Does the Cabibbo Angle Vanish in Fermi Matrix Elements of High-J States?

E. G. Adelberger, P. B. Fernandez, C. A. Gossett, J. L. Osborne,[a] and V. J. Zeps

Nuclear Physics Laboratory, University of Washington, Seattle, Washington 98195

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We have reinvestigated the β^+ decay of ^{24}Al(4^+) and find that the analog transition has $\mathscr{F}t = 3106 \pm 38$ sec. This is consistent with the value $\mathscr{F}t = 3081.7 \pm 1.9$ sec obtained from the $0^+ \rightarrow 0^+$ pure Fermi transitions but inconsistent with the value $\mathscr{F}t = 5715 \pm 13$ sec claimed for the Fermi component of ^{35}Ar decay. We find no evidence for the speculation that Fermi matrix elements of high-J states are anomalously large and conclude that the ^{35}Ar result is probably due to experimental error.

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The β-decay vector coupling constant G_β^V can be determined very precisely from the Fermi matrix elements of $(J=0) \rightarrow (J=0)$ transitions (see, for example, Hardy and Towner[1]). It is found that G_β^V is slightly smaller than G_μ, the coupling constant for μ decay. This is expected because $G_\beta^V = G_F \cos\theta_1$ where θ_1 is a quark mixing angle (we shall loosely refer to θ_1 as the Cabibbo angle) while $G_\mu = G_F$. (Radiative corrections to these expressions are considered below.) From the measured ratio of G_β^V/G_μ one finds[2] that $\cos\theta_1 = 0.9730 \pm 0.0024$. This value of θ_1 agrees well with that deduced from hyperon decay widths which are proportional to $\sin^2\theta_1$.

The constant G_β^V can also be measured in isospin-analog $J \rightarrow J$ transitions with $J > 0$, provided that one can subtract the contribution of the Gamow-Teller (GT) or axial-vector current to the decay rate. To separate the Fermi and GT matrix elements one needs to measure $J \cdot k_e$ or $k_e \cdot k_\nu$ angular correlations in the β decay. This is such a difficult task that precise values for the Fermi matrix element have been measured for only three $J > 0$ decays: $n \rightarrow p$, ^{19}Ne$\rightarrow$ ^{19}F, and ^{35}Ar$\rightarrow$ ^{35}Cl. The neutron and ^{19}Ne vector coupling constants agree very well with the value inferred from the $0^+ \rightarrow 0^+$ transitions. But the ^{35}Ar vector coupling constant is anomalous,[3] being 3% greater than G_β^V inferred from the $0^+ \rightarrow 0^+$ transitions. Each ingredient of the ^{35}Ar G_β^V measurement (angular correlation, half-life, branching ratio, energy release) has been checked in at least two experiments and the anomaly persists.[4]

Hardy and Towner[3] have pointed out that $G_\beta^V(^{35}\text{Ar})$ has the value one would expect if $\cos^2\theta = 1$ and called attention to Salam and Strathdee's prediction[5] that the Cabibbo angle should vanish in intense electromagnetic fields ($H \sim 10^{16}$ G) comparable[6] to those which occur in nuclei. Why should the anomaly occur only in ^{35}Ar? Hardy and Towner[3] speculated that perhaps in ^{35}Ar, which has $J = \frac{3}{2}$, the nucleons "see" a somewhat larger magnetic field than they do in all the other accurately measured nuclei which have $J \leq \frac{1}{2}$ and that this field was just large enough to drive the "phase transition" in which quark mixing disappears. In this Letter we report a measurement of the Fermi matrix element for a $(J=4) \rightarrow (J=4)$ transition where the magnetic moments are much greater than in the $A=35$ decay. We obtain a vector coupling constant which agrees with the $0^+ \rightarrow 0^+$ value. We conclude that the anomalous ^{35}Ar result is almost surely due to an as-yet-unidentified experimental error and that there is no evidence in nuclear β decay for the effect[5] predicted by Salam and Strathdee.

