Hot Toroidal and Bubble Nuclei

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As nuclear temperature increases, the surface-tension coefficient decreases and the Coulomb repulsion is effective in pushing the nuclear matter outward, leading to the formation of toroidal and bubble nuclei. We obtain the threshold temperatures above which these nuclei are stable against symmetry-preserving distortions. They are found to decrease with the mass number.

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Some time ago, Wheeler proposed nuclei with new types of topology and investigated the stability of toroidal nuclei.[1,2] Siemens and Bethe[3] showed that some spherical bubble nuclei with sufficiently large charge may be stable against a symmetry-preserving breathing deformation. Stability against other deformations was examined for these nuclei.[4-8] At zero temperature, nuclear shell effects[5] may stabilize some nuclei against toroidal expansion and contraction and may lead to bubble-type density for ^{200}Hg and ^{36}Ar. Experimental measurements on the properties of Hg isotopes show a peculiar discontinuity[9-12] at ^{200}Hg which warrants further studies.

Toroidal-shaped objects are common in hydrodynamical collisions.[13] Nuclei with a large angular momentum may also assume a toroidal shape.[14,15] Results from hydrodynamical calculations for nuclear collisions[16] show that as the collision energy increases, density voids between regions of normal density can develop. It is reasonable to expect that at some collision energies, the deceleration of the high-density region and the development of the density void may work together and lead to the formation of a bubble or bubbles. Indeed, bubble-type densities appear in the dynamics of an expanding nucleus in Thomas-Fermi theory[17] and finite-temperature Hartree-Fock theory.[18] Furthermore, there is recently considerable interest in the behavior of nuclear systems at high temperatures.[17-25] Bubble voids are found to be possible configurations of hot dense nuclear matter.[22,23] As the nuclear temperature T increases, the surface-tension coefficient $\sigma(T)$ decreases and may favor the formation of toroidal or bubble nuclei.

We consider a nucleus (Z,A) at a temperature T. The fissility parameter of the nucleus at zero temperature $x(0)$ is given in terms of $\sigma(0)$ and the nuclear density $n(0)$ at $T=0$ by[26]

$$x(0)=Z^2e^2n(0)/10\sigma(0)A. \tag{1}$$

Hence, the fissility parameter $x(T)$ of the nucleus at the temperature T is related to $x(0)$ by

$$x(T)=[n(T)\sigma(0)/\sigma(T)n(0)]x(0). \tag{2}$$

As the temperature increases, the decrease in the surface-tension coefficient is faster than the decrease in density so as to lead to an increase in the fissility parameter. Such an increase has been observed recently.[25] We would like to find the temperature T_{th} at which the fissility parameter $x(T_{\text{th}})$ reaches the threshold values of x_{th} for toroidal or bubble nuclei. For toroidal nuclei, x_{th}(toroidal) is 0.964 and the ratio of the major to the minor axis is 2.079. For bubble nuceli, the threshold x_{th}(bubble) is 2.03 and the ratio of the inner to the outer radius is 0.421.

To obtain the threshold temperature, we need an explicit relation between σ and T. Recently, Ravenhall, Pethick, and Lattimer[21] parametrized $\sigma(T)$ as follows:

$$\sigma(T)=\sigma(0)\left(\frac{1-T^2/T_c^2}{1+aT^2/T_c^2}\right)^p, \tag{3}$$

where the critical temperature T_c is 20 MeV, $a=0.935$, and $p=1.25$. We also need to evaluate the equilibrium density $n(T)$ by minimizing the free energy which leads to the equation for the equilibrium pressure p[21]:

$$pA/n+(-2E_s+E_c)/3=0, \tag{4}$$

where E_s and E_c are the surface and Coulomb energies of the nucleus for the shape corresponding to the toroidal or bubble threshold. In future refinement, we may wish to add a term for a bubble nucleus p_gA_g/n_g, where the subscript g refers to the gas matter contained inside the bubble. For numerical purposes, we choose an equation of state of the form

