Theory of Vortex Interaction with Thermoelectric Fields in a Thin-Film Superconductor

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(Received 14 September 1984)

The response of thermally activated free vortices in superconducting thin films to a crossed electric field and temperature gradient is shown to produce a static magnetic field perpendicular to the plane of the film. The magnetization, which results from an induced imbalance between the free vortex and antivortex densities, is proportional to the logarithmic derivative of the free-vortex density and to a vortex-nonequilibrium relaxation time.

PACS numbers: 74.20, 74.40+k, 74.50+r

In this Letter, we propose a new thermoelectric effect pertaining to thin superconducting films whose dynamical behavior is governed by thermally activated free vortices.[1] The basic idea of this effect is that a temperature gradient and an electric field can create an imbalance between the numbers of vortices and antivortices, resulting in a net magnetization perpendicular to the plane of the film. We propose a phenomenological model that relates this magnetization directly to the distribution function of free vortices and to the relaxation of the nonequilibrium vortex distribution.

Our model applies to a superconducting film whose width w is of order λ^2/d, where λ is the magnetic penetration depth, assumed much larger than the film thickness d. Such films are believed to exhibit a Kosterlitz-Thouless-type vortex unbinding transition[2] at a temperature T_c. In the temperature regime above T_c and below the fluctuation-corrected BCS transition temperature T_{c0}, bound vortex pairs dissociate to form equal numbers of free vortices and antivortices. The area density of free vortices is $n_{f0}(T)$, which is related to the order-parameter correlation length for $T > T_c$, $\xi_+(T)$, by

$$n_{f0}(T) = C_1/2\pi\xi_+^2(T), \tag{1}$$

where C_1 is of order unity.[2] The divergence of $\xi_+(T)$ at T_c leads to an exponentially activated free-vortex density with a characteristic temperature dependence near T_c of

$$\exp\{-2[b(T_{c0}-T_c)/(T-T_c)]^{-1/2}\},$$

where b is also of order unity.

Our phenomenological model makes several implicit assumptions about the dynamical properties of thermally activated vortices. First, we assume that the density of vortices is governed by the local value of the temperature at the vortex cores, where electron-phonon scattering couples the electrons to the lattice. In the Kosterlitz-Thouless regime, the circulating currents associated with a vortex spread over a distance λ^2/d, typically 1–10 mm. Although actual sample temperatures can vary by ~ 10 mK over this distance, we suppose that the mean (i.e., core) temperature governs the probability of thermal depairing.

Second, we assume that vortices driven out of equilibrium by thermoelectric fields relax with a simple relaxation time. Although we propose a general physical mechanism for this relaxation, our model assumes an *ad hoc* time constant for the process. Finally, we assume that the superfluid density falls to zero with a Ginzburg-Landau-type dependence as T_{c0} is approached from below. This assumption is used to make the effect vanish as superconducting order disappears.

As shown in the inset to Fig. 1, we consider a film with a transverse temperature gradient $\nabla T = (dT/$

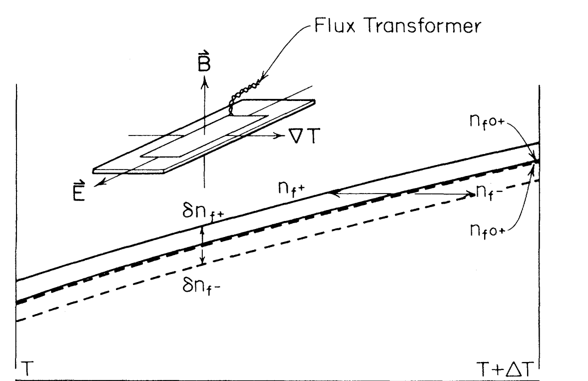

FIG. 1. A schematic illustration of the vortex imbalance produced in a thin superconducting film by the simultaneous application of an electric field and a temperature gradient. The inset shows the sample geometry and the direction of the resultant magnetic field.

