Amplitude-Dependent Properties of a Hydrodynamic Soliton

Junru Wu and Isadore Rudnick
Department of Physics, University of California, Los Angeles, California 90024
(Received 15 February 1985)

The finite-amplitude tuning curves of a surface-wave resonator have been determined for various liquid depths. It is found that the direction of the bending of the tuning curve depends on the liquid depth. The free decay rate of the soliton is measured and shows that solitons initially decay much faster than the corresponding transverse mode but have the same rate in the terminal decay.

PACS numbers: 47.35.+i

In a recent paper[1] we reported the discovery of a unique nonpropagating hydrodynamic soliton whose excitation is that of a transverse surface wave. The explanation which we offered at that time for the existence of this concentration of energy was that, like a polaron, the soliton is self-trapping. The envelope shape of the soliton is the hyperbolic secant, and self-trapping occurs because the cutoff frequency for the transverse vibration is a decreasing function of amplitude. Accordingly, the appropriate frequency for generation of the peak energy at the center of the soliton is below the cutoff frequency at the evanescent wings.

Since then there has been considerable progress in theory and experiment. The purpose of this Letter is to present the new experimental results which are central to our understanding of the self-trapping mechanism and which include data showing that this self-trapping behavior is absent when the water depth falls below a certain value, in agreement with theory.[2,3]

As pointed out in the previous paper, the necessary condition for creation of a nonpropagating hydrodynamic soliton is that a nonlinear dispersive system has an amplitude-dependent cutoff frequency such that the higher the amplitude, the lower the cutoff frequency. If ν_0 is the low-amplitude cutoff frequency, and ν is the amplitude-dependent cutoff frequency of a transverse surface wave resonator which has a rectangular section of width W and liquid depth d, the frequencies are given by[2,3]

$$\nu_0=\frac{1}{2\pi}\left\{\left[g\frac{\pi}{W}+\frac{\alpha}{\rho}\left(\frac{\pi}{W}\right)^3\right]\tanh\left(\frac{\pi}{W}d\right)\right\}^{1/2}, \tag{1}$$

$$\nu\simeq\nu_0\left(1-\frac{Ag^2a^2}{128\,W^4\nu_0^4}\right), \tag{2}$$

where g is the acceleration of gravity, a is the amplitude of the surface wave, α is the surface-tension coefficient, and A is the so-called nonlinear coefficient given by[2,3]

$$A=\tfrac{1}{8}(6T^4-5T^2+16-9T^{-2}), \tag{3}$$

where $T=\tanh(\pi d/W)$. When correction is made for surface tension, Eq. (3) becomes

$$A=\tfrac{1}{2}[1+(1-T^2)^2+\tfrac{1}{2}(1+4\alpha^*)^{-1}(1+T^2)^2 - \tfrac{1}{4}(T^2-4\alpha^*)^{-1}(3-T^2)^2], \tag{3a}$$

where $\alpha^*=\alpha(\pi/W)^2/\rho g.^2$

From Eq. (2), it is obvious that ν can be greater or less than ν_0 depending on the sign of A, which is in turn uniquely determined by the depth of the liquid. If we put $A=0$ in Eq. (3) or (3a) the theoretical cross-over depth can be calculated. Experimentally ν can be determined by measurement of the tuning curve of the transverse mode for various amplitudes of the surface-wave oscillation. From ν_0 determined by Eq. (1) and measured values of a and ν at the peak of the tuning curve, the nonlinear coefficient A can be calculated from Eq. (2) for different depths of the liquid.

The resonator used to measure the tuning curve is a sealed glass basin with the inside dimensions of L 2.0×W 2.6×H 7.0 cm. The resonator for measuring the low-amplitude oscillation of the transverse mode is a sealed glass channel with the inside dimensions of L 38×W 2.6×H 7.0 cm.

The reason for not using the long channel resonator to determine the tuning curve is that when the amplitude of the surface wave gets quite high, solitons appear in the long channel. The length of the short one is chosen to be smaller than the characteristic length of the soliton so that the soliton does not appear, and so that the fundamental resonant frequency of the longitudinal mode is well above that of the transverse mode. Then the tuning curve is that of the transverse mode and, as pointed out in Ref. 1, this is the equivalent of being at the first cutoff frequency for a waveguide of infinite length.

