Nonexistence of a Chiral Schwinger Model

The two-dimensional version of QED was first proposed by Schwinger[1] in 1962. Its solvability together with the fact that it has a massive "photon" has made it a useful vehicle for the discussion of a great number of topics in quantum field theory.

In 1973, this writer[2] examined the possibility of coupling to the vector potential not the vector current j^μ, but rather the chiral combinations $\frac{1}{2}(j^\mu \pm \epsilon^{\mu\nu} j_\nu)$, where $\epsilon^{\mu\nu}$ is the Levi-Civita tensor. It was found, however, that because of the well-known anomalies which occur in such theories, the consistency of the field equations could be ensured only if an intrinsic bare mass were given to the vector field. Calling that mass μ_0, it was found that the meson propagator has the form[3]

$$G^{\mu\nu} = \frac{1}{1-e^2/4\pi\mu_0^2}\left[\left(g^{\mu\nu} - \frac{\partial^\mu\partial^\nu}{\mu^2}\right) - \frac{e^2}{4\pi\mu_0^2}\frac{\partial^\mu\partial^\nu}{\mu_0^2}\right]\Delta(x) + \frac{e^2}{4\pi\mu_0^2}(\partial^\mu \pm \epsilon^{\mu\sigma}\partial_\sigma)(\partial^\nu \pm \epsilon^{\nu\tau}\partial_\tau)\frac{1}{\mu_0^2}[\Delta(x) - D(x)],$$

where the functions $D(x)$ and $\Delta(x)$ are, respectively, the propagators for fields of zero mass and mass

$$\mu^2 = \frac{\mu_0^2}{1-e^2/4\pi\mu_0^2}.$$

Using the complete set of propagators given in Ref. 2 one can proceed to demonstrate the fact that they constitute a solution which is entirely compatible with the field equations. It is to be noted that the limit of μ_0 going to zero is not allowed, however, since it would (a) imply tachyonic modes and (b) contradict the condition of current conservation which is built into the Maxwell equations.

Recently, however, Jackiw and Rajaraman[4] (JR) have attempted to construct a chiral Schwinger model. It is to be noted that although these authors start from a $\mu_0 = 0$ theory, they immediately arrive at an effective Lagrangean which has essentially a mass built into it through a parameter a. It is this theory which they proceed to solve, and it can be verified that the solution they obtain is identical to that quoted above when notational differences are taken into account. More specifically, this is accomplished by replacing the e^2 of JR with $e^2/4\pi$ and subsequently identifying a with $4\pi\mu_0^2/e^2$. Upon noting that the solution of Ref. 2 is known to satisfy the equations of motion with nonzero bare mass μ_0, it is then of interest to ask in what sense a corresponding statement applies for the JR solution. One finds that this necessarily requires the introduction of an additional term into their field equations which involves the parameter a and which is ultimately equivalent to an explicit bare-mass term.

In order to display this result in terms which are less dependent upon laborious calculations and clearer with regard to the underlying physics, it is of some interest to present an argument which suggests that the results of Ref. 4 are intrinsically implausible merely by virtue of the claimed mass spectrum. Thus one compares the bosonic particles in (a) the Thirring model, (b) the Schwinger model, and (c) the case of the massive vector meson coupled to the usual current operator, noting in advance the well-known fact that the current operator constructed from fermions has a zero-mass particle in its spectrum. Thus, the Thirring model has a single zero-mass boson which persists in the limit of zero coupling. Likewise the Schwinger model has a single (massive) boson which is actually the fermion-antifermion state with mass renormalized to a finite value. Since it is well known that the Maxwell field operators in two dimensions are merely functions of the current operator (i.e., they do not represent independent kinematical degrees of freedom), the existence of a *single* boson is reasonable.

Upon considering the vector-meson case one finds[5] the existence of both the massive and massless bosons. Such a result could be anticipated inasmuch as the massive vector field has a single degree of freedom of prescribed bare mass. It can consequently be asked whether the result of Ref. 4 could be considered plausible in view of these results. The answer to this question appears to be negative since in all the known solvable two-dimensional theories, the bosonic modes are all traceable either to the massless particle of the current operator or to an intrinsic boson put in at the outset. The claim of Jackiw and Rajaraman that a second boson exists in the Schwinger model as a dynamically generated rather than a kinematically specified (bare) mass is thus at variance with all previously known cases. While this argument is admittedly heuristic, its relevance is strongly borne out by the calculations presented above.

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