Spin Susceptibility of the Two-Dimensional Electron Gas with Open Fermi Surface under Magnetic Field

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Under perpendicular magnetic field, the spin susceptibility of a noninteracting two-dimensional electron gas with open Fermi surface shows a novel behavior due to the orbital effect. It exhibits a series of peaks at wave vectors which obey a quantized nesting condition. When the field increases, each of these peaks becomes in turn the absolute maximum. Their magnitude diverges logarithmically at low temperature. We discuss some experimental consequences in actual anisotropic metals.

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A rapidly growing number of papers examine the properties of the two-dimensional (2D) electron gas under magnetic field, either in the presence of impurities or in the presence of a periodic lattice potential. The reason is, of course, the study of the quantized Hall effect.[1] The vast majority of papers so far have dealt with isotropic 2D systems. However, the 2D anisotropic electron gas, such as can be found experimentally in weakly coupled chain systems,[2] has specific properties under magnetic field which deserve more attention. Prominent among them is the possibility of open Fermi surfaces, which result in a qualitative change of the quasiclassical motion of the electron wave packet. The occurrence of quasiparallel sheets of the Fermi surface also leads to the instability of the normal Fermi ground state versus formation of spin- or charge-density waves. The purpose of this Letter is to describe a spectacular consequence of the orbital effect of the magnetic field on the staggered static 2D spin susceptibility $\chi_0(\mathbf{Q},H,T)$ of the noninteracting electron gas with open Fermi surface. There is a characteristic area in the problem. This is the zero-field area between one sheet of the Fermi surface and the other sheet when it is translated by $\mathbf{Q}$ (Fig. 1). When a perpendicular field $\mathbf{H}$ is applied, the Fermi surface is destroyed but the susceptibility has maxima which correspond to integer values of this zero-field area in units of the area quantum in momentum space $A_0 = 2\pi eH$.

In quasi-1D conductors of the tetramethyltetraselenafulvalenium family [$(\mathrm{TMTSF})_2X$], the metallic phase stable in zero field can be destroyed by a field of order 3–4 T perpendicular to the most-conducting planes, and a spin-density wave (SDW) phase appears.[3,4] Gor'kov and Lebed gave the first theoretical analysis of this transition.[5] However, the observed field-induced SDW phase has a complex structure of subphases.[6–9]

By considering a field-dependent SDW wave vector, we were able to account for this structure.[10] The detailed investigation of $\chi_0(\mathbf{Q},H,T)$ discussed here fully confirms our interpretation. As already pointed out, the Hall effect in the $(\mathrm{TMTSF})_2X$ family exhibits plateaus which are reminiscent of the quantized Hall effect.[6,7] The results described here are relevant to the understanding of these plateaus.

Let us consider a 2D electron gas with an open Fermi surface and a dispersion relation which is linearized in the direction $\mathbf{a}$ of largest conductivity:

$$\epsilon(\mathbf{k}) = v(|k_{\parallel}| - k_F) + t_{\perp}(k_{\perp}b), \qquad (1)$$

where $t_{\perp}$ is a periodic function: $t_{\perp}(k_{\perp}b + 2\pi) = t_{\perp}(k_{\perp}b)$. The amplitude of $t_{\perp}$ is assumed to be small compared to vk_F. Linearity in the $k_{\parallel}$ dispersion is approximate but it can be shown that deviations from linearity can be accounted for by higher-order harmonics in $t_{\perp}$.[10,11] We now look for the wave-vector–dependent susceptibility $\chi_0(\mathbf{Q},H,T)$, where $\mathbf{Q}$