We have chosen to check on the ^{35}Ar anomaly by using the ^{24}Al(4^+)$\rightarrow$ ^{24}Mg decay for the following reasons: (1) The ^{24}Al$\rightarrow$ ^{24}Mg analog transition has an extremely small GT matrix element. (2) The magnetic moments of the ^{24}Al and ^{24}Mg states predicted by shell-model wave functions[7] are $2.83\mu_N$ and $2.27\mu_N$, respectively, much larger than the values of $0.64\mu_N$ and $0.80\mu_N$ for ^{35}Ar and ^{35}Cl, respectively. (3) The β^+-decay branching ratio of a 4^+ parent which decays to a daughter with a 0^+ ground state can be measured

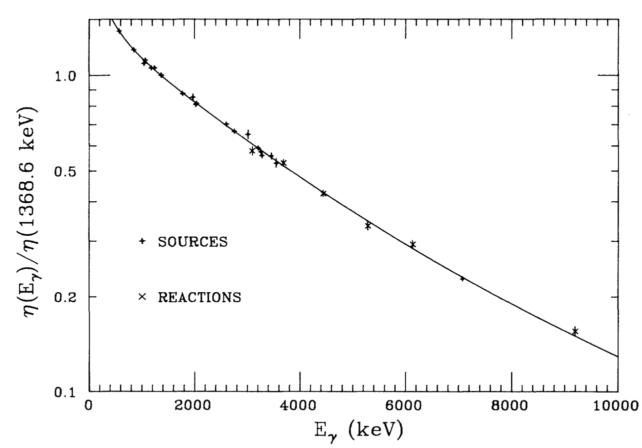

FIG. 1. Relative γ-ray detection efficiency. Points were measured as described in the text. The smooth curve is an analytic interpolation used to obtain the efficiency for γ rays of interest. Uncertainties in the interpolated values were assumed to be $\pm 0.5\%$ for $E_\gamma \leq 3.7$ MeV, $\pm 1\%$ for 3.7 MeV $< E_\gamma \leq 5$ MeV, $\pm 1.5\%$ for 5 MeV $< E_\gamma \leq 7.2$ MeV, $\pm 2\%$ for 7.2 MeV $< E_\gamma \leq 7.8$ MeV, $\pm 2.5\%$ for 7.8 MeV $< E_\gamma \leq 9$ MeV, and $\pm$ 3% for $E_\gamma > 9$ MeV.

TABLE I. Gamma transitions following $^{24}Al(\beta^+)^{24}Mg$.

