Microchips as Precision Quantum-Electrodynamic Probes

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We show that QED fluctuations for present metal-oxide-semiconductor field-effect transistor microcircuit devices are substantial and thus cannot be ignored. For example, the Casimir energy of a gate-region capacitor is typically $\frac{1}{10}$ of the electrostatic energy. Measurement of their electronic transport properties is suggested as a tool to learn about some nonperturbative aspects of QED which cannot be reached via high-energy or atomic physics.

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It is the purpose of this Letter to note that the *present* microchip technology has already reached a stage where purely quantum-electrodynamic (QED) energy functions (e.g., the Casimir energy) are significant ($\sim 10\%$) in comparison to the stored Coulomb energy. Thus, in any theoretical analysis, we must of necessity include QED effects. In fact, present microelectronics technology may be exploited with little effort to provide a beautiful probe into the detailed structure of the field theory underlying QED—at a level which is unimaginable in high-energy physics.

Consider a generic field-effect transistor (FET) device with a gate-region capacitor with surface charge density per unit area $\sigma = ne$ and distance d between the plates. The (stored electrostatic energy)/area in the capacitor is given by

$$u_1 = 2\pi\sigma^2 d/\epsilon, \tag{1}$$

where ϵ is the dielectric constant. On the other hand, the (Casimir energy)/area due to QED fluctuations is given by

$$u_2 = -(\hbar c/8\pi^2\sqrt{\epsilon}d^3)\pi^4/90. \tag{2}$$

The dimensionless ratio

$$\beta = |u_2/u_1| = (\pi/1440\alpha)\sqrt{\epsilon}/(nd^2)^2 \tag{3}$$

measures the relative weight of photon oscillation in QED versus electrostatic contributions to the energy. The fine structure constant $\alpha = e^2/\hbar c \cong \frac{1}{137}$. Typically, we have $\epsilon \sim 4$, electron density $n \sim 10^{11}/\text{cm}^2$, and $d \sim 500$ Å. This gives

$$\beta \cong 0.1,$$

the figure quoted in the introduction. This proves our claim that QED effects are indeed sizable and must be retained. Moreover, as β increases further ($\beta \gtrsim 1$), the above Casimir analysis breaks down since other nonlinear QED effects of the type discussed below must be included.

The next interesting discovery is that not only are QED effects of significant size for a metal-oxide-semiconductor field-effect transistor device, but that the latter can provide a probe into those structures of QED which are unreachable by standard high-energy experiments or by low-energy but highly accurate atomic physics experiments.

High-energy physics experiments essentially test (rather accurately) the perturbative aspects of QED in the scattering of one charged particle by another. Precision measurements of atomic energy levels test small QED corrections to the binding energy. Another celebrated example is the $g-2$ measurement, where once again perturbative QED corrections are successfully compared with data. In all of the above, one is probing perturbative QED. On the other hand, microelectronic devices can allow us to investigate accurately some *nonperturbative, collective* aspects of QED as shown below.

In standard computations of electronic transport properties at low energies, typically in the millielectronvolt to electronvolt range, QED is justifiably ignored on the grounds that energies in question are so very much smaller than the "threshold" $2m_ec^2$ ($\sim 10^6$ eV) necessary for sizable QED fluctuations. However, there are well-known cases where this perturbative argument does not apply. One example countering the threshold-energy-barrier argument is already provided by the (nonperturbative) Casimir term u_2 in Eq. (2), which is independent of the charged particle mass (as well as independent of e^2 or α). For microchips, since both n and d are small, β becomes large.

Another example that violates the threshold-barrier rule has been extensively discussed by us in connection with the electronic transport properties of FET's subjected to external electromagnetic fields.[1] Starting from the physical, four-component massive Dirac equation, it has been shown that the time rate of change of the electronic spin density is independent of the electron mass. In particular, we find that for a capacitor at voltage V with a magnetic flux Φ passing through the capacitor plates, spin waves exist that involve positron degrees of freedom down to virtually zero excitation energy.[2]

The chirality phase θ of gauge theories[3] finds here a transparent physical meaning also. For the QED problem mentioned in the last paragraph, θ is related to the ratio of charge to flux within the capacitor by[2]

$$\theta = (4\pi^2/\alpha) Q/\Phi. \qquad (4)$$

The presence of α in the denominator is proof positive that we are dealing with nonperturbative QED. The physical energies are periodic in θ. The implications of the generated vacuum current are far reaching for practical devices. They cause real, measurable effects in macroscopic quantities like voltage, current, charge, etc. A specific example is considered below. Theoretically, we would be finding out what QED really predicts for the *inside* of a macroscopic dipole. Such understanding in Abelian QED could be of considerable help in solving the much more difficult problem of non-Abelian QCD for example.

Our last application illustrates how the energy threshold barrier can be overcome in yet another way. The notion of periodicity in θ implies that the chiral energy $W(\theta+2\pi) = W(\theta)$, so that quite generally the effective energy for the macroscopic capacitor can be written as

$$W(\theta) = \sum_{n=1}^{\infty} h\nu_n [1-\cos(n\theta)], \qquad (5)$$

$$U(V,\Phi) = \min_Q [Q^2/2C - QV + W(4\pi^2 Q/\alpha\Phi)], \qquad (6)$$

where C is the geometric capacitance and θ is as given in Eq. (4).

The expression for chiral energy in Eq. (6) is more accurate than the one-loop calculation giving the Casimir energy. The tunneling frequencies ν_n for this geometry can be obtained from the Dirac equation. We find $h\nu_n \sim (m_e c^2)\sqrt{\alpha}(\Phi/\Phi_0) d/(\epsilon A)^{1/2}$, where A is the area of the capacitor. However, observe that while the first two terms in Eq. (6) are extensive, i.e., depend upon all electrons, the coefficient of the last term is intensive—it depends upon the mass of a single electron. Evidently, there can be situations where the last term is of the same order of magnitude as the other two. In such cases, the equation of state exhibits hysteresis.

