Measurements of Macroscopic Quantum Tunneling out of the Zero-Voltage State of a Current-Biased Josephson Junction

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The escape rate of an underdamped ($Q \approx 30$), current-biased Josephson junction from the zero-voltage state has been measured. The relevant parameters of the junction were determined *in situ* in the thermal regime from the dependence of the escape rate on bias current and from resonant activation in the presence of microwaves. At low temperatures, the escape rate became independent of temperature with a value that, with no adjustable parameters, was in excellent agreement with the zero-temperature prediction for macroscopic quantum tunneling.

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The observation of macroscopic quantum tunneling is regarded as a test of whether quantum mechanics is valid for macroscopic variables, a fundamental question[1] that has only recently been addressed experimentally. The necessary conditions for the observation of macroscopic quantum tunneling can be realized in the current-biased Josephson tunnel junction, where the phase difference between the two superconductors is the macroscopic variable, and the tunneling occurs from the zero-voltage state to the nonzero-voltage state. Previous experiments on a current-biased Josephson junction[2–4] or on a superconducting ring interrupted by a Josephson junction[5–7] have yielded results that have been interpreted as being consistent with the theoretical predictions for macroscopic quantum tunneling. In this Letter, we present results of experiments on a current-biased junction that differ from earlier measurements primarily in that we determine *in situ* all of the relevant parameters using *classical* phenomena. In particular, we measure the impedance shunting the junction at the relevant microwave frequencies. We are thus able to compare the experimental results quantitatively with theoretical predictions with no adjustable parameters.

The current-biased Josephson junction can be represented as a particle moving in a one-dimensional tilted cosine potential.[8] The zero-voltage state of the junction corresponds to the confinement of the particle to one well of this potential. After the particle escapes from this metastable state, it runs freely down the tilted cosine potential, and a voltage appears across the junction. For a constant bias current I slightly less than the critical current I_0, the well from which the particle escapes is given by a cubic potential with barrier height[9] $\Delta U = (2\sqrt{2}I_0\Phi_0/3\pi)(1-I/I_0)^{3/2}$, where $\Phi_0 = h/2e$ is the flux quantum. We have designed the experiment so that the total admittance across the junction, including contributions from the current and voltage leads, can be represented to a good approximation by a capacitance C and a resistance R in parallel. In the zero-voltage state, the plasma frequency $\omega_p/2\pi$ of small oscillations of the particle at the bottom of the well is $\omega_p = (2\pi I_0/C\Phi_0)^{1/2}[(1-(I/I_0)^2]^{1/4}$, while the damping factor is $Q = \omega_p RC$.

In the thermal regime ($k_B T >> \hbar\omega_p$), the escape of the particle from the well occurs via thermal activation at a rate[10]

$$\Gamma_t = a_t(\omega_p/2\pi)\exp(-\Delta U/k_B T), \tag{1}$$

where $a_t = 4/[(1+Qk_B T/1.8\Delta U)^{1/2}+1]^2$ is of the order of unity in our experiment, k_B is Boltzmann's constant, and T is the temperature. In the quantum regime ($k_B T << \hbar\omega_p$), to lowest order in $1/Q$ the escape is predicted to occur via macroscopic quantum tunneling at a rate[11]

$$\Gamma_q = \frac{a_q\omega_p}{2\pi}\exp\left[-\frac{7.2\Delta U}{\hbar\omega_p}\left(1+\frac{0.87}{Q}\right)\right] \tag{2}$$

at $T=0$, where $a_q \approx [120\pi(7.2\Delta U/\hbar\omega_p)]^{1/2}$.