$$p(n,T)=(\hbar^2/5m)(1.5\pi^2)^{2/3}n^{5/3}[1+(5\pi^2/12)(T/\epsilon_{\text{F}})^2]+\tfrac{3}{8}[t_0+t_3n^{\alpha}/6]n^2, \tag{5}$$

where $t_0=-3390$ MeV fm^3, $t_3=21\,662$ MeV fm$^{3(1+\alpha)}$, and $\alpha=0.1479$, corresponding to an equilibrium nuclear-matter density[27] $n_{\text{eq}}=0.1533$ fm^{-3}, binding energy[27] $W=-16.1$ MeV, and compressibility[28] $K=220$ MeV. The temperature dependence is assumed to be that of a degenerate Fermi gas and an expansion is made up to second order in (T/ϵ_{F}). We use a Fermi energy ϵ_{F} of 40 MeV, a surface-energy coefficient a_2 of 18.011 MeV, and a surface symmetry coefficient[27] $\kappa_s=1.59$. The set of Eqs. (1)–(5) is employed to solve for T and $n(T)$ when $x(T)$ is set equal to x_{th}. This gives the threshold temperature T_{th} and the corresponding density.

We plot in Fig. 1 the ratio T_{th}/T_c and the equilibrium density $n(T)$ as a function of Z for nuclei along the beta-stability line of spherical nuclei. One observes that for either the toroidal or the bubble case, the threshold temperature is a decreasing function of Z while $n(T)$ is an increasing function of Z. For $Z=80$, the threshold temperature is $0.41T_c$ for a toroidal shape and is $0.76T_c$ for a bubble shape. Toroidal nuclei can be formed at a lower temperature than bubble nuclei.

The surface tension vanishes when the temperature exceeds the critical temperature. In that case, it is not meaningful to speak of a toroidal or a bubble nucleus as there is no distinction between the different phases beyond the critical temperature. Therefore, these nuclei can be formed only in the temperature window $T_{\mathrm{th}} \leq T \leq T_c$. For heavier nuclei, the threshold temperature is lower and the temperature window is wider. A toroidal nucleus rotating about its central symmetry axis will experience a centrifugal force pushing the nuclear matter outward. The threshold temperature for the toroidal nucleus in such a rotation is lower than that for a nonrotating toroidal nucleus.

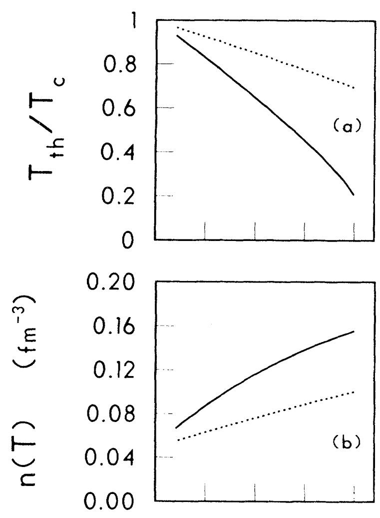

FIG. 1. (a) The ratio of the threshold temperature T_{th} for the formation of toroidal and bubble nuclei to the critical temperature T_c, as a function of the atomic number Z along the beta-stability line (of spherical nuclei). The solid curve is for toroidal nuclei and the dotted curve is for bubble nuclei. (b) The equilibrium nuclear density at the threshold temperature T_{th} for toroidal and bubble nuclei, as a function of the atomic number Z along the beta-stability line. The solid curve is for toroidal nuclei and the dotted curve is for bubble nuclei.