Peak no.	Energy[a] (keV)	Assignment[b]	Intensity(%)[c] Present work	Ref. 10	Ref. 11
1	775	6010→5235		0.053(8)	
2	822	8439→7616		0.021(8)	
3	860	9301→8439		0.022(11)	
4	997	5235→4238		0.137(7)	
5	1060	10576→9516		0.258(17)	
6	1077	9516→8439	15.31(15)	14.84(31)	14.5(7)
7	1091	8439→7349		0.140(7)	
8	1275	10576→9301		0.106(6)	
9	1369	1369→0	95.76(13)	96.0(2.5)	96.1(1.0)
10	1705	9516→7812		0.016(4)	
11	1772	6010→4238	0.55(8)	0.40(1)	0.3(1)
12	1888	6010→4123		0.056(6)	
13	1900	9516→7616	0.66(8)	0.82(2)	0.8(2)
14	1952	9301→7349	0.14(9)	0.094(6)	
15	2137	10576→8439	0.24(9)	0.168(9)	
16	2381	7616→5235		0.037(10)	
17	2429	8439→6010	0.79(10)	0.774(18)	0.9(2)
18	2577	7812→5235		0.030(12)	
19	2754	4123→1369	44.33(28)	41.19(90)	44.6(5)
20	2870	4238→1369	1.21(8)	1.097(28)	1.1(2)
21	3204	8439→5235	3.51(9)	3.085(66)	3.3(2)
22	3378	7616→4238		0.043(7)	
23	3493	7616→4123		0.04(1)	
24	3506	9516→6010	2.21(10)	1.98(6)	2.3(2)
25	3866	5235→1369	5.81(13)	5.26(22)	5.6(2)
26	4201	8439→4238	4.25(11)	4.02(22)	4.2(3)
27	4238	4238→0	3.94(11)	3.61(21)	3.9(2)
28	4281	9516→5235	0.82(11)	0.66(4)	0.7(2)
29	4316	8439→4123	15.99(21)	14.20(86)	15.6(3)
30	4641	6010→1369	3.70(12)	3.42(25)	4.1(4)
31	5061	9301→4238		0.036(13)	
32	5178	9301→4123	0.85(15)	0.98(10)	1.1(2)
33	5340	10576→5235		0.115(13)	
34	5393	9516→4123	19.27(33)	18.3(18)	21.1(3)
35	5980	7349→1369		0.093(9)	
36	6247	7616→1369		0.54(4)	0.5(2)
37	7070	8439→1369	38.76(63)	43.0(1.3)	40.9(5)
38	7348	7349→0	0.17(7)	0.153(16)	
39	7615	7616→0	0.12(6)	0.224(15)	0.2(1)
40	7931	9301→1369	1.09(5)	1.34(10)	1.0(2)
41	8146	9516→1369		0.028(7)	
42	9450	10821→1369	0.09(3)	0.110(20)	
43	9943	11314→1369		0.027(6)	

[a]From Ref. 10.
[b]^{24}Mg $E_i \rightarrow E_f$ in kiloelectronvolts.
[c]Normalized such that the flux into the ground state of ^{24}Mg is 100.

quite precisely. All β^+ transitions produce nuclear γ rays so that one does not need to count annihilation radiation in order to determine the total number of β^+ decays. [In fact, 96% of all ^{24}Al β^+ decays feed the 1369-keV ($2^+ \rightarrow 0^+$) γ ray.]

We produced ^{24}Al by bombarding 0.7-mm-thick Mg-metal targets enriched to > 99.9% in ^{24}Mg with an 18-MeV proton beam from the University of Washington tandem accelerator. Targets were shuttled between the bombardment station and a heavily shielded Ge(Li) detector by use of the "rabbit" system described by Hoyle *et al.*[8] Targets were bombarded for ~3 sec and counted for a period of 3.0 sec beginning 1 sec after the end of bombardment. This allowed the 130-msec 1^+ isomer of ^{24}Al to decay to insignificance before counting began. The detector was located 14 cm from the source. A 11.7-cm-thick Lucite positron stopper was placed between the source and the detector. The efficiency of the Ge(Li) detector for γ-ray energies between 570 and 3548 keV was measured with ^{24}Na, ^{56}Co, and ^{207}Bi radiative sources with use of the intensities of Yoshizawa *et al.*[9] The efficiency at higher energies was found as follows. The efficiency at 7069 keV was determined relative to that at 1369 keV by an argument based on the assumption that ^{24}Al(4^+) decays cannot directly feed the 1369-keV 2^+ state (details may be found in Warburton *et al.*[10]). Finally, the efficiency of the Ge(Li) detector was compared to that of a 25.4-cm×25.4-cm NaI spectrometer whose efficiency had previously been measured by use of "tagged" protons from the ^{12}C(^{3}He,$p\gamma$) and ^{13}C(^{4}He,$p\gamma$) reactions. Since the efficiency of the NaI detector varies quite slowly for E_γ between 2 and 9 MeV it provided a good measure of the relative efficiency of the Ge(Li) detector. The comparison was made with 6.13-, 9.17-, 5.28-, 4.43-, and 3.68-MeV γ rays produced by a ^{13}C + ^{238}Pu source and the ^{13}C(p,γ), ^{15}N($p,p'\gamma$), ^{12}C($p,p'\gamma$), and ^{13}C($p,p'\gamma$) resonances at $E_p = 1748$, 7300, 5370, and 4525 keV, respectively. The measured detection efficiency is shown in Fig. 1. Efficiencies were interpolated between the measured points with use of an analytic expression. We verified that pileup and summing effects in our ^{24}Al γ-ray spectra were less than 1% by subsidiary measurements taken at twice the count rate and then at one half the detector solid angle.