To express the experimental measurements of the escape rate in a way that is as independent as possible of the parameters of the junction, we introduce the "escape temperature" T_{esc} defined through the relation

$$\Gamma = (\omega_p/2\pi)\exp(-\Delta U/k_B T_{esc}). \tag{3}$$

In the thermal regime, the theoretical prediction is

$$T_{esc} = T/(1-p_t), \tag{4}$$

where the magnitude of $p_t = (k_B T/\Delta U)\ln a_t$ is small compared with unity. In the quantum regime at $T=0$, the prediction is

$$T_{esc} = \frac{\hbar\omega_p/k_B}{7.2(1+0.87/Q)(1-p_q)}, \tag{5}$$

where $p_q \approx (\hbar\omega_p/7.2\Delta U)\ln a_q$. The crossover temperature at which the escape rate changes from thermal (temperature dependent) to quantum (temperature independent) is predicted[12] to be $\hbar\omega_p/2\pi k_B$ in the limit $Q >> 1$. We will determine T_{esc} from the measured values of Γ, ω_p, I_o, and I, and compare these experimental values with that predicted by Eq. (4) or (5).

The experimental configuration has been described previously.[13] The 10×10-μm^2 Nb-NbO_x-PbIn tunnel junction was mounted at one end of an attenuating coaxial line. The mount and the last of a series of low-pass filters for the bias circuitry were thermally anchored to the mixing chamber of a dilution refrigerator. The critical current of the junction could be reduced by a magnetic field. A separate, heavily filtered coaxial line, capacitively coupled to the bias leads of the junction, could be used to inject a microwave bias current, thereby enabling us to measure $\omega_p(I)$ and $Q(I)$ *in situ* in the classical limit using resonant activation.[13] We determined the escape rates by ramping the bias current and measuring the value of current at which the junction switched to the nonzero-voltage state.[9] By turning off the current within 30 μs after the transition, we were able to make heating effects negligible. We collected typically 10^5 switching events for each measurement of Γ.

The critical current was determined in the thermal regime by the exponential dependence of the escape rate on the bias current. As is evident from the expressions for ΔU and Γ_t, a plot of the experimentally determined quantity $\{\ln[\omega_p(I)/2\pi\Gamma(I)]\}^{2/3}$ vs I should, with neglect of departures of a_t from unity, be a straight line with slope scaling as $T_{esc}^{2/3}$ that intersects the current axis at I_0. Figure 1 shows three examples of such plots out of the seven obtained in the thermal regime over the temperature range from 102 to 800 mK. In addition, we show two sets of data obtained at 46 and 19 mK where quantum corrections are expected to be important. We note that the slope

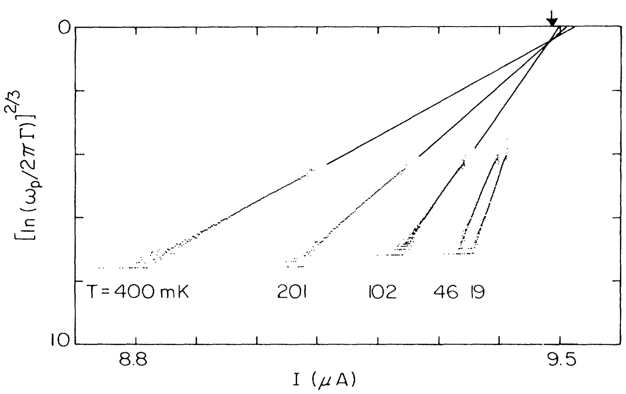

FIG. 1. $[\ln(\omega_p/2\pi\Gamma)]^{2/3}$ vs I for five values of temperature. Lines that intersect the current axis have been drawn through the data in the thermal regime, at the three highest temperatures. The arrow indicates the value of I_0 obtained after corrections for the prefactor were made.

changes very little as the temperature is lowered from 46 to 19 mK, indicating that T_{esc} is nearly the same at these two temperatures. In fact, two sets of data obtained at 30 and 24 mK (not shown in Fig. 1) were indistinguishable from the data at 19 mK. As expected, the lines drawn through the data at the three higher temperatures intersect the current axis at very nearly the same point. The values of I_0 obtained from the seven sets of data in the thermal regime ranged from 9.498 to 9.535 μA. We then corrected these values of I_0 for the departure of a_t from unity, using the value $Q = 30 \pm 15$ obtained from resonant activation.[13,14] These corrections were small, varying from -12 ± 4 nA at 100 mK to -47 ± 12 nA at 800 mK. After these corrections were made, the critical current was independent of temperature to within the experimental uncertainties, with the value 9.489 ± 0.007 μA. Quantum corrections[15] to I_0 were negligible.