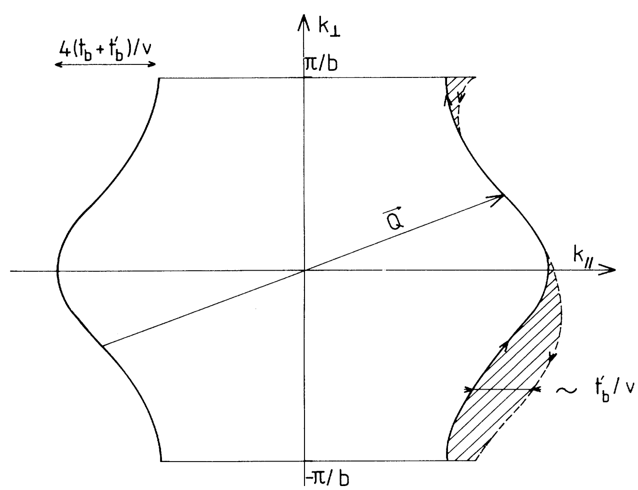

FIG. 1. Open Fermi surface of a quasi-1D electron gas (solid curves). In zero field, the best nesting vector $\mathbf{Q}$ connects the inflection points of the two sheets (Ref. 3). It leaves an area A, the size of which is characterized by t_b', between one sheet of the Fermi surface and the other sheet translated by $\mathbf{Q}$ (dashed curve). In a field, the susceptibility is maximum when this zero-field area is quantized in terms of $A_0 = 2\pi eH$.

is written as $\mathbf{Q}=(2k_F+q_{\parallel},q_{\perp})$. In the mixed representation[5] which is appropriate in the presence of a magnetic field $\mathbf{H}=(0,0,H)$, χ_0 is written as

$$\chi_0(\mathbf{Q},H,T)=T\sum_{\omega_n}\int(dk_{\perp}/2\pi)\int dx\, G_{++}(i\omega_n,k_{\perp},0,x)G_{--}(i\omega_n,k_{\perp}-q_{\perp},x,0), \tag{2}$$

where G_{++} (G_{--}) propagates an electron (a hole) with longitudinal wave vector k_F ($-k_F-q_{\parallel}$).[5] With the Landau gauge $\mathbf{A}=(0,Hx,0)$, one has

$$G_{++}=\frac{\operatorname{sgn}\omega_n}{iv}\exp\left[-\left(\frac{\omega_n}{v}-ik_F\right)x+\frac{i}{v}T_{\perp}(k_{\perp}b-eHbx)\right]\quad(\omega_n x>0), \tag{3}$$

$$\chi_0(\mathbf{Q},H,T)=\frac{1}{4\pi b v}\int_0^{2\pi}\frac{dp}{2\pi}\int_d^{\infty}\frac{dx/x_T}{\sinh x/x_T}\left(\exp\left\{iq_{\parallel}x+\frac{i}{eHbv}[T_{\perp}(p-eHbx)+T_{\perp}(p-q_{\perp}b-eHbx)-T_{\perp}(p)-T_{\perp}(p-q_{\perp}b)]\right\}+\exp\{x\to -x\}\right), \tag{4}$$

with $T_{\perp}(p)=\int_0^p t(p')dp'$ and $x_T=v/2\pi T$. We have set $k_B=\hbar=c=1$. $d=1/2\gamma k_F$, where γ is the Euler constant. We turn now to the analysis of this susceptibility. First, we show that it exhibits maxima for quantized values of the wave vector.

In the presence of a field, there is a characteristic length $x_0=1/eHb$, which for an open surface plays the role of a cyclotron radius. The oscillatory factor $T_{\perp}$ in Eq. (4) has a spatial wavelength of $\lambda=2\pi x_0$. $\chi_0(\mathbf{Q},H,T)$ is maximum if there is a condition of phase coherence in the integrand, namely, $\exp(iq_{\parallel}\lambda)=1$, so that

$$q_{\parallel}=neHb=n/x_0 \quad \text{with } n \text{ integer.} \tag{5}$$

This condition has a simple physical meaning. Let us introduce A, the zero-field area between one sheet of the Fermi surface and the other sheet translated by $\mathbf{Q}$ (Fig. 1). $A=2\pi q_{\parallel}/b$, so that $A=nA_0$, where $A_0=2\pi eH$ is the area quantum in reciprocal space. *The susceptibility is maximum, every time that A is quantized.*

In order to make this point more quantitative, we have performed numerical calculations of $\chi_0(\mathbf{Q},H,T)$ for two kinds of Fermi surfaces. First, we use the dispersion relation $t_{\perp}(p)=-2t_b\cos p$ which, in zero field, leads to perfect nesting, i.e., χ_0 diverges logarithmically at low T for $\mathbf{Q}=(2k_F,\pi/b)$. For this dispersion relation, the field-dependent susceptibility is

$$\chi_0(\mathbf{Q},H,T)=\frac{1}{2\pi b v}\int_{d/2x_0}^{\infty}\cos(2\nu y)J_0(2z\sin y)\frac{dy/r}{\sinh y/r}, \tag{6}$$

with $\nu=q_{\parallel}x_0$, $z=(4t_b/eHvb)\cos(q_{\perp}b/2)$, and $r=x_{\perp}/2x_0$. Figure 2 shows the evolution of $\chi_0(\mathbf{Q},H,T)$ when a field is applied. It clearly shows several novel features. As expected from the considerations above, it exhibits relative maxima, the positions of which are strictly quantized. The distance between maxima increases proportionally with the field as found in Eq. (5). We have checked that each maximum of the susceptibility diverges logarithmically at low T. But despite this new structure of relative maxima in a field, the absolute maximum is always located at $(2k_F,\pi/b)$.