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$dx)\hat{\mathbf{x}}$ directed along the width and a uniform electric field $\mathbf{E}=E\hat{y}$ directed along the length. In response to the electric field, free vortices are driven across the width of the film by the Lorentz force produced by the superfluid current $\mathbf{J}_s=\rho_s\mathbf{E}$, where ρ_s is the flux flow resistivity. In the absence of pinning forces, vortices and antivortices drift in opposite directions, their velocity limited by eddy-current damping in the vortex cores. For a film at uniform temperature, the two polarities of vortices maintain equal and spatially uniform densities characteristic of the equilibrium distribution (ignoring spatial fluctuations).

In the presence of a temperature gradient, however, the equilibrium free-vortex density is determined by the local temperature and hence varies with position, being higher on the hot side of the film than on the cold side. Because of this temperature gradient, the vortex drift resulting from the electric field produces a vortex polarity imbalance at each point in the film, i.e., an excess of vortices of positive circulation and a deficit of vortices of negative circulation with respect to the local equilibrium number. Because each vortex carries a quantum of magnetic flux $\Phi_0=hc/2e$, this imbalance produces a net film magnetization that, as we shall show, is uniform across the film to lowest order in ∇T.

In order to calculate this magnetization, we separate the free-vortex density into vortex and antivortex components, designated n_{f+} and n_{f-}, respectively. Similarly, the equilibrium value of the vortices and antivortices are n_{f0+} and n_{f0-}. It is assumed that $n_{f0+}=n_{f0-}=n_{f0}/2$ in the absence of external magnetic fields. In steady state, we require the time derivative $dn_f/dt=0$:

$$\frac{dn_f}{dt}=\frac{\partial n_{f\pm}}{\partial T}\mathbf{v}_{d\pm}\cdot\nabla T-\frac{(n_{f\pm}-n_{f0\pm})}{\tau_f}=0. \quad (2)$$

In Eq. (2), $\mathbf{v}_{d\pm}$ is the vortex drift velocity of each polarity. The first term of this equation corresponds to the creation of a nonequilibrium free-vortex density by diffusion, and the second term describes the relaxation of the free-vortex distribution to equilibrium. As discussed previously, we describe the free-vortex relaxation as a simple exponential process characterized by time constant τ_f. Solving for the excess-vortex density $\delta n_{f+}=(n_{f+}-n_{f0+})$ we obtain to first order in ∇T,

$$\delta n_{f\pm}=(\partial n_{f0\pm}/\partial T)\tau_f\mathbf{v}_{d\pm}\cdot\nabla T. \quad (3)$$

The drift velocity $\mathbf{v}_d$ can be obtained from the Josephson relation $\hbar d(\Delta\phi)/dt=2e\Delta V$, where $\Delta\phi$ and ΔV are the phase and voltage differences across a segment of the film of length L. Noting that each vortex contributes a phase difference of 2π as it moves across the film, we set $d(\Delta\phi)/dt=2\pi n_f v_d L$, from which it follows that

$$\mathbf{v}_{d\pm}=\pm(1/\Phi_0)(\mathbf{E}\times\hat{\mathbf{z}})/n_{f0}. \quad (4)$$

The magnetic field resulting from the nonequilibrium vortices exhibits both spatial and temporal fluctuations. However, on a scale comparable to the average spacing between vortices, of order $n_f^{-1/2}$, the field is uniform with a value proportional to the vortex imbalance:

$$B=\Phi_0(\delta n_{f+}-\delta n_{f-}). \quad (5)$$

Combining Eqs. (3)–(5), and setting $n_{f0}=(n_{f0+}+n_{f0-})$, we obtain the principal result of our model:

$$\mathbf{B}=\tau_f(\partial\ln n_{f0}/\partial T)\mathbf{E}\times\nabla T. \quad (6)$$

The generation of the vortex polarity imbalance is illustrated schematically in Fig. 1, where the equilibrium densities n_{f0+} and n_{f0-} across the width of the film are shown as solid and dashed lines, respectively. The nonequilibrium distributions n_{f+} and n_{f-} created by the electric field are seen to arise from the transverse drift of the vortices. At each point in the film, the steady-state density n_{f+} is essentially given by the local equilibrium density n_{f0+} shifted from an adjacent higher-temperature region of the film, while n_{f-} is characteristic of the local equilibrium density n_{f0-} shifted from a lower-temperature region. The combined effect of the electric field and temperature gradient thus results in an antisymmetric imbalance in the number of vortices and antivortices, with $\delta_{nf+}=-\delta_{nf-}$.

