Role of Deformation in the Intrusion of the $h_{9/2}$ Levels below the $Z=82$ Proton Shell

J. A. Bounds, C. R. Bingham, and P. Juncar[(a)]
The University of Tennessee, Knoxville, Tennessee 37996

H. K. Carter, G. A. Leander, R. L. Mlekodaj, and E. H. Spejewski[(b)]
UNISOR, Oak Ridge Associated Universities, Oak Ridge, Tennessee 37831

and

W. M. Fairbank, Jr.[(c)]
Vanderbilt University, Nashville, Tennessee 37235
(Received 26 April 1985)

The hyperfine structure and isotope shifts of $^{189,191,193}Tl^m$ and ^{193}Tl have been observed by means of collinear atom beam laser spectroscopy. Deduced deformations for the $\frac{9}{2}^-$ isomers are larger than for the $\frac{1}{2}^+$ ground states and increase with decreasing neutron number. This explains a strong neutron-number dependence of the odd-proton energy levels in these nuclei near the $Z=82$ shell closure. Despite different deformations, rotational properties are nearly identical in $^{185-199}Tl$. Microscopic theory ascribes this to a systematic balance between changing deformation and neutron pairing.

PACS numbers: 21.10.Ft, 21.10.Ky, 27.80.+w

The transition region below the $Z=82$ shell gap has been a topic of considerable interest for the last decade.[1] The $\frac{9}{2}^-$ states in the odd Tl isotopes were discovered in 1963[2] and were interpreted as an intruder $h_{9/2}$ from above the shell gap soon thereafter.[3] The level systematics were extended as low as $A=185$ by UNISOR decay work[4] and showed that the energy difference between the $h_{9/2}$ state and the $s_{1/2}$ ground state reaches a minimum at $A=189$. A simple explanation of this behavior is that deformation is producing the lowering of the $h_{9/2}$ level.[3] However, in-beam spectroscopy[3] on the high-spin states of Tl revealed that the rotational bands based on the $\frac{9}{2}^-$ states have very constant level spacing, i.e., the moment of inertia of the $\frac{9}{2}^-$ state is apparently not changing as the neutron number is reduced and thus gives no evidence of deformation change. This apparent discrepancy has been resolved in this work.

The present experiment was designed to observe differences in the deformation of a series of Tl isotopes by measurement of their isotope shifts. The isotopes were produced by the reaction $^{181}Ta(^{16}O,xn)^{197-x}Tl$ and mass separated by the UNISOR isotope separator.[5] The collinear fast-atom beam laser spectroscopy technique[6] was used to record the atomic hyperfine structure (hfs) of the 535-nm transition $6\,^2P_{3/2}$ to $7\,^2S_{1/2}$. Details of the laser system have been reported elsewhere.[7] The hfs spectrum for the transitions observed in ^{193}Tl, shown in Fig. 1, spans approximately 20 GHz. In order to measure absolute frequencies of the Doppler-shifted Tl transitions, a saturated-absorption reference spectrum of I_2 and frequency markers from a passively stabilized etalon were simultaneously recorded. Similar spectra were obtained for $^{189-192}Tl$.

Examination of Fig. 1 reveals that both the characteristic three hfs transitions due to a Tl $\frac{1}{2}^+$ ground state and the six transitions due to a $\frac{9}{2}^-$ isomeric state are observed in ^{193}Tl. The $(^{16}O,xn)$ reactions used here predominantly populate the $\frac{9}{2}^-$ isomer. Because of the isomeric decay ($t_{1/2}\approx 2$ min) to the $\frac{1}{2}^+$ ground state in ^{193}Tl, both states can be studied. In $^{189,191}Tl$ the $\frac{9}{2}^-$ isomer has fallen below the $\frac{3}{2}^+$ state and isomeric decay is insufficient to permit the ground states to be observed. Figure 1 clearly shows the isomer shift in ^{193}Tl.

The observed frequency shifts were corrected for the different Doppler shifts of the isotopes in order to obtain the rest-frame isotope shifts. The isotope shift is composed[8] of the normal mass shift of ~ 8 MHz between adjacent masses, the specific mass shift, which also has a small value for large masses, and the field shift. We obtained excellent approximations to the field shifts by subtracting the normal mass shifts from the isotope shifts while ignoring the specific mass shifts which are smaller than the experimental error. The resulting field shifts for mass $A\leqslant 193$ are plotted in Fig. 2 along with values obtained earlier by other groups[10] for $A\geqslant 194$.