New results occur if the dispersion relation does not lead to perfect nesting. We took the dispersion relation $t_{\perp}(p)=-2t_b\cos p-2t_b'\cos(2p)$, the second term being the first correction to perfect nesting. For such a dispersion, the zero-field susceptibility is maximum for a noncommensurate nesting vector.[3] But *the maximum no longer diverges at low T.* We have computed the susceptibility in the presence of the field. We find

$$\chi_0(\mathbf{Q},H,T)=\frac{1}{2\pi b v}\int_{d/2x_0}^{\infty}\frac{dy/r}{\sinh y/r}\int_0^{2\pi}\frac{dp}{2\pi}\cos(2\nu y-2z\sin y\cos p-2z'\sin 2y\cos 2p), \tag{7}$$

with $z=(4t_b/eHvb)\cos(q_{\perp}b/2)$ and $z'=(2t_b'/eHvb)\cos(q_{\perp}b)$. As in the previous case, $\chi_0(\mathbf{Q},H,T)$ exhibits a series of maxima, the abscissas of which are still quantized: $q_{\parallel}=neHb$. Moreover, the absolute maximum appears at a field-dependent wave vector $\mathbf{Q}(H)$ with $q_{\parallel}=n_0eHb$ (Figs. 3 and 4). We summarize below the essential features of this new behavior.

At a given field, there is a main series of maxima; their positions $\mathbf{Q}(H)$ are close to the *zero-field* continuous line of maxima which corresponds to the condition that the two sheets of the Fermi surface are tangent. The maxima deviate from this line when the field increases. This series is shown by arrows and labeled by quantum number n in Fig. 3. In this figure, the absolute maximum is labeled by $n_0=2$. When the field is varied, *each of these peaks becomes in turn the absolute maximum* as shown in Fig. 4. For each value of the field, we have looked for the absolute maximum of $\chi_0(\mathbf{Q},H,T)$. In Fig. 5, the latter is plotted as function of the field. It is a succession of segments

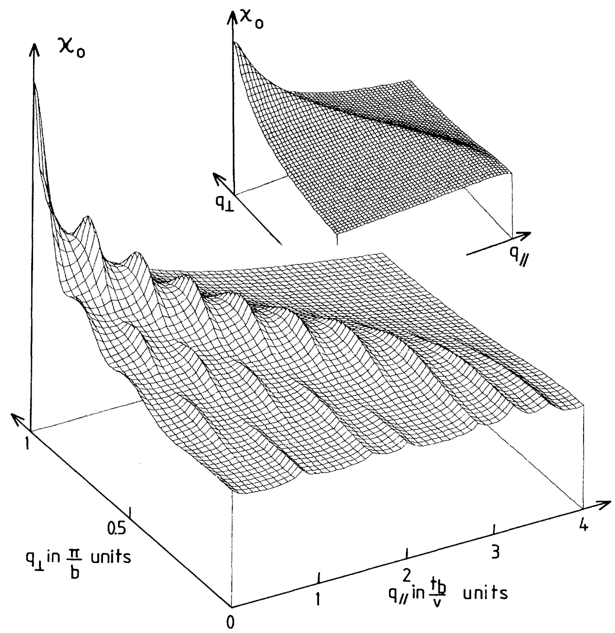

FIG. 2. $\chi_0(\mathbf{Q},H,T)$ vs $\mathbf{Q}$ in the case $t_b'=0$, $E_c/t_b=0.4$, $T/t_b=1/50\pi=0.006\,37$. It exhibits peaks at quantized values of the longitudinal wave vector $q_{\parallel}=neHb$, but the absolute maximum still stays at the zero-field best nesting vector $(2k_F,\pi/b)$. Inset: susceptibility in lower field and higher temperature, $E_c/t_b=0.08$, $T/t_b=1/5\pi=0.0637$.