The resonators are cleaned with a solution of potassium hydroxide in ethyl alcohol, followed by washing with hot tap water and ethyl alcohol. Then they are filled with 100% ethyl alcohol up to the depth d. The resonator is put on an improvised table on rollers, which is driven horizontally by a 5-in. loudspeaker in a direction normal to the sides of the trough so that sloshing occurs across the width of the resonator (Fig. 1). An accelerometer (4367, Bruel & Kjaer) is used to

© 1985 The American Physical Society

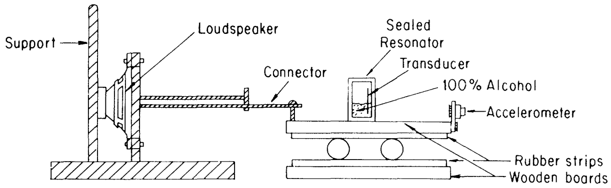

FIG. 1. Schematic of the apparatus with direct drive.

measure the displacement of the oscillating table. It is found that the displacement is quite sinusoidal—the second-harmonic component is at least 40 dB lower than the fundamental. In order to keep the displacement amplitude constant when the frequency is changed, a feedback loop is used.

The transducer for measuring the displacement of the surface wave is a pair of vertical wire electrodes that dip into the liquid. The high-frequency ac conductance between the electrodes is modulated by the low-frequency surface wave motion. A lock-in amplifier picks up the slow modulation, amplifies it, and sends it to the ac-dc converter. The X-Y recorder is used to plot out the tuning curves. The block diagram of the electronics is shown in Fig. 2.

It is of interest to measure the free decay rate of the soliton and this was done in two different ways. One is the direct excitation method, which means that the channel is driven by the oscillating table in its width direction. Once the soliton is created and reaches a steady state, the driving sound is turned off. The free-decaying oscillation data were stored in a wave analyzer and plotted out by an X-Y recorder. The other method is a parametric excitation in which the channel is oscillated vertically at twice the soliton frequency by a vibration exciter (4809, Bruel & Kjaer). Identical decay rates were obtained for both methods.

Typical frequency sweeps for several different drive amplitudes for each of three different depths d are shown in Fig. 3. The dashed curves are drawn through

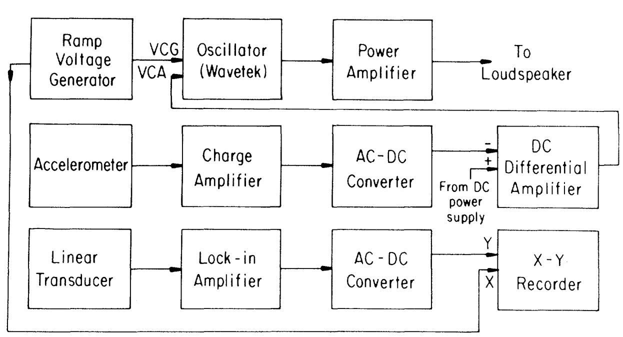

FIG. 2. The block diagram of the electronics.

the measured peak frequencies, and the dash-dotted curves are for the predicted peak frequencies from Eq. (2) when the nonlinear coefficient A of Eq. (3) is used.

The upper set of plots is for the case of $d = 0.5$ cm. It is evident that the peak frequency increases as the amplitude of the surface wave becomes bigger; the