The field shift can be expanded as an electronic factor multiplied by a series of terms λ, where $\lambda=\sum(C_n/C_1)\delta\langle r^{2n}\rangle$, with $\delta\langle r^{2n}\rangle$ being the difference in the mean $2n$th radial moment between two masses.[8] It has been found with Pb and Hg (Thompson *et al.*[11] and Bonn *et al.*[12]) that the series λ is proportional to $\delta\langle r^2\rangle$) to a good approximation, and thus we assume

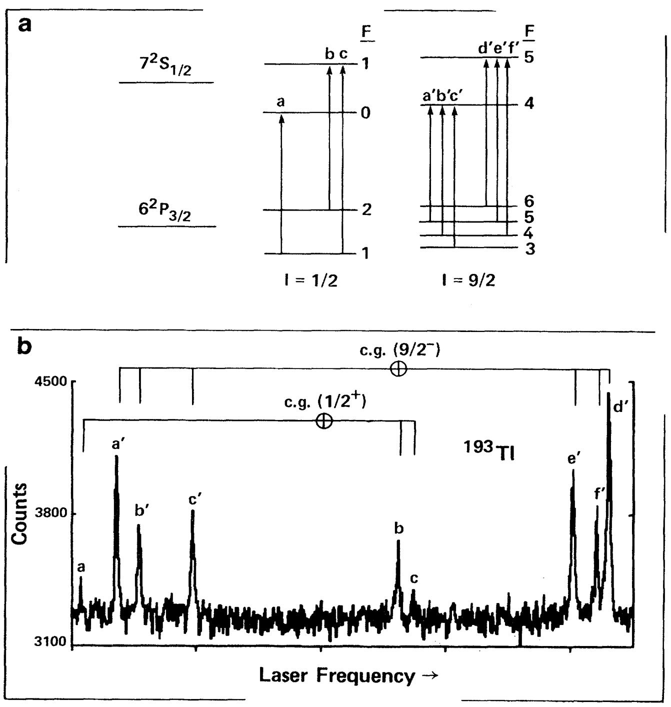

FIG. 1. (a) Hyperfine structure for the $6\,^2P_{3/2}$ and $7\,^2S_{1/2}$ atomic states for $I=\frac{1}{2}$ and $\frac{9}{2}$ nuclei (hfs level spacings are not to scale). The transitions observed in ^{193}Tl are identified. (b) Sample of data taken in the present experiment. Hyperfine spectrum from $^{193}\mathrm{Tl}^g$ $(I=\frac{1}{2}^+)$ and $^{193}\mathrm{Tl}^m$ $(I=\frac{9}{2}^-)$. The frequency shift between the centers of gravity of the transitions for the two isomers is apparent. Sensitivity is such that 1 atom per 1000 in the mass-separated beam is detected at the strongest transitions.

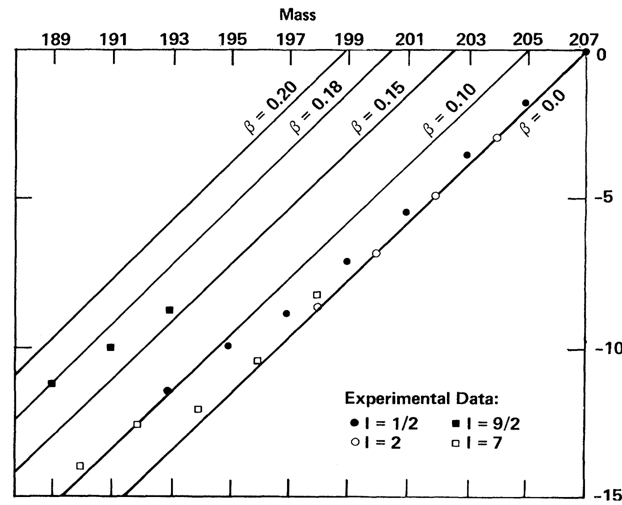

FIG. 2. Experimental field shifts in Tl compared to predictions with different droplet-model deformations (Ref. 9). Data for $A \leqslant 193$ are from the present work; results for $A \geqslant 194$ are cited in Ref. 10. All errors are $\pm(\leqslant 120$ MHz).