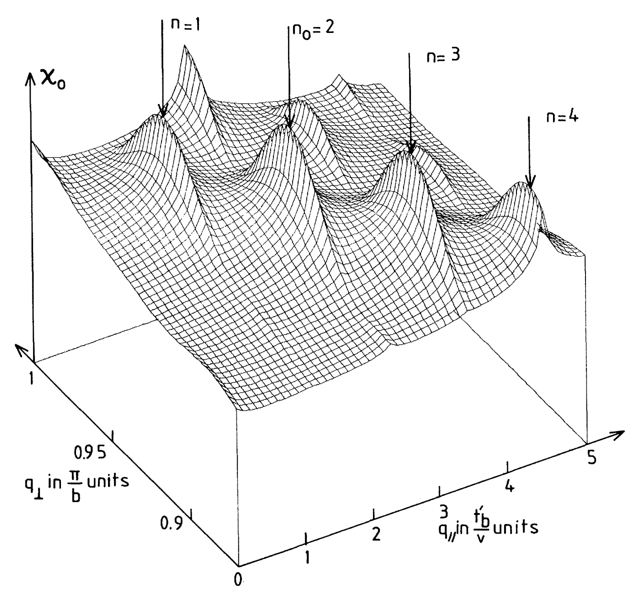

FIG. 3. $\chi_0(\mathbf{Q},H,T)$ vs $\mathbf{Q}$ in the case $t_b'/t_b=0.1$, $E_c/t_b'=1.158$, $T/t_b'=1/40\pi=0.007\,96$. It exhibits a main series of peaks labeled by quantum numbers n. The absolute maximum in this case is labeled by $n_0=2$.

characterized by successive values of the quantum number n_0. The wave vector jumps at the transition field between each segment.

For open Fermi surface, it is possible to define a "cyclotron" frequency and thus a characteristic energy of the magnetic field: $E_c=\omega_c=v/x_0=evbH$. Figure 5 shows that small quantum numbers n_0 are reached when E_c is of the same order of magnitude as t_b', the characteristic energy which describes deviation from perfect nesting. This shows that lattice periodicity has dramatic consequences on the orbital effect at much smaller fields than in isotropic lattices.

Figure 3 shows other relative maxima which never become absolute whatever the field. They correspond to a less good nesting of the Fermi sheets.

We have shown that *each maximum diverges logarithmically at low T.* The minima are nearly constant in temperature since they are mainly determined by the

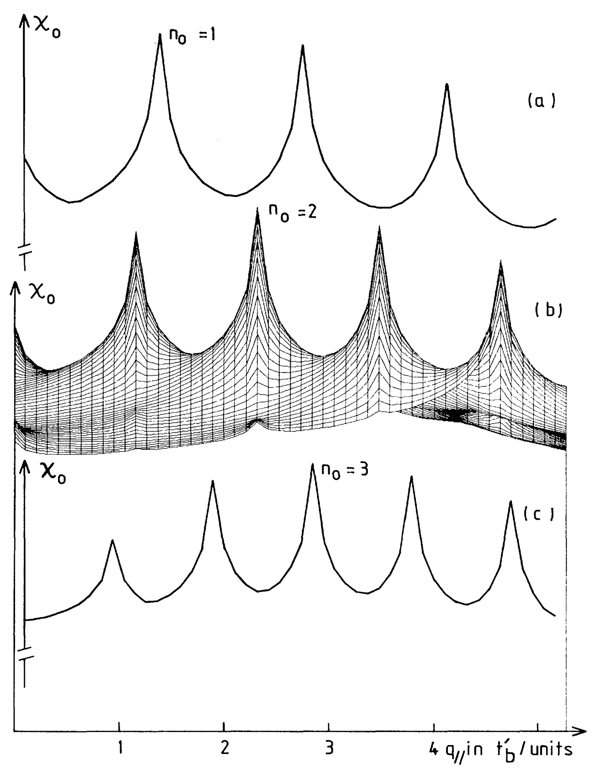

FIG. 4. (a) The envelope of χ_0 in a case where $n_0=1$ ($E_c/t_b'=1.368$). (b) Projection of Fig. 3 along the direction $q_\perp$ on the $q_\perp=\pi/b$ plane. The different ridges of $\chi_0(\mathbf{Q},H,T)$ are now superimposed on the same plane and the envelope shows the main series of peaks. The abscissa of the absolute maximum characterized by $n_0=2$ increases linearly with the field until the peak $n_0=1$ becomes in turn the absolute maximum. (c) At a lower field ($E_c/t_b'=0.947$), the peak labeled by $n_0=3$ is the main one. We have checked with a smaller spacing that the shape around each maximum is quadratic at finite temperature.