We used the measured values of $\omega_p(I)$ and $Q(I)$ at numerous values of I and three values of I_0 to determine C and R as functions of $\omega_p/2\pi$ over the frequency range from 2 to 8 GHz; a typical value of $\omega_p/2\pi$ for the high-critical-current junction in the quantum limit was 4 GHz. We found that the measured value of both C and R varied somewhat over this frequency range, presumably because of standing-wave resonances in the line connected to the junction. However, the variations in C were sufficiently small that we feel justified in using an average value with an error bar that includes most of the variation, $C = 6.35\pm 0.4$ pF. The frequency dependence of the value of R was more pronounced, and we have taken the value $R = 190\pm 100\Omega$, where the quoted error again includes most of the variation. Since the damping in these experiments is relatively weak, the large uncertainty in R does not lead to a significant error in the predicted value of T_{esc} at $T=0$. As in our previous experiments,[13] the value of R was dominated by the conductance of the coaxial line to which the junction was attached, while C was dominated by the self-capacitance of the junction.

Using the measured values of I_0, C, and R, we can compute[16] T_{esc} from Γ as a function of I and T. In Fig. 2, we plot T_{esc} vs T for a junction with $I_0 = 9.489 \pm 0.007\mu$A. Since, as we shall see, T_{esc} is weakly dependent on the bias current, we have plotted these data at a bias current chosen so that $\ln(\omega_p/2\pi\Gamma) = 11$. The predicted crossover temperature of 30 mK is indicated by a solid arrow in Fig. 2. At temperatures above about 100 mK, the measured value of T_{esc} is very close to the temperature T as we expect in the thermal regime. At temperatures below about 25 mK, on the other hand, T_{esc} becomes independent of temperature, with a value of 37.4 ± 4 mK. The Caldeira-Leggett prediction at $T=0$ is $T_{esc} = 36.0\pm 1.4$ mK, which is in very good agreement with the tem-

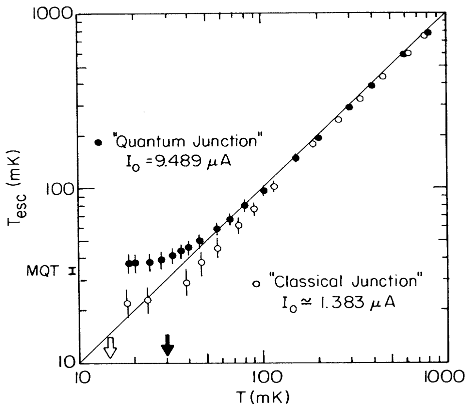

FIG. 2. T_{esc} vs T for two values of critical current for $\ln(\omega_p/2\pi\Gamma)=11$. The solid and open arrows indicate the predicted crossover temperatures for the higher and lower critical currents, respectively. The prediction of Eq. (5) for the higher critical current is indicated at the left.

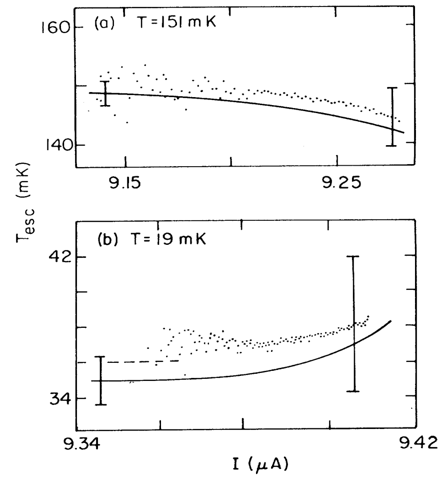

FIG. 3. T_{esc} vs I for a junction with $I_0=9.489\pm0.007\mu$A (a) in the classical regime and (b) in the quantum regime. Points are the experimental data and solid lines are the theoretical prediction. The dashed line in (b) is the prediction for zero damping. The error bar on the left and the right of each figure represents the possible shift in the theoretical and experimental curves, respectively, due to uncertainties in the experimental parameters. The solid line represents $T_{esc}=T$.