the same for Tl. Since the electronic factor is not directly calculable for Tl (King[13]) but should be virtually the same for all of its isotopes,[6] the field shift should then be proportional to $\delta\langle r^2\rangle$. To deduce the proportionality factor in our analysis we employ the droplet model,[9] which reproduces rms radii throughout the periodic table where deformations are known, and we assume zero deformations for ^{207}Tl (which has 126 neutrons) and ^{200}Tl (which from Fig. 2 apparently has the least deformation of the other isotopes). In this model the change in the mean-square radius depends on the mass difference and the change in shape and thus it was used to extract deformations from the field shifts. Droplet-model values of $\delta\langle r^2\rangle$ (Eq. 21 in Ref. 9) with $\beta=0$ are normalized to the field shifts of ^{207}Tl and ^{200}Tl. Predictions with different deformations using the same normalizing factor are shown as solid lines in Fig. 2. If the deformation of ^{200}Tl is not zero but some small value, the lines would be slightly steeper and would result in slightly higher values for the deformations in all isotopes except ^{207}Tl. Thus the deformations deduced from Fig. 2 may be slightly underestimated. The deformation of the odd-A Tl $\frac{1}{2}^+$ states is seen to increase with decreasing A to a value of 0.1 in ^{193}Tl while the deformation of the $\frac{9}{2}^-$ state exceeds 0.15 in ^{193}Tl and increases to 0.18 by ^{189}Tl. The isomer shift is ascribed to quadrupole deformation under the assumption that the monopole radial contribution from the change of single-particle orbit is relatively small, as suggested by spherical shell-model calculations of Speth, Zamick, and Ring[14] which include the giant monopole resonance.

The deformations of the isomers were also deduced in another way which assumes that they can be characterized by a single parameter β_2. A least-squares fit to the measured frequency intervals for each mass produced the standard hyperfine constants A and B[15] from which the magnetic dipole and spectroscopic quadrupole moments, μI and Q_s, can be deduced. Since no quadrupole moments have been previously measured in any Tl isotope, a direct calibration cannot be made. Instead, calculations were made by use of the procedure outlined by Lindgren and Rosen.[16] Their relativistic calculations were used with the measured B's to give the values of Q_s listed in Table I. The strong-coupled character of the $h_{9/2}$ band[3] corresponds to oblate deformation and suggests that the spectroscopic and intrinsic quadrupole moments are related by[17]

$$Q_s = Q_0 \frac{3K^2 - I(I+1)}{(I+1)(2I+3)}. \tag{1}$$

Negative values of Q_0 are in fact obtained for $I=\frac{9}{2}$,

TABLE I. Moments and deformations of light Tl isotopes.

Tl	Q_s ($e \cdot$b)	Q_0 ($e \cdot$b)	β_2 from Q_0	$\langle \beta_2^2 \rangle^{1/2}$ from isomer shift
193g	$\cdots$	$\cdots$	$\cdots$	0.099(1)
193m	−2.20(2)	−4.02(3)	−0.144(1)	0.158(1)
191m	−2.27(3)	−4.16(5)	−0.150(2)	0.170(2)
189m	−2.29(4)	−4.19(7)	−0.153(3)	0.181(2)

$K=\frac{9}{2}$ and are listed in Table I, as are the deformations β_2 obtained by the relation[18]

$$Q_0=\frac{3}{(5\pi)^{1/2}}ZA^{2/3}r_0^2\beta_2(1+0.36\beta_2). \qquad (2)$$

The deformations thus obtained are found to be somewhat smaller than those obtained from the isotope shifts. The errors shown represent measurement uncertainties but not uncertainties stemming from the method of Ref. 16, which would be proportionally the same for all points. A disagreement in derived β_2's from the isotope shift and from Q_s similar to ours has been previously observed in the neutron-deficient Hg isotopes, where it is believed that Q_0 is underestimated from Q_s because of violation of strong coupling and axial symmetry.[11] The differences in Tl are larger for the lighter isotopes and may be in part due to mixing with a coexisting state of opposite and larger deformation, which could simultaneously reduce the quadrupole moment and increase the effective radius.

A theoretical calculation was carried out to see if all the observed features of the Tl isotopes will emerge from the deformed shell model. It is an *a priori* microscopic calculation in the sense that the parameter values are standard. First the equilibrium shape of the intrinsic mean field was determined for both the $\frac{1}{2}^+$ and the $\frac{9}{2}^-$ configurations of $^{185-199}$Tl. The energy as a function of quadrupole (ϵ_2) and hexadecapole (ϵ_4) deformation was calculated in the Strutinsky approximation to Hartree-Fock theory as described by Nilsson *et al.*[19] The only difference in the present calculation compared to the older ones[20,21] is the use of updated spin-orbit and orbit-orbit strengths in the single-particle potential. The new strengths are a universal set proposed by de Wieclawik *et al.*[22] for use in all regions of nuclei with several improvements over the older sets. The main difference in the present results is that the $\frac{9}{2}^-$ excitation energy now has a minimum near $A=189$ as deduced from experiment [Fig. 3(a)]. The quadrupole deformation coordinate ϵ_2 is shown by the dashed line in Fig. 3(c) for the $\frac{9}{2}^-$ configuration (for the $\frac{1}{2}^+$ configuration, $\epsilon_2 \approx -0.07$). Both the value and the rate of change of the deformation in

