Solution of the Strong *CP* Problem by Color Exchange

Stephen M. Barr[(a)] and A. Zee[(b)]
Department of Physics, University of Washington, Seattle, Washington 98195
(Received 20 September 1985)

We present a new way to solve the strong *CP* problem in models with a spontaneously broken *CP* invariance. It is simpler than existing non-Peccei-Quinn approaches. It predicts the existence of light (i.e., weak scale) colored Higgs bosons which could be seen in colliders.

PACS numbers: 11.30.Er, 11.30.Qc, 14.80.Gt

In this Letter we propose a new way of solving the strong *CP* problem.[1] The two key ingredients of our approach are that (1) *CP* invariance is spontaneously broken, and (2) light colored scalar fields mediate the *CP*-nonconserving interaction.

One of us (A.Z.) proposed[2] recently the introduction into the standard $SU(3) \otimes SU(2) \otimes U(1)$ theory of an SU(2)-singlet scalar field χ, thus allowing Yukawa couplings such as $d_R C d_R \chi$, $d_R C s_R \chi$, and $s_R C s_R \chi$. The field χ has to transform as $\mathbf{6}^*$ under color in order for the couplings to be symmetric in family. This proposal was part of a program[2–7] to look for new physics in the scalar sector of the standard theory.

It was also suggested[2] that χ exchange could generate a $\Delta S=2$, *CP*-nonconserving interaction. A similar suggestion was made independently by Nièves[8] who constructed a specific model[9] at the $SU(3) \otimes SU(2) \otimes U(1)$ level. We note, however, that the spontaneous breaking of *CP* invariance at the weak scale leads to a nasty domain-wall problem. We thus propose models at the grand-unified level instead, counting on inflation to resolve the domain-wall problem.

The central point of this paper is that the color-exchange approach "naturally" solves the strong *CP* problem. In our approach, the entire quark-mass matrix (and not only its determinant) is real at tree level. This idea was first proposed some time ago by one of us (S.B.) and Seckel,[10] who were unable to implement it in a technically natural way, however. Indeed, the vital element missing in Ref. 10 is the existence of light colored scalars, a point which will be discussed in detail below.

We impose *CP* invariance on the Lagrangean so that the Yukawa couplings are real at tree level. Thus the tree-level quark-mass matrix $M_Q^{\rm tree}$ is completely real. [The phase of the vacuum expectation value (VEV) of the Weinberg-Salam doublet ϕ is meaningless and may be chosen to be real by a global hypercharge rotation.] Note that this is a consequence of there being only one ϕ. Were there two or more, their VEV's could and would have *CP*-nonconserving relative phases that would show up in the tree-level quark-mass matrix. The tree-level value of $\Theta_{\rm QCD}$ is also zero by *CP* invariance; thus

$$\bar{\Theta}^{\rm tree} = \Theta_{\rm QCD}^{\rm tree} + \arg\det M_Q^{\rm tree} = 0. \qquad (1)$$

Most remarkably, as we will see, the quantum numbers of χ are such that the one-loop correction to the phase of the quark-mass matrix vanishes identically. A small two-loop correction is generated, and for a certain range of parameters, this correction may be detectable by electric-dipole-moment measurements.

CP nonconservation in the χ-exchange process $d+d \to \chi \to s+s$ can come either from the χ propagator or from the χ-quark vertex. In the first case, there must be more than one χ, or else the propagator is real by Hermiticity.

We now construct grand-unified models to illustrate these two possibilities. The moment that we mention grand unification (GU), we are faced with the hierarchy problem, of course. In line with standard practice, we will have to simply decree that certain components of a grand multiplet are light (i.e., with mass $\ll M_{\rm GU}$) while other components are heavy (i.e., with mass $\sim M_{\rm GU}$). Both of our models are based on SU(5) with fermions of each family in $\mathbf{10}_L + \mathbf{5}_L^*$.