We have examined the consequences of a decreasing surface tension and its relation to the formation of toroidal or bubble nuclei. What needs to be further examined is the evolution of such objects after they are formed. The results here may also depend on the equation of state and the temperature dependence of σ, which are the subjects of current interest. Whatever theoretical extrapolations there can be, in the final analysis the formation and evolution of these nuclei is an experimental question. It is of interest to see whether there are ways to produce and to detect these nuclei and whether there are already possible candidates for which these shapes may play a role in their eventual breakup. Nearly-head-on central hydrodynamical collisions lead to the formation of toroidal objects.[13] Hydrodynamical calculations and other dynamical calculations also indicate density voids.[16–18] Furthermore, in central collisions of heavy nuclei at intermediate energies of many tens of megaelectronvolts per projectile nucleon, nucleon-nucleon collisions lead to a rapid thermalization of the system and a temperature of the magnitude of 5–20 MeV can be reached. Central collisions of heavy nuclei at intermediate energies may allow the formation of hot toroidal or bubble nuclei. However, as these nuclei can be formed only within this temperature window, there is a corresponding energy window for their formation at a given impact parameter. It will be of interest to map out these windows. On the other hand, relativistic peripheral collisions at very high energies may deposit sufficient energy to either the projectile or the target nucleus to reach the temperature window discussed here. Thus, in a relativistic nuclear peripheral collision, toroidal or bubble nuclei may also be formed. One way to check whether a toroidal nucleus or a bubble nucleus has been formed is to measure the temperature of the fragments from the fragmenting nucleus to see if the temperature corresponds to the range of temperature of $T_{\mathrm{th}} \leq T \leq T_c$ discussed here. One can use the giant dipole gamma rays to study the shape of a hot nucleus before it breaks up.[29] One can look at the fragment angular and mass distributions as the fragments from a toroidal or a bubble nucleus are expected to distribute themselves in a more symmetric way. The dominant modes of instability for a toroidal nucleus are probably those of sausage distortions of small order. One expects that two, three, four, or more fragments ("superclusters") of approximately equal masses will distribute in a ringlike manner.[5,30] The total kinetic energy of the fragments will reveal the approximate geometry of the fragments before separation. The dominant modes of instability for a bubble nucleus are probably spheroidal distortions of low multipolarities.[5] The fragment mass and angular distribution should show the characteristics of superclusters in the form of a few pieces of large fragments at an intermediate stage. In contrast, a fragmentation without going through the doorway of these peculiar shapes would have quite different characteristics.

It is worth noting that a metastable rotating toroidal nucleus or a bubble nucleus will manifest itself as a nucleus with a large collisional cross section and will have an anomalously short mean free path in subsequent collisions, not unlike the anomalons which have been reported recently.[31,32] In this connection, it is of interest to discuss whether the anomalons are toroidal or bubble nuclei. One can envisage the possibility that a projectile nucleus in a relativistic peripheral collision is heated up and has acquired an angular momentum after the collision to lead to the formation of a hot toroidal or a bubble nucleus. Subsequent cooling and loss of angular momentum by particle and gamma-ray emission bring the nucleus down to a temperature where nuclear shell effects become operative. The shell effects may be responsible for stabilizing the nucleus to make it a long-lived exotic nucleus. Although shells exist for light bubble nuclei, stability of such nuclei has not been examined in detail. On the other hand, previous investigation of the toroidal single-particle states indicates that shell effects may stabilize light toroidal nuclei with mass $\sim 20 < A < \sim 70$ against toroidal expansion and contraction. If these toroidal nuclei are anomalons, one expects that as the toroidal aspect ratios R/d increase with mass number (Fig. 16 of Ref. 5), the mean free path decreases rapidly with the mass of the anomalon which is presumably related to the mass of the incident projectile. Recently, it has been suggested that some special cases of these toroidal nuclei with configurations of $N\times(^4\mathrm{He}+2n)$ may be anomalons.[33] If nuclear shell effects are responsible for the stability of the anomalons, our previous analysis in Ref. 5 indicates that the anomalons will encompass a large region of mass number and atomic number in the light-nuclei region. They include but need not be restricted to toroidal nuclei with such configurations as the $N\times(^4\mathrm{He}+2n)$ toroids. There are, however, many conflicting experimental results concerning the observations of the anomalons.[32] Further definitive studies of the anomalons will be of interest.

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