It should be noted that the total number of free vortices as well as the number of bound-vortex pairs is not changed by the thermoelectric fields. As a result, the system cannot be restored to equilibrium by means of vortex pair breaking or recombination, as would be the case for a symmetric $(\delta_{nf+}=\delta_{nf-})$ excitation mode, as in the laser irradiation experiment of Bancel and Gray.[3] Instead, the relaxation of the vortices requires the restoration of the balance between the vortex and antivortex densities; the rate for this process, $1/\tau_f$, governs the magnitude of the steady-state film magnetization.

It is possible to visualize this relaxation process by considering the two polarities of vortices as separate interpenetrating gases. Under the influence of $\mathbf{E}$ and ∇T, one gas is compressed (i.e., the mean spacing between vortices decreases), while the other is expanded, leaving the total (combined) density unchanged. The relative compression and expansion of the free-vortex gases, and the resulting magnetization, is accompanied by an increase of free energy and a force which tends to restore the distribution to equilibrium. A similar force is responsible for the migration of vortices into a bulk type-II superconductor when an external magnetic field $B>H_{c1}$ is applied.[4] For this case, the rate of flux entry from the edges of the sample is determined by the vortex drift velocity, which is limited by eddy-current damping and pinning at defect sites. In thin, high-sheet-resistance films, however, both damping and pinning effects are weak and the vortices can respond at a rate limited by the intrinsic supercurrent acceleration time[5]

$$\tau(T)=\frac{2k_b T_{c0}\hbar}{\pi\Delta^2(T)}=\frac{5.2\times 10^{-13}\ \mathrm{K}}{T_{c0}-T}\ \mathrm{sec}. \qquad (7)$$

It is probable that this time also governs the polarity-imbalance relaxation. Note that each vortex need move only a distance of order $v_d\tau_f \approx 10^{-9}$ cm to reach its equilibrium position.

Figure 2 illustrates the temperature dependence predicted by Eq. (6) for two choices of τ_f. For the solid curve, τ_f is assumed constant over the entire temperature interval, while for the dashed curve, τ_f is given the Ginzburg-Landau $(T_{c0}-T)^{-1}$ dependence of Eq. (7). To facilitate a comparison with the experiment, we plot the total magnetic flux Φ produced by a supercurrent I_s and temperature difference ΔT. The experimental parameters are given in the figure and are appropriate to typical granular films. The explicit expression for the thermoelectric flux is given by

$$\Phi(T)=\frac{\tau_f R_n}{(T_{c0}-T_c)}f\left(\frac{T-T_c}{T_{c0}-T_c}\right)\left(1-\frac{T^4}{T_{c0}^4}\right)I_s\Delta T, \qquad (8)$$

$$f(x)=2.70\frac{\cosh(x^{-1/2})}{\sinh^3(x^{-1/2})}x^{-3/2}. \qquad (9)$$

Equation (8) was obtained from Eq. (6) by setting $\mathbf{E}=\rho_s\mathbf{J}$, with ρ_s given by the Halperin-Nelson expression for the flux-flow resistivity of dirty superconducting films[5]:

$$\rho_s=2.7\rho_n(\xi_{\mathrm{GL}}/\xi_+)^2, \qquad (10)$$

where ξ_{GL} is the Ginzburg-Landau coherence length and ρ_n is the normal-state resistivity. In Eq. (9), the Halperin-Nelson interpolation formula[2] has been used for ξ_+ to calculate the equilibrium free-vortex density over the temperature range $T_c < T < T_{c0}$:

$$\xi_+(T)=b^{-1/2}\xi_{\mathrm{GL}}\sinh\left(\frac{b(T_{c0}-T_c)}{(T-T_c)}\right)^{1/2}. \qquad (11)$$

Note from Fig. 2 that the predicted flux is about $10\Phi_0$, for $I_s=10\ \mu$A and $\Delta T=10$ mK, and is well within the sensitivity of SQUID magnetometers.