In *model A*, the Higgs bosons are in $\mathbf{5}_H$, $\mathbf{24}_H$, $\mathbf{15}_H^A$, and R_H^B $(A,B=1,2,\ldots)$. We decree that the SU(2) doublet in $\mathbf{5}_H$ and the color-$\mathbf{6}$ χ^A in each of $\mathbf{15}_H$ are light, while the rest are all heavy. Here R can be any representation containing an $SU(3) \otimes SU(2) \otimes U(1)$ singlet. *CP* invariance is broken spontaneously by the VEV of the singlet components of R^B, which can have nontrivial relative phases. This *CP* nonconservation will be communicated to the entire Higgs sector (through couplings such as $A_{ABCD}\mathbf{15}^{A*}\mathbf{15}^B R^{C*} R^D$). In particular, the propagator of the light color-$\mathbf{6}$'s, χ^A, will be *CP* nonconserving.

[A particularly economical choice, given essentially by Nièves,[8] involves taking R_H to be just an SU(5) singlet which transforms as $R_H \to -R_H$ under *CP*. The coupling $(i\sigma \mathbf{15}^{\dagger A}\mathbf{15}^B R + \text{H.c.})$, with σ real and $B \neq A$, generates *CP* nonconservation when R_H acquires a VEV.]

SU(5) allows the Yukawa coupling

$$\mathscr{L} = f_{mn}(\mathbf{10}_{mL} C \mathbf{10}_{nL})\mathbf{5}_H + g_{mn}(\mathbf{10}_{mL} C \mathbf{5}_{nL}^*)\mathbf{5}_H^* + h_{mn}^A(\mathbf{5}_{mL}^* C \mathbf{5}_{nL}^*)\mathbf{15}_H^A. \qquad (2)$$

Here m and n label generations.

Writing Δ^2/M^2 for the measure of CP nonconservation in the χ propagator (here Δ^2 is essentially the imaginary part of $A_{ABCD}\langle R^{C*}\rangle\langle R^D\rangle$, M^2 is the average mass squared of the χ, and h_{11} and h_{22} are some combinations of the h_{11}^A and h_{22}^A), we find that the experimental value of the ϵ parameter in the K_L-K_S system requires

$$(h_{11}h_{22}/M^2)(\Delta^2/M^2) \sim 3\times 10^{-15}\ \mathrm{GeV}^{-2}. \qquad (3)$$

(We can use either the vacuum-insertion method or the bag model to estimate the matrix element $\langle K_0|\bar{d}_R\bar{d}_R s_R s_R|\bar{K}_0\rangle$.)

It is not unreasonable to imagine the couplings h_{mn}^A to be of the same order as the standard Higgs couplings g_{mn}. Thus with $h_{11}^A \sim g_{11} \cong 2\times 10^{-5}$, $h_{22}^A \sim g_{22} \cong 5\times 10^{-4}$, and $\Delta^2/M^2 \cong 10^{-1}$, Eq. (3) gives $M \cong 550$ GeV. We note that this model can accommodate "maximal" CP nonconservation, $\Delta^2/M^2 \sim O(1)$.

How big does Θ come out to be in such a model? Either chirality or group theory guarantees that there is *no one-loop diagram* that contributes to $\bar{\Theta}$. For instance, at the SU(5) level the $\mathbf{15}_H$ has to be emitted and absorbed to have CP nonconservation. The quark-mass terms always involve the $\mathbf{10}_L$, but the $\mathbf{15}_H$'s do not couple to the $\mathbf{10}_L$. The lowest-order graph that contributes to the CP-nonconserving part of the quark-mass matrix involves two loops as shown in Fig. 1. Crudely, it is of order

$$\delta M_{nn} \sim \sum_{l,m} \frac{\lambda g_{nl} h_{lm}^* h_{mn}}{64\pi^4}\left(\frac{\Delta^2}{M^2}\right). \qquad (4)$$

Roughly then (with the assumption that phases of diagonal components of M_Q dominate),

$$\bar{\Theta} \cong \Theta_{\mathrm{QFD}} = \arg\det M_Q$$
$$\sim \frac{1}{64\pi^4}\sum_{n,l,m}\lambda\left(\frac{g_{nl}}{g_{nn}}\right)\left(\frac{h_{lm}^* h_{mn}}{h_{11}h_{22}}\right)(3\times 10^{-15}\ \mathrm{GeV}^{-2})\,M^2 = (5\times 10^{-14})\left[\sum_{n,l,m}\lambda\left(\frac{g_{nl}}{g_{nn}}\right)\left(\frac{h_{lm}^* h_{mn}}{h_{11}h_{22}}\right)\left(\frac{M}{300\ \mathrm{GeV}}\right)^2\right] \qquad (5)$$

(where QFD refers to quantum flavor dynamics).

