# Self-Reflected Wave inside a Very Dense Saturable Absorber

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The numerical solution of the Maxwell-Bloch equations describing the propagation of a plane monochromatic wave through a saturable absorber shows the appearance of a new kind of reflected wave generated inside the medium. This new phenomenon is described and its physical origin, related to the saturation-induced refractive index changes, is discussed.

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The propagation of a laser beam through a nonlinear medium can display certain characteristic features such as self-focusing[1] and self-induced transparency.[2] In this Letter a new result concerning the propagation of a plane monochromatic traveling wave that penetrates a very dense saturable absorber is presented. I show now that under appropriate conditions a reflected wave can be generated well inside the nonlinear medium. The generation mechanism is not the coupling to other beams, as in four-wave mixing. It is not a boundary reflection either. Its origin is due to the saturation-induced refractive index changes across the absorber.

Two clearly defined regions appear in the study of the propagation of an intense incident field through a saturable absorber.[3] In the first region, close to the boundary, the field saturates the medium and the relative absorption is not too high. When the decreasing amplitude of the wave arrives at nonsaturating values, absorption grows quickly and determines the beginning of the second region, the unsaturated region. In this Letter I analyze situations—a saturable medium with a very large absorption coefficient—in which the pass from one to the other region is accomplished over less than a wavelength. If the properties of a system change in such a way, a reflected wave may appear. I show here, by a numerical calculation, the existence of this reflected wave generated inside the nonlinear medium. To employ a standard denomination it will be called a self-reflected wave.

The study of the laser-absorber interaction is made here in the most simple case where this phenomenon occurs. The field is considered as a plane monochromatic wave, varying only along the $z$ axis. It can be written in the form $E(z)\exp(-i\omega_L t)$; I do not try to give an explicit spatial dependence to the field. On the other hand, a great density of homogeneously broadened two-level systems constitute the absorbing medium. I assume that it fills completely the half-space $z \geqslant 0$, and that every two-level absorber acts completely independent from the others.

The propagation of the field is described by the Maxwell-Bloch equations.[4] The steady-state solution is given by

$$\gamma D = -(i\mu/\hbar)(EP^* - E^*P) + \gamma N, \tag{1}$$

$$2(\gamma - i\Delta)P = (i\mu/\hbar)DE, \tag{2}$$

$$d^2E/dz^2 = -k_0^2(E + 4\pi\mu P). \tag{3}$$

Here $\mu$ is the transition dipole moment, taken to be parallel to the field; $\mu P$ is the polarization; $D$ is the population difference; $N$ is the density of absorbers; $k_0 = \omega_L/c$ is the field wave number, in vacuum; and $\Delta = \omega_L - \omega_T$ is the detuning, $\omega_T$ being the transition frequency. To simplify, only a homogeneous relaxation rate $\gamma$ is considered. This relaxation parameter will be considered as constant regardless of the field intensity, although this assumption has been recently questioned.[5]

I introduce a dimensionless function, proportional to the field at each point $z$, defined by

$$F(z) = \mu E(z)/2\hbar\gamma. \tag{4}$$

The substitution of (1), (2), and (4) into the propagation Eq. (3) gives

$$\frac{d^2F}{dz^2} = -k_0^2 F\left[1 + \psi\frac{i\gamma^2 - \Delta\gamma}{\gamma^2 + \Delta^2 + 4\gamma^2 FF^*}\right], \tag{5}$$

where

$$\psi = 2\pi\mu^2 N/\hbar\gamma \tag{6}$$

is a dimensionless parameter that accounts for the strength of the coupling with the field and for the density of absorbers. Obviously, Eq. (5) describes the field propagation only inside the medium (i.e., for $z \geqslant 0$). For $z < 0$ the field will be a solution of the vacuum wave equation [the same as (5) but with $\psi = 0$]. At this point it is common to introduce the slowly varying envelope approximation. Nevertheless, given the nature of the effects we are interested in, we will not make this approximation and we will solve numerically the second-order differential Eq. (5).

As boundary conditions of (5) at $z = 0$ we take $F$ and its derivative to be continuous at this plane:

$$F(0)=F_I+F_R, \tag{7}$$

$$dF/dz(0)=(F_I+F_R)ik_0, \tag{8}$$

where the parameters $F_I$ and $F_R$ are related to the incident, $E_I$, and reflected, $E_R$, field complex amplitudes, respectively, by expressions analogous to (4).