It is useful to make a few comments about the relevance of our model to experiments on real films. Although the calculation yields either a maximum or a disappearance of the flux at T_{c0} (depending on the assumed relaxation time), the vortex picture on which the model is based fails near T_{c0} as superconducting order weakens and fluctuations in the order parameter become important. Although our model accounts for the disappearance of superconducting order through a $(1-t^4)$ dependence of n_s [this term appears explicitly as the last term on the right in Eq. (8)], this choice undoubtedly oversimplifies the real experimental situation close to T_{c0}. Accordingly, the detailed temperature dependence predicted by our model is most valid near T_c, where the exponentially increasing free-vortex density dominates the contribution from more slowly varying processes.

An additional complicating factor pertaining to experiments arises from the influence of nonuniform film resistances on the observed flux. A nonuniform film resistance can arise either from intrinsic inhomogeneities in the deposited film, or from the temperature gradient itself. In this second case, the vortices induced by the applied temperature gradient create a transverse gradient in the film resistance, leading to a higher current density along the cold side of the film than along the hot side. We have calculated the effect of this nonuniform current flow under realistic experimental conditions and find that it produces an additional "classical" contribution to the magnetic flux, distinct from that arising from vortex imbalance. This contribution is most significant near T_c, where the relative inhomogeneity in n_{f0} is greatest. In practice, however, this classical effect can be suppressed by applying a small external magnetic field (10–50 mG) perpendicular to the plane of the film. The background of vortices produced by this field will smooth out the variations in film resistance, suppressing the spurious classical effect.

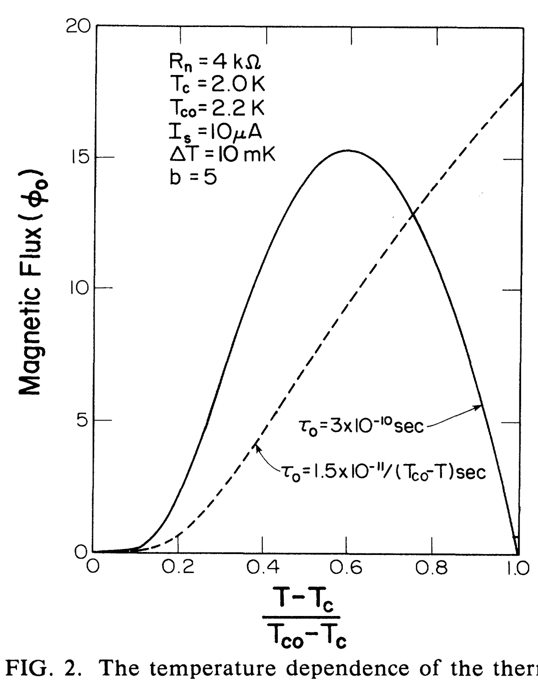

FIG. 2. The temperature dependence of the thermoelectric flux predicted by Eq. (8). The dashed line and solid lines correspond respectively to a temperature-independent relaxation time and a relaxation time having the temperature dependence of Eq. (7).

In conclusion, we propose that a net magnetization will be generated in a thin superconducting film by the interaction of an electric field with a transverse temperature gradient. The thermoelectric effect arises from an induced imbalance between free vortices and antivortices and provides a means to investigate the dynamics and relaxation of vortices in two-dimensional Kosterlitz-Thouless superconductors.

The authors are grateful for the hospitality of John Clarke and the Department of Physics at the University of California at Berkeley, and for the assistance of Hu Jong Lee. Valuable discussions with T. R. Lemberger are also acknowledged. This work was supported by the Low Temperature Physics Program of the National Science Foundation under Grant No. DMR-8106121, and the University of Illinois Materials Research Laboratory under Grant No. DMR-8020250. The use of facilities of the Ohio State University Materials Research Laboratory under Grant No. DMR-8119368 is also acknowledged.

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