This can be seen through the following simple example. If we only retain $n=1$ transitions, the gate voltage as a function of the filling factor $\nu = \theta/2\pi$ may conveniently be written as

$$V_g(\nu) = V_1\nu + V_2\sin(2\pi\nu), \qquad (7)$$

where $V_1 = 2d\alpha B/\epsilon$ and $V_2 = 4\pi^2 h\nu_1/\alpha\Phi$. For fixed electron density (σ constant) the second term rapidly oscillates to zero as $B \to 0$. Also, in the classical limit ($h \to 0$) the second term vanishes. For usual large-area capacitors ($A \to \infty$) the second term once again vanishes in comparison with the first. On the other hand, for a microcapacitor, as the second term becomes substantial, interesting nonlinearities develop as shown in Fig. 1(a) for various values of $\gamma = V_2/V_1$. $V_g'(\nu)$ changes sign for the critical value $\gamma^* = 1/2\pi$, which signals the onset of hysteresis. The Maxwell construction, as in Fig. 1(b), is required for $\gamma > \gamma^*$. This is our explanation for the effect observed in a metal-oxide-semiconductor field-effect transistor device by Pudalov, Semenchinsky, and Edelman.[4]

We note in passing that the work of Pruisken[5] on quenched averaging over impurity potentials also yields θ states, and so we indicate the similarities and differences between the above considerations and the impurity-averaging approach. (i) The electrodynamic U(1) gauge group ultimately determines the θ-state periodicity, although other groups are involved. In our work chiral rotations of the Dirac equation are used, and in the work of Pruisken the non-Abelian group $U(n,n)/U(n)\times U(n)$ is involved in the intermediate stages of the impurity-averaging process. The final θ-

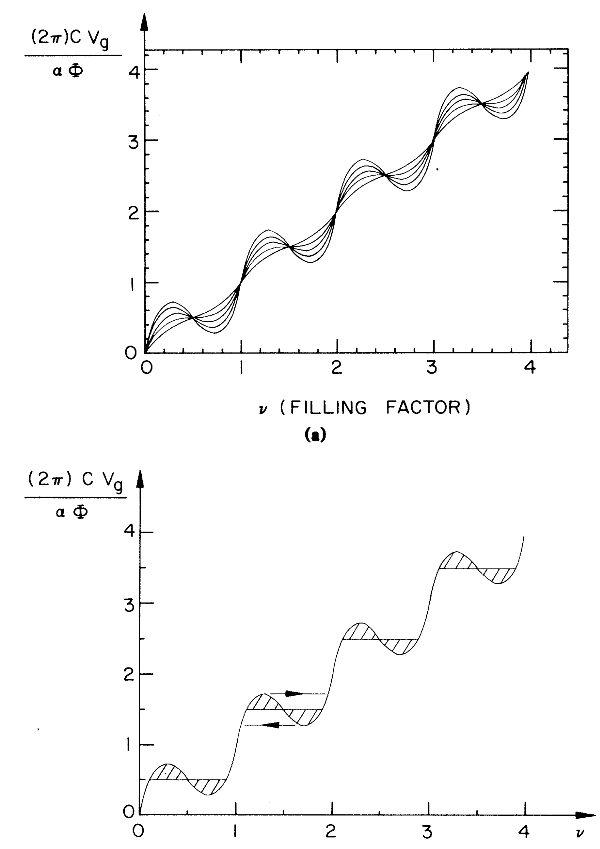

FIG. 1. (a) Plot of V_g vs ν (filling factor) as in Eq. (7) for various values of γ. (b) Same as in (a) for $\gamma > \gamma^*$ along with the Maxwell construction (diagonal lines).

state periodicity is *virtually identical* if the step Hall-conductance values are related to the filling factor in the usual manner. (ii) In both approaches there is the notion that *all* of the hole-state Landau levels below the Fermi level enter into the determination of the Hall conductance. This last point allows for a concise statement of the differences between the two approaches. (iii) *All* of the hole-state Landau levels below the Fermi level means (to us) the inclusion of the Landau levels of genuine positron states implicit in the four-component Dirac equation in three spatial dimensions. In all other theories, the nonrelativistic Pauli equation limit is used and in strictly two spatial dimensions. This quantitatively effects the *strength* of the θ-state energy periodicities since our strengths scale with the rest mass of the electron and, inversely, as the area of the gate capacitor, while in the impurity-averaging approach the θ-state energy variations scale with impurity energy fluctuations *intensively;* e.g., *per unit area* a microchip or huge gate capacitors of several farads would have the *same* intensive oscillating θ-state energy. (iv) Finally, consistent with our view of treating an FET device as a quantum electrodynamic circuit element, the whole notion of conductance enters into the theory as the self-energy part of the photon propagator. This approach is not taken by any other workers in the field (to our knowledge) although it is a well-known approach previously used to describe the Casimir effect by Lifshitz and Pitaveski,[6] and also used by Schwinger, DeRaad, and Milton[7] in the Casimir-effect context.

In summary, we have shown that even present FET devices exhibit sizable QED fluctuations. Experimentally observed hysteresis and phase transitions for such devices are interpreted here as being due to topological θ oscillations of QED. This analysis can be naturally extended to study other facets of vacuum polarization and Dirac sea fluctuations induced by external electromagnetic fields on FET's. Details shall be presented elsewhere.

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