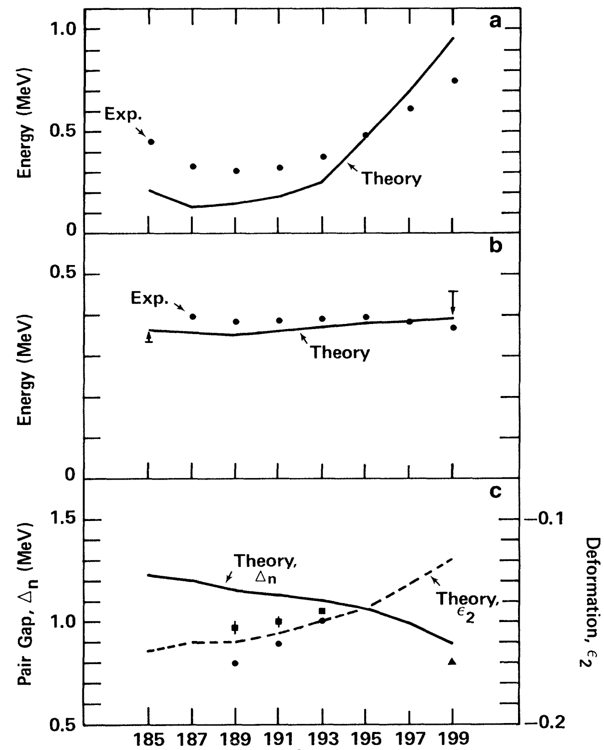

FIG. 3. Results of the theoretical calculations for $^{185-199}$Tl described in the text in comparison with experiment. (a) Energy of the $\frac{9}{2}^-$ isomer with respect to the $\frac{1}{2}^+$ ground state. (b) Energy of the first state in the rotational band based on the $\frac{9}{2}^-$ isomer [experimental energies for (a) and (b) are cited in Ref. 1]. (c) Deformation, ϵ_2, from theory (dashed line) and from the experimental Q_s (squares) and isotope shifts (circles) obtained in this work. The odd-even energy difference, Δ_n, from BCS theory (solid line) and from experimental masses (triangles).

^{189,191,193}Tl are seen to be compatible with the experimental results from the present work. Clearly the rate of change of the deformation is more than enough to account for the A dependence in the $\frac{9}{2}^-$ excitation energy, through the deformation dependence of the lowest Nilsson orbit from the $h_{9/2}$ shell. The mechanism for the upturn in the lightest Tl isotopes is more subtle, namely, a difference in the rate at which the Strutinsky shell correction changes as a function of A at the respective deformations of the $\frac{1}{2}^+$ and $\frac{9}{2}^-$ configurations.

The main reason for doubting the role of deformation in the past was based on the notion that a simple correspondence exists between the deformation and the rotational moment of inertia. The observation in $^{185-199}$Tl of almost identical strong-coupled bands based on the $\frac{9}{2}^-$ level would then be incompatible with changing deformation. In order to investigate this we calculated microscopic moments of inertia for the $\kappa = \frac{9}{2}$ bands at their respective equilibrium deformations. The method is described by Andersson *et al.*[23] and Arve, Chen, and Leander.[24] The essence of the method (see Sect. 6.5h of Ref. 17) is to achieve self-consistency between collective motion and single-particle response to that motion with use of the random-phase approximation (RPA). For rotations, the change in the single-particle Hamiltonian, H_{sp}, is proportional to $-[H_{sp}, J_+]/\kappa$. Here, the coupling constant κ can be fixed by the physical requirement of a zero-energy solution to the RPA dispersion relation, which corresponds to a reorientation of the band-head spin in space (M degeneracy). The lowest excited solution represents the first excited member of the κ band.

