to the orbital plane. I understand that an effect of this type on planetary orbits has not been looked for. Let us work out the displacement $\epsilon \equiv \delta/r$ to lowest order in ϵ. The upward tug of the $\mathbf{v}\times\mathbf{B}$ force

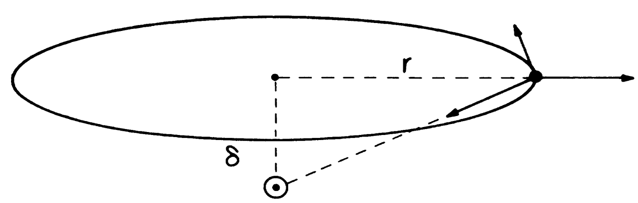

FIG. 1. Were there a gravitipole inside the sun, the sun would no longer lie in the orbital plane of the planet.

is balanced by the small downward component of the Newtonian attraction:

$$\frac{v\gamma}{4\pi r^2}=(GM_{\odot}/r^2)(\delta/r). \tag{12}$$

As usual, we have $v^2=GM/r$, and so

$$\epsilon\equiv\delta/r=\gamma/4\pi(GM_{\odot}r)^{1/2}. \tag{13}$$

The displacement is governed by the ratio of the Compton wavelength corresponding to m^* to the geometric mean of the sun's Schwarzschild radius and the planetary orbit radius. We find

$$\epsilon=7\times10^{-5}[(10^{-8}\ \mathrm{eV})/m^*]. \tag{14}$$

If m^* is much smaller than 10^{-8} eV, then celestial mechanics already rules out the presence of a gravitipole inside the sun. [We imagine that a primordial gravitipole in the solar system would have fallen into the sun. If we imagine a gravitipole inside the Earth, the Earth-moon system would yield a bound on m^* better than the one in Eq. (14) by 10^4.] An elliptical planetary orbit will be distorted by a gravitipole inside the sun since the $\mathbf{v}\times\mathbf{B}$ force varies as $r^{-3/2}$. The problem of the determination of planetary motion around a sun containing a gravitipole is formally equivalent to the problem of the determination of the motion of an electric charge around a dyon.[7]

(9) The equivalence principle still holds for a particle falling in the field of a gravitipole. The particle still follows a geodesic. It is just that the gravitipole produces a rather unusual gravitational field. Note that the metric cannot be defined globally around a gravitipole in the same way that the gauge potential cannot be defined globally around a magnetic monopole. The movement of the gravitipole in an external gravitational field, however, is presumably not governed by the standard action in Eq. (8) and violates the equivalence principle.

(10) Friedman and Sorkin[8] constructed topological solutions of Einstein gravity which, surprisingly, carry half-integral angular momentum. It is not inconceivable that in some modified or generalized theory of gravity the gravitipole may exist as a topological solution.

(11) Montonen and Olive[9] speculated that the magnetic monopole and the photon form a triplet under some "dual magnetic group." Does this mean that the gravitipole and the graviton similarly form a representation under some dual group?

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[1]A particularly clear discussion has been given by S. Weinberg [*Gravitation and Cosmology* (Wiley, New York, 1972), Chap. 9] whose notation we use. In particular, $c=1$.

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