With $h_{mn} \sim g_{mn}$, the term $n = l = m = 3$ dominates the sum, and $|h_{33}|^2/h_{11}h_{22}$ is of order $m_b^2/m_d m_s \sim 2\times 10^4$. For λ near unity and M near 300 GeV, this would give $\bar{\Theta} \sim 10^{-9}$, corresponding to an electric dipole moment of[11] $\sim 5\times 10^{-25}$ $e\cdot$cm as compared to the current bound[12] of $(2.3 \pm 2.3)\times 10^{-25}$ $e\cdot$cm.

In *model B,* we take the same particle content as in model A with these changes: There is only one $\mathbf{15}_H$, the $R_H^{(A)}$ are taken to be a singlet $\mathbf{1}_H$, and we add some number of $\mathbf{1}_L$ fermions. Thus this model is more economical than model A. The coupling $i\tau \mathbf{1}_H \mathbf{15}_H \mathbf{5}_H^* \mathbf{5}_H^*$ can become CP nonconserving when $\mathbf{1}_H$ acquires a vacuum-expectation value. The CP nonconservation shows up in one-loop order as indicated in Fig. 2, generating a phase in the effective h_{mn}.

Equations (3) and (4) still apply with Δ^2/M^2, which measures the CP nonconservation in the χ propagator in model A, replaced by $\arg(h_{11}^* h_{22})$, which measures the CP nonconservation in the χ vertices in model B. Again the lowest-order graph that contributes to $\bar{\Theta}$ is Fig. 1. The estimate in Eq. (5) is still valid if we replace (Δ^2/M^2) by $\arg(h_{lm}^* h_{mn})$.

Let us try to put the approach suggested above in historical perspective. The various approaches to the strong CP problem are charted in Table I. Virtually all successful models based on spontaneously broken CP invariance have been based on having M_Q complex at tree level, so as to generate a sufficiently large ϵ through the Kobayashi-Maskawa mechanism, and yet having $\det M_Q$ real at tree level to make $\bar{\Theta}$ sufficiently small. This requires a special form for M_Q. The simplest and most satisfactory of these models are those of the type discovered by Nelson.[19] A simpler ap-

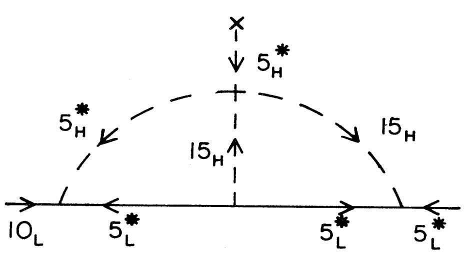


FIG. 1. The lowest-order contribution to $\bar{\Theta}$ in the models described in the text comes from such two-loop diagrams.

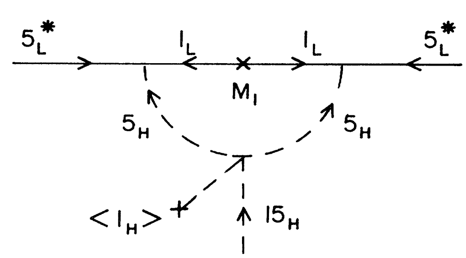
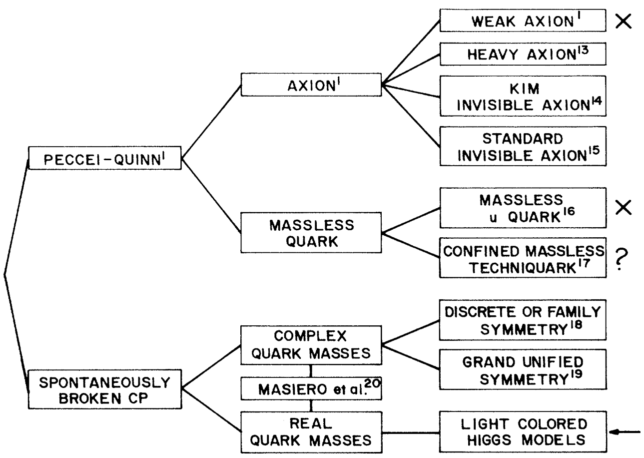


FIG. 2. An effective CP-nonconserving piece of the $(5_L^* C 5_L^*)15_H$ vertex comes from this diagram in model B in the text.