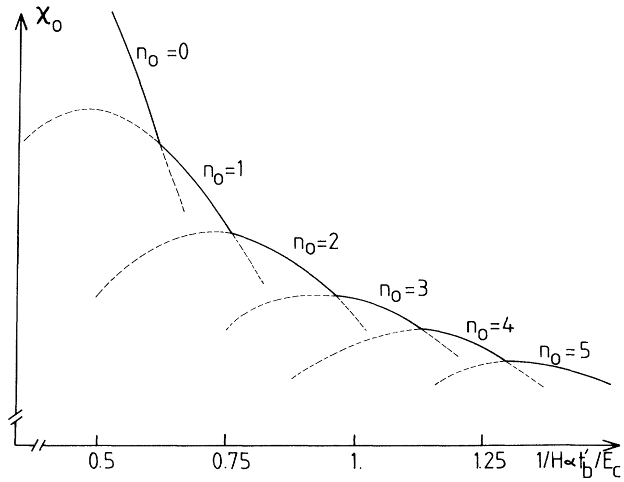

FIG. 5. Absolute maximum of $\chi_0(\mathbf{Q},H,T)$ as a function of $1/H$ (scale is in units of t_b'/E_c). It is a succession of segments characterized by successive values of the quantum number n_0. At the transition fields between different values of n_0, the wave vector jumps discontinuously ($t_b'/t_b = 0.1$, $T/t_b' = 1/40\pi$).

cutoff. Thus the "contrast" of the structure that we have found increases at low T as expected from Eq. (4). At $T=0$ and when H goes to zero, $\chi_0(\mathbf{Q},H,T)$ exhibits a fine structure made of an infinity of logarithmically divergent peaks. The restoration of the logarithmic divergence for each maximum can be qualitatively understood in the following way: In zero field, the susceptibility $\chi_0(q_\parallel,q_\perp,T)$ of the 2D electron gas involves two degrees of freedom, the two components of the wave vector which have a continuous set of eigenvalues. In a field, the wave vector is no longer a good quantum number. Because of quantization of orbits, there is a discrete set of eigenvalues. For each value of n, there remains only one degree of freedom so that $\chi_0(n,q_\perp,H,T)$ recovers the characteristic 1D logarithmic divergence. Finally, we have shown that the structure described in this paper remains from $T=0$ K up to temperatures such that $E_c/k_B T \gtrsim 2\pi$. In the same way, if there is a mean free path for electron motion, due to impurities, the structure in the maxima remains as long as $\omega_c \tau \gtrsim 1$, where τ is the collision time.

The effect of a small 3D coupling of conducting sheets can be studied by taking into account a dispersion relation

$$t_c(k_z c) = -2t_c \cos(k_z c) - 2t_c' \cos(2k_z c)$$

in the third direction. We have shown that the quantized structure of the susceptibility remains in the domain of wave vectors $(q_\parallel,q_\perp,q_z)$ where q_z is close to the zero-field best-nesting vector, as long as $E_c > t_c'$.

Our results on $\chi_0(\mathbf{Q},H,T)$ have strong consequences on the phase diagram of the quasi-2D interacting anisotropic electron gas, as can be seen from Stoner's criterion[5,10] or from a microscopic Landau expansion.[12] The critical line of the metal-SDW transition is given by $\chi_0(\mathbf{Q},H,T) = 1/I$, where I is the effective interaction constant. As seen from Fig. 5, in a real system with metallic ground state in zero field, a cascade of field-induced phase transitions necessarily occurs, since each peak diverges logarithmically at low T. The transition line is second order. It is a succession of segments, each characterized by a quantum number which also labels the SDW phase at lower T. Our work proves that the quantization condition holds already on the transition line in the $(TMTSF)_2X$ compounds. The meaning of that condition in the ordered phase is the following: The size of the pocket of carriers is such that it contains an integer number of Landau levels. The transition between SDW phases is a first-order one, due to the jump of the nesting vector.

In conclusion, we have proved that under magnetic field, the susceptibility of an electron gas with open Fermi surface exhibits a new and rich structure governed by quantization of nesting of the Fermi surface.

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