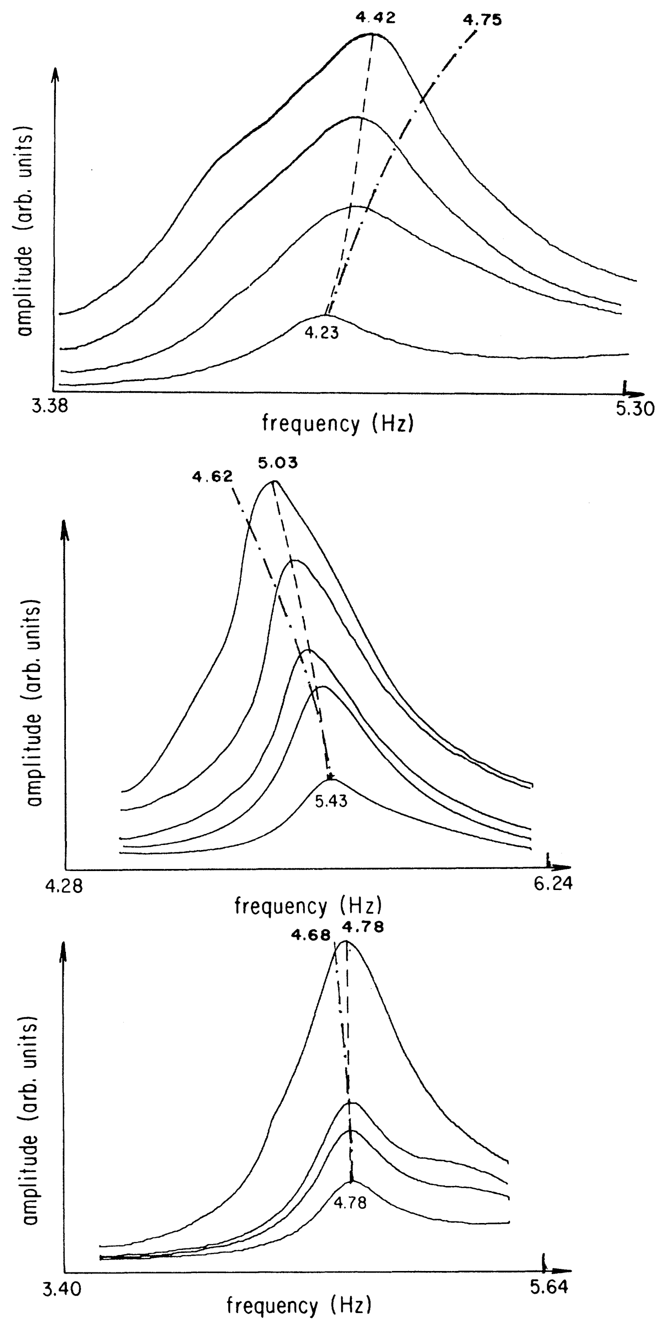

FIG. 3. The transverse-mode amplitude-dependent tuning curves of a rectangular section resonator ($W = 2.6$ cm).

peak frequency shifts up from 4.23 to 4.42 Hz when the amplitude of the wave increases from 0.1 to 0.4 cm in steps of 0.1 cm. ν_0 is 4.22 Hz from Eq. (1) for $d = 0.5$ cm.

The middle set of plots is for the case of $d = 2.0$ cm. What is of overriding importance is that now the peak frequency decreases as the amplitude is increased; the peak frequency shifts down from 5.43 to 5.03 Hz as the amplitude of the wave goes from 0.2 up to 1.0 cm in steps of 0.2 cm. For this depth, ν_0 is calculated to be 5.52 Hz by Eq. (1).

The lower set of plots is for the case of $d = 0.8$ cm. Within experimental error the peak frequency remains constant at 4.78 Hz, as the amplitude increases from 0.1 to 0.8 cm. It was experimentally found that at $d = 0.8$ cm, changes in depth of 0.1 cm produced hardly any detectable shift of peak frequency. We concluded that it is at this depth ($d = 0.8 \pm 0.1$ cm) that the crossover from right leaning to left leaning occurs. Equation (1) gives $\nu_0 = 4.81$ for this depth.

In an experiment performed to investigate this crossover we find stable solitons down to depths as shallow as 0.95 cm, although at this depth the soliton is only marginally stable. We attempted but failed to observe a stable soliton at shallower depths. The discrepancy between this experimentally observed crossover and the crossover depth of 0.8 cm may be due to the fact that the stability of the soliton requires a small but finite degree of left leaning.

Figure 4 contains plots of the nonlinear coefficient A versus fluid depth d from Eqs. (3) and (3a). The points are experimental data. From the two curves we can get an independent determination of the crossover depth to compare with the above experimental value. This is the value of d at which A vanishes. The intersection of a horizontal line at $A = 0$ with the dash-dotted curve yields a value of $d = 0.85$ cm, and for the solid curve $d = 1.03$ cm. From Figs. 3 and 4, we can see that Eq. (3a), which includes surface-tension effects, fits the experimental results in the deep-water region better than Eq. (3). The reverse is true for the crossover depth. Over all, both equations work better at the deep-water region than the shallow-water region. We believe the reason for this is the breakdown of the perturbation theory as the system approaches the nondispersive limit at zero depth.[4] As to the discrepancy between theory and experiment, Putterman and Larraza have expressed their satisfaction with the extent of agreement. They mention that the theory was developed for waves which though nonlinear are much weaker than those used in this experiment. Moreover, viscous effects are not included.

Figures 5(a) and 5(b) are plots of typical free decays of, respectively, a soliton and a transverse mode in the same channel. Clearly, the soliton decays faster than the transverse mode initially. In Fig. 6, it is apparent that the two decay rates become identical when the amplitude of the soliton becomes small after 1.5 s.

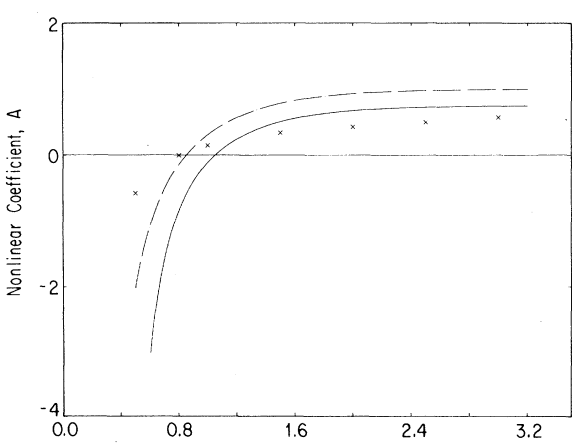

FIG. 4. Plots of the nonlinear coefficient A vs fluid depth d given by Eqs. (3) and (3a); the dash-dotted curve is from Eq. (3), and the solid curve is from Eq. (3a).