TABLE I. Chart of various ways to solve the strong *CP* problem. The arrow indicates the approach in this paper.



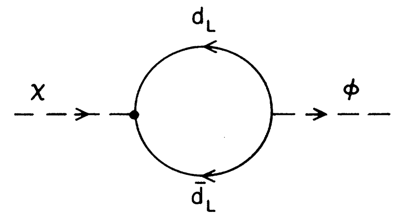


FIG. 3. If χ is colorless, it will generally be forced to have a VEV by $\chi^\dagger\phi$ terms in the Higgs potential which cannot be ruled out by symmetries since diagrams such as this must exist.

proach would seem to be to have M_Q completely real at tree level and have ϵ generated through some non-Kobayashi-Maskawa mechanism such as a $\Delta S=2$ Higgs boson exchange. This was first suggested in Ref. 10. That paper, however, only considered the case in which the Higgs bosons χ which mediated the superweak force were color-singlet, weak doublets like the Weinberg-Salam Higgs bosons. This leads to a fatal difficulty. Such χ will naturally acquire a VEV when $SU(2)\otimes U(1)$ breaks. Such a VEV must contribute, of course, to M_Q. And, since the VEV of the ordinary Weinberg-Salam doublet, ϕ, and that of χ will (naturally) have a *CP*-nonconserving relative phase, M_Q will be complex at tree level and $\overline{\Theta} >> 10^{-9}$. Attempts to protect χ from acquiring a VEV by imposition of symmetries are futile as shown in Ref. 10. The essence of the argument is simple: Both ϕ and χ couple to the quarks. Therefore, diagrams like Fig. 3 exist, and therefore, it cannot be possible to rule out terms like $\chi^\dagger\phi$ in the Higgs potential, which will induce $\langle\chi\rangle\neq 0$.

The idea that we put forward here is essentially that of Ref. 10 with the crucial difference that the χ here are taken to be colored. Obviously, the difficulty encountered there does not occur here: As long as color is unbroken, $\langle\chi\rangle=0$ and cannot contribute to M_Q.

The crucial feature of this model is that all phases are drained from M_Q at tree level, thus severing the link between $\overline{\Theta}$ and ϵ until two-loop order.

A model which has some similarities to the present idea was proposed by Masiero, Mohapatra, and Peccei.[20] In their left-right model $\Theta_{\rm QFD}\neq 0$ even at *tree* level; however, it is of order $(M_{W_L}/M_{W_R})^2$, the ratio of the squares of scales of the breaking of $SU(2)_L$ and $SU(2)_R$, which is itself, of course, a technically unnatural hierarchy. In common with the present mechanism their model has the features that M_Q as a whole is nearly real, and that the $\Delta S=2$, superweak, *CP*-nonconserving force is mediated by color-**6** fields. (See Table I.)

Since our mechanism requires the existence of light colored scalars, we must look at the effect[21] on $\sin^2\theta_{\rm W}$. Introduction of one light scalar in a $(6,1,-\frac{2}{3})$ of $SU(3)\otimes SU(2)\otimes U(1)$ will change $\sin^2\theta_{\rm W}$ by -0.02. This is tolerable, but perhaps makes $\sin^2\theta_{\rm W}$ a little small for comfort. With two such light $(6,1,-\frac{2}{3})$ representations as in model A (or a unified version of Nièves's model), $\delta(\sin^2\theta_{\rm W})=-0.04$, which gives $\sin^2\theta_{\rm W}$ too small. However, one can improve matters if in the $\mathbf{15}_H$ both the $(6,1,-\frac{2}{3})$ and the $(1,3,1)$ are light [the $(3,2,\frac{1}{6})$ can cause proton decay and must be superheavy]. Then $\delta(\sin^2\theta_{\rm W})=-0.0077$ for each such $\mathbf{15}_H$.