Since we are dealing with an absorber and the incident field comes forward from $z=0$, the only solutions of (5) that will have a physical meaning to us will be those satisfying $|F(z)|\to 0$ as $z\to\infty$. Therefore, we can assume that $|F(z)|<<1$, at large enough values of $z$. The saturation term in (5) is then negligibly small and the wave equation becomes linear. The solutions of this linear wave equation vanishing at large values of $z$ are forward propagating waves,

$$F(z)=F(z_0)\exp[ik_0n(z-z_0)], \tag{9}$$

$n$ being the complex refractive index,

$$n^2=1+\psi\gamma(i\gamma-\Delta)/(\gamma^2+\Delta^2). \tag{10}$$

Concerning the numerical computation of the solutions of (5), it is not feasible to start it at the vacuum-absorber interface since we do not know the value of the wave reflected at this plane. Instead, our strategy implies the beginning of the computation at a large value $z_0$, introducing a small value of the rescaled field, $|F(z_0)|<<1$, and of its derivative, $ik_0nF(z_0)$, according to (9). The calculation of the solution determined by these starting values has been given by a simple numerical procedure based on a Taylor expansion up to the sixth order around every computed point, ending at $z=0$. From the values of the field, and its derivative at $z=0$, Eqs. (7) and (8) give the values of the incident and reflected waves.

To obtain a self-reflected wave the medium must have a high absorption coefficient, so high that the imaginary part of the complex refractive index must be much greater than the real one. The results to be shown in this Letter correspond to a particular significant case where this wave appears. The wave can also appear at lower values of the absorption coefficient, but it always vanishes when the parameter $\psi$ approaches unity because in this case, even in resonance, the real and imaginary parts of the refractive index become similar.

Figure 1 shows the computed values of the field amplitude at each point of the absorber, corresponding to $\psi=100$ and $\Delta=2.8\gamma$, for three different values of the incident field. According to (10) the complex refractive index in this case is $n=1.00+i5.63$. The detuning $\Delta$ has been chosen adequately to work in a purely absorptive case. When the incident field is low enough for the linear approximation to be valid, the field in the absorber is given by (9). Those values of $\psi$ and $\Delta$ imply a strong attenuation of the field, as shown in Fig. 1(a), for $F_I=6$, one of the highest values of $F_I$ where nonlinearities are still nondominant. On increase of the incident field the linear approximation fails completely and the field inside the absorber displays two clearly defined regions [Figs. 1(b) and 1(c)]. Next to the boundary there is a region which presents a node-antinode pattern and corresponds to values of the field that introduce a high saturation degree. The second region begins as soon as saturation is not dominant and the field is quickly absorbed. These two regions correspond to the ones indicated at the beginning, the saturated region, next to the boundary, and the unsaturated region.

The high value of the coupling parameter $\psi$ needed precludes the use of the slowly varying envelope approximation in the solution of the wave equation, since the absorption over a wavelength is not negligible. We can, however, approximately fit the field inside the absorber in the saturated region by a superposition of two counterpropagating waves of variable amplitude. The best fit to the field given in Fig. 1(c) is