Our measured γ-ray intensities are listed in Table I, along with the two most precise previous measurements.[10,11] Agreement is reasonable. We obtained the absolute ^{24}Al(4^+) β^+ branching ratios listed in Table II from our results (using the intensities of Ref. 10 for the weakest transitions which we could not detect reliably). Small corrections were applied for α decay by use of the branching ratios of Ref. 11. We combine our branching ratio for the superallowed transition with those from Refs. 10 and 11 to obtain a "best value" of (38.2 ± 0.4)%.

We measured the ^{24}Al(4^+) lifetime using the multiscaling technique. The multiscaler was triggered by a NaI detector which counted γ rays with 3 MeV < E_γ < 8 MeV. The multiscaler period was measured with a precision frequency meter. The ^{24}Al decay was followed for 18 sec and a half-life of $t_{1/2} = 2.053 \pm 0.004$ sec was obtained. This agrees well with the most recent measurement[12] of $t_{1/2} = 2.054 \pm 0.009$ sec. We combine these two results to obtain a "best value" of $t_{1/2} = 2.053 \pm 0.004$ sec.

Are these results consistent with the ^{35}Ar anomaly reported in Refs. 3 and 4? For a mixed Fermi-GT transition one has, using the notation of Ref. 1,

$$ft(1+\delta_R) = K/(G_\beta^V)^2(M_V)^2(1+\rho^2),$$

with $K = 2\pi^3 \ln 2\hbar^7 c^6/(mc^2)^5$,

$$(G_\beta^V)^2 = (G_F \cos\theta_1)^2(1+\Delta_R),$$

$$(M_V)^2 = 2T(1-\delta_c), \quad \rho = g_A M_A / g_V M_V,$$

where T is the isospin of the parent state, ρ is the ratio of axial-vector to vector matrix elements, and δ_R and Δ_R are nucleus-dependent and nucleus-independent radiative corrections, respectively. We obtain G_β^V from the relation

$$(G_\beta^V)^2 = K/[\mathscr{F}t(2T)(1+\rho^2)],$$

where $\mathscr{F} = f(1+\delta_R)(1-\delta_c)$.

Towner[13] has kindly calculated the quantities f, δ_R, and δ_c with the same procedures that he employed in Ref. 1. The statistical rate function f was computed by use of the energy release deduced from Refs. 14 and 10. Towner obtains $f(1+\delta_R) = 580.8 \pm 3.0$ and finds that δ_c, the correction for the isospin-nonconserving difference between the ^{24}Al and ^{24}Mg analog wave functions, is $\delta_c = (0.57 \pm 0.10)\times 10^{-2}$. We can obtain a lower limit on δ_c from experiment. One of the main mechanisms for generating a nonzero δ_c is isospin mixing in the daughter state. This isospin impurity is expected to be dominated by analog-antianalog mixing as discussed in Ref. 8. The analog-antianalog mixing probability inferred from an ^{24}Al β-γ circular-polarization correlation experiment[8] is $(1.1 \pm 0.8)\times 10^{-2}$.

TABLE II. Positron decay of ^{24}Al.

^{24}Mg level[a] J^π	E_x (keV)	Positron yield (%) Present work	Ref. 10	Ref. 11
4^+	4123	8.12(51)	7.7(1.0)	6.9(7)
3^+	5235	1.39(19)	1.40(13)	0.7(4)
4^+	6010	1.35(21)	1.2(1)	1.2(5)
4^+	8439	47.88(70)	50.0(2.0)	50.0(1.0)
4^+	9301	2.04(18)	2.5(2)	2.1(3)
4^+	9516	38.01(41)	37.0(1.5)	39.1(8)
$(3,4,5)^+$	10576	0.744(89)	0.67(6)	
$(3,4,5)^+$	10821	0.086(38)	0.11(1)	

[a]J^π and E_x from Ref. 10.