As a first concluding remark, we emphasize that the preceding models are only meant as illustrations of a very general mechanism, the key ingredients of which are three: (1) *CP* a spontaneously broken symmetry; (2) a light colored scalar to mediate a superweak force; (3) a single Weinberg-Salam doublet (or some other assumption) that ensures that $\Theta_{\rm QFD}^{\rm tree}=0$. Obviously these requirements are not very restrictive and can be satisfied in a wide class of theories.

The second remark regards baryon- and lepton-number–nonconserving effects. As is well known,[3] the minimal SU(5) conserves $B-L$ because of a global symmetry with quantum numbers $X(\mathbf{10}_L)=1$, $X(\mathbf{5}_L^*)=-3$, and $X(\mathbf{5}_H)=-2$. This symmetry is preserved in our models with $X(\mathbf{15}_H)=6$. If desired, one can also readily break[3,22] $B-L$ by a cubic Higgs coupling $\rho\mathbf{5}_H^*\mathbf{5}_H^*\mathbf{15}_H$. This violates $B-L$ by two units and generates a neutron-antineutron oscillation matrix element, $M_{n\bar{n}}$, of order

$$M_{n\bar{n}}\sim h_{11}(g_{11})^2\rho M_n^6/M^2M_5^4 \qquad (6)$$

(where M and M_5 are the masses of χ and the color triplet in $\mathbf{5}_H$). It has been emphasized[23] that with scalar fields transforming as color **6** one can construct a model with observable n-$\bar{n}$ oscillation without excessive proton decay. The breaking of $B-L$ also allows the appearance of neutrino Majorana masses, the mechanism being that the $\langle 5_H \rangle$ induces a vacuum expectation value for the field which is a (1,3,1) of SU(3) ⊗ SU(2) ⊗ U(1) contained in the $\mathbf{15}_H$. The neutrino Majorana masses are of order $\rho h_{mn} \times \langle 5_H \rangle^2 / M_{\text{triplet}}^2$. If M_{triplet} is around 10^{15} GeV, then M_{ν_e} comes out typically fairly small $\{M_{\nu_e} \cong (h_{11}/g_{11})[\rho/(10^{15}\text{ GeV})] \times 10^{-7}\text{ eV}\}$. However, with a large value of ρ [which is probably required by Eq. (6) to give observable n-$\bar{n}$ oscillations], one could also get observable neutrino masses. On the other hand, if the (1,3,1) of Higgs bosons is light as suggested above to get a better fit for $\sin^2\theta_W$, then ρ had better be zero (e.g., by the global symmetry), or else M_{ν_e} willl be far too large.

The third remark is that a characteristic feature of the models described here is that CP nonconservation is absent in the D-$\bar{D}$ system as has been noted independently by Nièves.[8]

Fourthly, the color-**6** field, χ, may be produced in pairs by a gluon in hadronic collisions.[24] Once produced, the χ may grab two quarks to form a new class of hadron $(\chi\overline{qq})$ with mass of order of hundreds of gigaelectronvolts. If h_{33} is the largest coupling, as we have assumed, then these hadrons will tend to decay rapidly into the bottom channel. Abandoning our theoretical prejudice that the χ mass is weak scale, we can also speculate on the phenomenology of new hadrons of the form $(\chi\overline{dd})$, $(\chi\overline{du})$, $(\chi\overline{uu})$, and so forth with masses in the tens of gigaelectronvolt range and whose decays into channels such as $(ss\overline{uu}) = (K^-K^-)$ and $(sb\overline{uu}) = (B^-K^-)$ provide striking signatures.[25] In this case, according to Eq. (3), the couplings h_{11} and h_{22} would have to be smaller by a factor of 10 than what we had stated. $\bar{\Theta}$ would be smaller by a factor of 10^2.

Finally, one might consider making such a model supersymmetric. Several potential difficulties would have to be dealt with. Since there are two light Higgs doublets, H and H', one would have to ensure that the phase of μ (in $\mu HH'$) was sufficiently small. At tree level this would be the guaranteed by CP invariance. Also one must worry about the phases of the masses of the fermionic partner of the χ $(\tilde{\chi})$ and about gluino-quark couplings. The color-**6** fermion, $\tilde{\chi}$, could be of use in condensing to generate the weak scale.[26] These questions are being investigated.