The results are plotted and compared to experiment in Fig. 3(b). The agreement in overall magnitude between theory and experiment is somewhat arbitrary since the scaling of the theoretical results is sensitive to the prescription for the strength of the pairing interaction (for the RPA calculations it was determined by the average-gap method with $\tilde{\Delta} = 14/\sqrt{A}$ MeV). The significant result is that the rotational excitation energy stays almost as constant for $^{185-199}$Tl in theory as in experiment, despite the changing deformation. The microscopic mechanism for this lies in changing neutron-pairing correlations, as can be verified by varying ϵ_2, Δ_p, Δ_n, and N one at a time, holding the others fixed. The self-consistent neutron gap parameter, Δ_n, is plotted in Fig. 3(c) and is seen to increase with decreasing A. Thus, *when the number of valence neutron holes increases, both the deformation and the neutron-pairing correlations increase with opposite and compensating effects on the moment of inertia.* The latter point is illustrated quantitatively by the arrows on the end points of the theoretical curve in Fig. 3(b). These arrows show the shift that comes from the use of the self-consistent and N-dependent values of Δ_n, rather than a single intermediate value.

In summary, the present work clearly demonstrates the fallacy of assuming that a constant moment of inertia infers a constant deformation. The magnitude and the effect of deformation changes in light Tl isotopes have been established and there has been demonstrated an interesting aspect of the competition between quadrupole and pairing correlations in near-singly-closed-shell nuclei.

The authors wish to thank C. E. Bemis, Jr., for many helpful discussions, C. A. Reed and C. N. Thomas for their technical aid, and the Holifield Heavy Ion Research Facility operations staff for delivering the intense ion beams required for these experiments. Support for this work was furnished by the U.S. Department of Energy under Contracts No. DE-AS05-76ER04936, No. DE-AC05-76OR00033, and No. DE-AS05-76ER05034, as well as a NATO fellowship.

---

[a]Present address: Laboratoire Aimè Cotton, Centre National de la Recherche Scientifique II, F-91405, Orsay Cedex, France.

[b]Present address: Energy/SDS, Oak Ridge Associated Universities, Oak Ridge, Tenn. 37831.

[c]Present address: Colorado State University, Ft. Collins, Colo. 80523.

[1]K. Heyde *et al.*, Phys. Rep. **102**, 291 (1983).

[2]R. M. Diamond and F. S. Stephens, Nucl. Phys. **45**, 632 (1963).

[3]J. O. Newton *et al.*, Nucl. Phys. **A148**, 593 (1970).

[4]A. G. Schmidt *et al.*, Phys. Lett. **66B**, 133 (1977).

[5]E. H. Spejewski, R. L. Mlekodaj, and H. K. Carter, Nucl. Instrum. Methods **186**, 71 (1981).

[6]R. Neugart, Nucl. Instrum. Methods **186**, 165 (1981).

[7]H. K. Carter *et al.*, Proc. SPIE **426**, 60 (1983).

[8]K. Heilig and A. Steudel, At. Data. Nucl. Data Tables **14**, 613 (1974).

[9]W. D. Myers and K. H. Schmidt, Nucl. Phys. **A410**, 61 (1983).

[10]R. J. Hull and H. H. Stroke, J. Opt. Soc. Am. **51**, 1203 (1961); D. Goorvitch, S. P. Davis, and H. Kleiman, Phys. Rev. **188**, 1897 (1969); R. Neugart, private communication.

[11]R. C. Thompson *et al.*, J. Phys. G **9**, 443 (1983).

[12]J. Bonn *et al.*, Z. Phys. A **276**, 203 (1976).

[13]W. H. King, *Isotope Shifts in Atomic Spectra* (Plenum, New York, 1984).

[14]J. Speth, L. Zamick, and P. Ring, Nucl. Phys. **A232**, 1 (1974).

[15]H. Kopfermann, *Nuclear Moments,* translated by E. E. Schneider (Academic, New York, 1958).

[16]I. Lindgren and A. Rosen, Case Stud. At. Phys. **4**, 93 (1975).

[17]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, London, 1975), Vol. 2.

[18]K. E. G. Lobner, M. Vetter, and V. Honig, Nucl. Data Tables, Sect. A **7**, 495 (1970).

[19]S. G. Nilsson *et al.*, Nucl. Phys. **A131**, 1 (1969).

[20]W. de Wieclawik *et al.*, in Proceedings of the International Conference on Nuclei Far From Stability, Cargèse, France, 1976, CERN Report No. CERN 76-13 (unpublished), p. 419.

[21]J. S. Dionisio *et al.*, J. Phys. G **2**, L183 (1976).

[22]As tabulated by T. Bengtsson and I. Ragnarsson, Phys. Scr. **T5**, 165 (1983), except $\kappa_n, \mu_n = 0.095, 0.18$, respectively, for $N = 3$ and 0.080,0.22, for $N = 4$.

[23]C. G. Andersson *et al.*, Nucl. Phys. **A361**, 147 (1981).

[24]P. Arve, Y. S. Chen, and G. A. Leander, Phys. Scr. **T5**, 157 (1983).