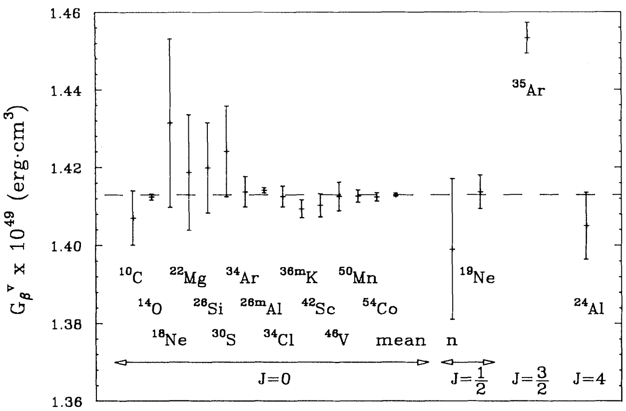

FIG. 2. Measured values of G_β^V. The $0^+ \to 0^+$ values are inferred from Ref. 1. The $J=\frac{1}{2}$ and $J=\frac{3}{2}$ values are inferred from Refs. 3 and 4.

An estimate of 0.5×10^{-2} for the analog-antianalog mixing probability can be obtained from the observed mass splitting of the isospin multiplets around $A=24$ (see Ref. 8). Both of these values are consistent with the δ_c computed by Towner. We therefore take δ_c from Towner's calculation but, to be conservative, increase his uncertainty by a factor of 3, which leads to $\mathscr{F}=577.5\pm3.5$. Our "best value" for the superallowed decay of ^{24}Al is $\mathscr{F}t=3106\pm38$ sec.

To extract G_β^V from the measured $\mathscr{F}t$ value we must know ρ^2. In ^{24}Al(4^+) decay ρ^2 is so small that it gives a negligible contribution to the decay rate. This extraordinary smallness of M_{GT} is easily understood. In the asymptotic Nilsson model (^{24}Mg is known to have a sizeable deformation), M_{GT} vanishes for the analog transition (see Ref. 8).[15] A complete $0\hbar\omega$ shell-model calculation[7] (which is very successful in accounting for the other GT decays of ^{24}Al) predicts $\rho^2=3.5\times10^{-3}$ in qualitative accord with the simplified model.

If we accept the shell-model value for ρ^2, our results lead to a value $G_\beta^V(^{24}\text{Al})=(1.4050\pm0.0086)\times10^{-49}$ erg·cm^3 which agrees well with $G_\beta^V=(1.4129\pm0.0004)\times10^{-49}$ erg·cm^3 inferred from the $0^+\to0^+$ transitions[1] and disagrees strongly with the value $G_\beta^V=(1.4533\pm0.0040)\times10^{-49}$ erg·cm^3 inferred from the ^{35}Ar results.[4] These results are summarized in Fig. 2. The conclusion that our experiment is inconsistent with the ^{35}Ar result does not depend on the shell-model ρ^2. If ρ^2 were actually larger than predicted, the extracted $G_\beta^V(^{24}\text{Al})$ would decrease —increasing the disagreement with $G_\beta^V(^{35}\text{Ar})$. Thus we conclude that there is no evidence for an anomaly in the Fermi matrix elements of nuclei with $J>\frac{1}{2}$. The anomalous ^{35}Ar result probably arises from an unidentified error in the very difficult measurement of ρ.

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[15]In addition the asymptotic Nilsson model predicts a strong M_{GT} for the ^{24}Al(4^+) decay to the 4^+ 8.4-MeV antianalog state. It also predicts a strong M_{GT} for the ^{24}Al(1^+) superallowed transition and a vanishing M_{GT} for the ^{24}Al(1^+) transition to the 1^+ antianalog state. All of these predictions are in qualitative accord with the data.