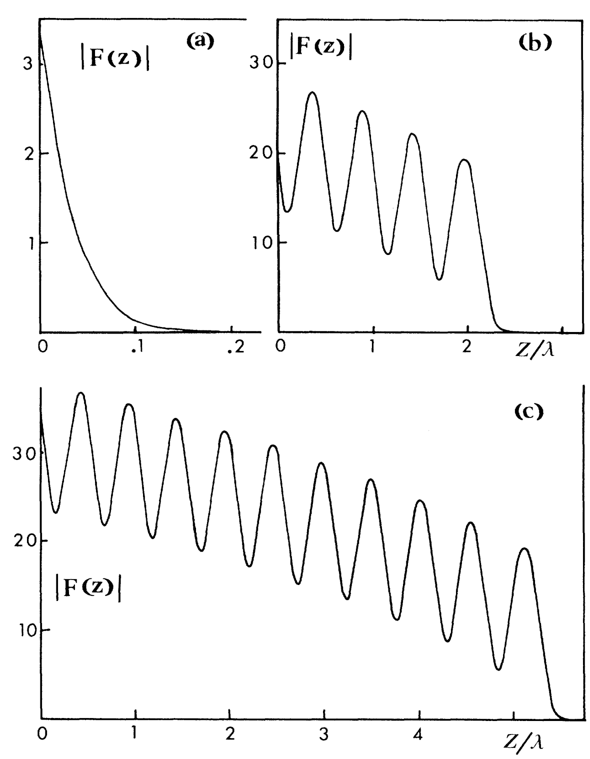


FIG. 1. Spatial profile of the rescaled total amplitude inside the absorber, showing clearly the node-antinode structure that evidences its standing-wave nature. The three figures correspond to the same absorber ($\psi=100$ and $\Delta=2.8\gamma$), but to three different values of the incident field amplitude: (a) $F_I=6$; (b) $F_I=15$; and (c) $F_I=30$. Notice that (a) has both scales enlarged by a factor 10.

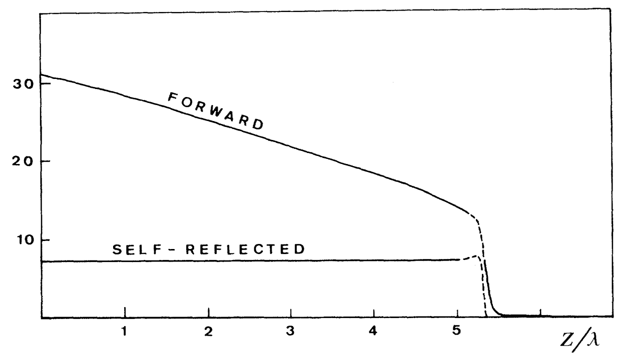


FIG. 2. Approximate values of the forward and self-reflected field amplitudes inside the absorber (solid lines). Broken lines indicate the region where these two components are undefined. This figure corresponds to the same parameters as in Fig. 1(c).

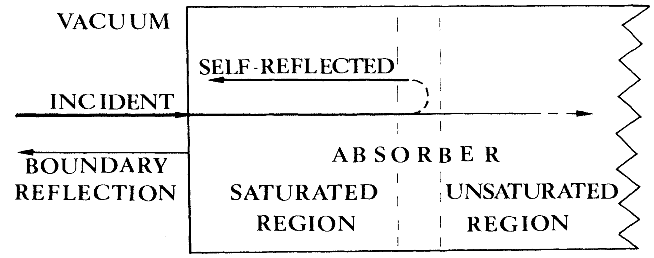


FIG. 3. Schematic representation of the phenomenon. The only thing that we can actually measure is the total reflected wave, resulting from the superposition of the wave reflected at the boundary and the transmitted part of the self-reflected wave. Obviously, this wave experiences a reflection at the $z=0$ boundary (induced by the boundary itself) and contributes to some extent to the forward field, but for simplicity this is not indicated in the figure.

shown in Fig. 2. When we arrive at the unsaturated region this kind of fitting is not valid.

It is obvious that a phenomenon like self-reflectivity cannot be explained at all within the slowly varying envelope approximation because there is not any polarization at all that allows the coupling between the forward and backward components of the field.

Up to now we have considered the forward and self-reflected wave only inside the absorber. To relate them with the incident and reflected fields outside the absorber—the only two fields we can actually measure—we have to take into account the boundary reflection at $z=0$. Notice that it is described merely by (7) and (8) since we do not introduce any reflecting surface other than the polished face of the absorber boundary itself. So the forward field can be understood as the superposition of the incident field that enters the medium plus the reflection of the backward wave at this plane. The total reflected field, $F_R$, calculated by (7) and (8) can be understood as the superposition of the field reflected by the boundary itself plus the transmitted part of the self-reflected wave, as shown schematically in Fig. 3. As will be shown elsewhere[6] the total reflected intensity exhibits strong nonlinearities and even bistable loops can appear when it is plotted versus the input light intensity.

Finally let us point out that to our knowledge this effect has not been expermentally observed. Perhaps this is due to the high saturation and absorption needed for its appearance. We think, however, that in certain situations, like the excitonic resonances or even the sodium vapor, it could be observed. Another possible difficulty for its observation is the influence of other effects neglected here, mainly interactions between the absorbers, that can strongly modify the propagation of the field.[7]

In conclusion, I have shown that in the theoretically simple case of the propagation of a plane monochromatic wave through a saturable absorber, a reflected wave can be generated inside it due to the saturation-induced refractive index changes.

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