(a)Present address: Brookhaven National Laboratory, Upton, N.Y. 11973.

(b)Present address: Institute for Theoretical Physics, University of California, Santa Barbara, Cal. 93106.

[1]R. Peccei and H. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).

[2]A. Zee, University of Washington Report No. 40048-18 P5 (to be published).

[3]F. Wilczek and A. Zee, Phys. Lett. **88B**, 311 (1979).

[4]A. Zee, Phys. Lett. **93B**, 389 (1980).

[5]J. Nièves, Nucl. Phys. **B189**, 189 (1981).

[6]V. Silveira and A. Zee, Phys. Lett. B (to be published).

[7]A. Zee, Nucl. Phys. B (to be published).

[8]J. Nièves, to be published.

[9]See also Taejin Lee, to be published.

[10]S. Barr and D. Seckel, Nucl. Phys. **B233**, 116 (1984).

[11]V. Baluni, Phys. Rev. D **19**, 227 (1979); R. Crewther, P. DiVecchia, G. Veneziano, and E. Witten, Phys. Lett. **88B**, 123 (1979).

[12]N. Ramsay, Phys. Rev. **43**, 409 (1978).

[13]S.-H. Tye, Phys. Rev. Lett. **47**, 1035 (1985); B. Holdom, Nucl. Phys. **B208**, 397 (1982), and Phys. Lett. **154B**, 316 (1985), and **156B**, 452(E) (1985).

[14]J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979).

[15]A. Vainshtein and V. Zakharov, Nucl. Phys. **B166**, 493 (1980); M. Dine, W. Fischer, and M. Srednicki, Phys. Lett. **104B**, 199 (1981); H. P. Nilles and S. Raby, SLAC Report No. SLAC-Pub-2743, 1981 (unpublished); M. Wise, H. Georgi, and S. L. Glashow, Phys. Rev. Lett. **47**, 402 (1981). For an attempt to hide the weak axion, see D. Kaplan, to be published.

[16]H. Georgi and A. MacArthur, to be published.

[17]S. Dimopoulos and L. Susskind, unpublished.

[18]M. A. B. Beg and H.-S. Tsao, Phys. Rev. Lett. **41**, 278 (1978); R. N. Mohapatra and G. Senjanovic, Phys. Lett. **79B**, 283 (1978); H. Georgi, Hadronic J. **1**, 155 (1978); G. Segrè and H. A. Weldon, Phys. Rev. Lett. **42**, 1191 (1979); S. Barr and P. Langacker, Phys. Rev. Lett. **42**, 1654 (1979). A more recent idea is in H. Georgi, A. Nelson, and M. Shin, Phys. Lett. **150B**, 306 (1985).

[19]A. Nelson, Phys. Lett. **136B**, 387 (1984), and **134B**, 422 (1984); S. Barr, Phys. Rev. D **30**, 1805 (1984), and Phys. Rev. Lett. **53**, 329 (1984); A. Nelson, Phys. Lett. **143B**, 165 (1984).

[20]A. Masiero, R. N. Mohapatra, and R. D. Peccei, Nucl. Phys. **B192**, 66 (1981).

[21]K. Hagiwara, F. Halzen, and K. Hikasa, Phys. Lett. **92B**, 103 (1980), and **94B**, 557(E) (1980).

[22]L. N. Chang and T. P. Chang, Phys. Lett. **92B**, 103 (1980).

[23]D. Seckel and A. Zee, in Proceedings of the Harvard Conference on Neutron Oscillations, 1982 (unpublished), p. 33.

[24]G. Karl, Phys. Rev D **14**, 2374 (1976); F. Wilczek and A. Zee, Phys. Rev. D **16**, 860 (1977); A. Yu. Ignatiev, V. A. Kuzmin, and M. E. Shaposhnikov, Institute for Nuclear Research, Moscow, Report No. P-0283, 1983 (unpublished).

[25]We thank H. Lubatti for a discussion on the experimental propects.

[26]We thank P. Ramond for this remark. See W. Marciano, Phys. Rev. D **21**, 2425 (1980).