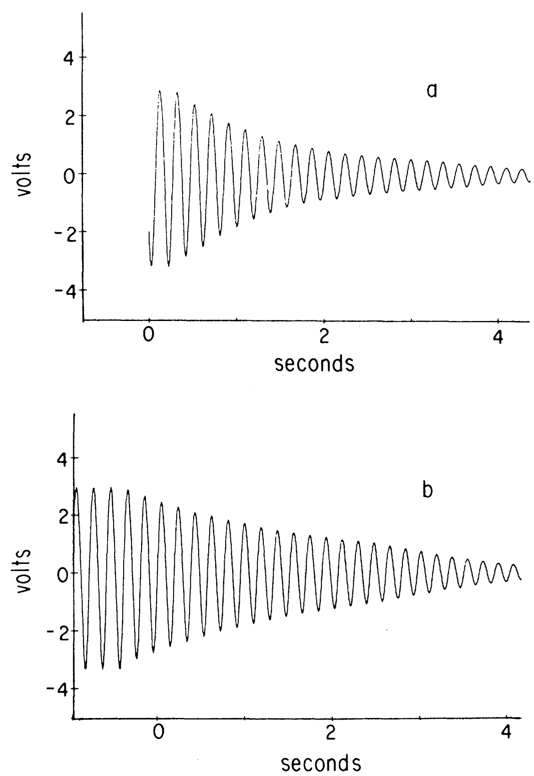

FIG. 5. Plot of the free decay. (a) A decaying soliton with characteristic amplitude of 1.8 cm and frequency of 5.1 Hz. (b) A decaying transverse mode wave with amplitude of 0.5 cm and frequency of 5.1 Hz.

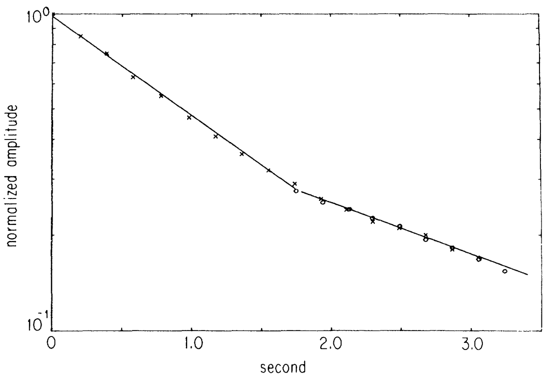

FIG. 6. Plots of the amplitude vs time. Crosses represent the amplitude of the same soliton as in Fig. 5(a), and circles the amplitude of the same transverse wave as in Fig. 5(b).

The initial decay rate of the soliton (frequency 5.1 Hz, and amplitude 1.8 cm) is 0.7 s^{-1}, and changes to 0.4 s^{-1} as its amplitude drops below 0.5 cm, which is seen to be the same as the decay rate of the small-amplitude transverse mode. The theoretical analysis by Miles[2] yields a value for the decay rate of the small-amplitude decaying wave between 0.2 and 0.5 s^{-1}, depending on the extent of contamination of the liquid surface.

Experimental measurements show that a rectangular-section fluid resonator ($W = 2.6$ cm) has an amplitude-dependent tuning curve with a peak frequency ν_p, such that

$$d\nu_p/da < 0 \text{ for } d > 0.8 \text{ cm},$$
$$> 0 \text{ for } d < 0.8 \text{ cm}.$$

The nonlinear coefficient, A, as a function of liquid depth is found to be in good agreement with the theoretical results of Miles[2] and Larraza and Putterman.[3] The value of A can then be used to determine peak frequency of the tuning curve as a function of amplitude of the wave, and again reasonable agreement is obtained. All things considered, the agreement of the experiment with theory is quite satisfactory. Solitons initially decay much faster than the corresponding transverse mode, but have the same rate in the terminal decay.

We wish to acknowledge useful conversations with R. Keolian, A. Larraza, and S. Putterman. This work was supported in part by the U. S. Office of Naval Research.

[1]J. Wu, R. Keolian, and I. Rudnick, Phys. Rev. Lett. **52**, 1421 (1984).

[2]J. W. Miles, J. Fluid Mech. **148**, 450 (1984).

[3]A. Larraza and S. Putterman, J. Fluid Mech. **148**, 443 (1984), and Phys. Lett. **103A**, 15 (1984).

[4]This observation was made by A. Larraza in the course of a discussion of these results.