perature-independent value observed in our experiment. The contribution of the damping to the predicted value of T_{esc} is -1.5 mK, which is less than the combined uncertainty of the theoretical prediction and experiment. Thus we cannot presently make any statement about the effect of dissipation on quantum tunneling. We note that the error in the measured values of T_{esc} in the quantum limit is dominated by the uncertainty in ΔU, which arises, in turn, from the uncertainty in I_0. On the other hand, the error in the predicted value of T_{esc} arises predominantly from uncertainties in ω_p and Q.

Although the low-temperature values of T_{esc} plotted in Fig. 2 are in good agreement with the $T=0$ prediction, nevertheless one should demonstrate that the flattening of T_{esc} is not due to an unknown, spurious noise source. To establish that the effective temperature of the dissipative element was close to T down to the lowest temperatures of the experiment, we applied a magnetic field to the junction to reduce the critical current. After we had corrected the data for the temperature dependence of a_t, we found that this reduced critical current still varied very slightly with temperature, from 1.376 ± 0.005 μA at 800 mK to 1.388 ±0.002 μA at 20 mK. The temperature dependence of I_0 may have arisen because of the sensitivity of I_0 to magnetic field and the fact that the applied field possibly changed with temperature. In Fig. 2, we have also plotted T_{esc} for the junction with the lower critical current for $\ln(\omega_p/2\pi\Gamma)=11$. At each temperature, we calculated T_{esc} using the value of I_0 measured at that temperature. The predicted crossover temperature, 14 mK, is indicated with an open arrow. We observe that T_{esc} is equal to T to within the experimental error, although there is a suggestion that T_{esc} is beginning to flatten off at the lowest temperature, where quantum effects are likely to become significant. Thus we conclude that the flattening of T_{esc} for the junction with the higher critical current did not arise from spurious noise sources.

An important difference between the thermal and quantum regimes may be observed through the weak dependence of T_{esc} on the bias current, which arises from the different forms of a_t and a_q and from the current dependence of ω_p. This behavior is illustrated in Fig. 3 for $I_0=9.489\mu$A. In Fig. 3(a) we plot T_{esc} vs I in the thermal regime ($T=151$ mK), together with the prediction of Eq. (4). The decrease of T_{esc} with increasing bias current arises because $a_t<1$. Within the uncertainties, the data are in good agreement with theory. Figure 3(b) shows T_{esc} vs I in the quantum regime ($T=19$ mK), together with the prediction of Eq. (5). In this limit, T_{esc} increases with increasing bias current through the current dependence of ΔU because $a_q \gg 1$; the current dependence of ω_p is relatively unimportant. Again, within the experimental uncertainties, the data are in good agreement with theory. The very different current dependence of T_{esc} at low and high temperatures lends further support to the claim that the escape mechanisms are different in the two temperature regimes.

In summary, we have measured the escape rate of a current-biased, underdamped ($Q\approx30$) Josephson tunnel junction from the zero-voltage state for two values of critical current, the lower value being achieved by means of an external magnetic field. At the lower critical current, T_{esc} followed the classical prediction to within experimental error, indicating that the effects of extraneous noise were negligible. For the higher value of critical current, the value of T_{esc} was equal to T at high temperatures, but began to flatten off at temperatures below 50 mK and became independent of temperature below 25 mK. Within the experimental uncertainties, the low-temperature value of T_{esc} was in excellent agreement with the theoretical prediction for $T=0$, with all the relevant parameters measured *in situ* in the classical limit.

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