Pore-Scale Viscous Fingering in Porous Media

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A simple computer model of viscous fingering in a porous medium is presented, together with experimental results for fingering of oil into glycerine in etched-glass networks. The model treats the medium as a square lattice of connected tubes with randomly chosen radii. By Monte Carlo simulation it is demonstrated that for a narrow size distribution of tubes the fingers form ordered patterns (dendrites) with growth mostly along the coordinate axes, but for a wide size distribution the fingers form a chaotic structure with a fractal dimension around 1.72. Qualitatively similar behavior is seen in the experiments.

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Viscous fingering in porous media is a phenomenon which results when a less viscous fluid is displacing a more viscous one; a planar boundary between the fluids is unstable against small perturbations, and in the course of time the interface adopts a fingered configuration. In the oil industry this viscous fingering can be a serious problem when one is trying to displace a viscous oil by pumping water into the reservoir (water flooding), because it leads to poor recovery of the hydrocarbon. Since interfacial tension provides a stabilizing effect at short distances, the fingering of immiscible fluids in a porous medium is usually a macroscopic phenomenon in which the fingers are large compared to the pore scale. For this reason, viscous fingering in porous media is often compared to that which occurs in a Hele-Shaw cell,[1] in which the flow takes place between two parallel plates. There, the two-dimensional velocity field $\mathbf{v}$ of a fluid of viscosity μ is related to the pressure p by

$$\nabla p = -(\mu/k)\mathbf{v}, \tag{1}$$

which is the same macroscopic equation (Darcy's law) as holds in a porous medium. In the Hele-Shaw cell the permeability k is related to the gap thickness b by $k = b^2/12$. In the classical Hele-Shaw fingering problem, first considered by Saffman and Taylor,[2] the displacing fluid is inviscid, and flow in the displaced fluid satisfies (1). If the displaced phase is perfectly wetting, then the interfacial pressure difference between the displacing phase, labeled 2, and the displaced phase, labeled 1, is given by

$$p_2 - p_1 = \gamma(2/b + 1/R), \tag{2}$$

where γ is the interfacial tension, and R is the macroscopic curvature of the interface.

Hele-Shaw fingering is an interesting and deep problem which has been discussed extensively in the literature.[2–4] However, in the present authors' opinion, it is of doubtful relevance to the question of fingering in a porous medium for several reasons: (1) In a porous medium, the interface between the two fluids is not sharp, but rather there exists a region of partial saturations in which the macroscopic flow behavior is more complex than in (1). (2) The surface tension in a porous medium does not act via the macroscopic curvature of the interface as in (2). Rather, at the pore level it acts via the curvature of individual menisci, and at the macroscopic level through the capillary pressure function.[5] (3) The classical Hele-Shaw fingering analysis requires a perfect cell, while natural porous materials are extremely disordered.

The purpose of the present Letter is to investigate the third issue—the effect of randomness on the fingering process. The randomness in a porous medium can exist on many scales, from the pore level (microns) to the reservoir scale (kilometers). Here we will focus on viscous fingering at the microscopic scale in a medium which is statistically homogeneous, but random on the pore scale. The model that we consider for such a medium is a square network of cylindrical tubes of fixed length L and variable radius r. The radii are uniformly distributed in the interval $[1-\lambda, 1+\lambda]$, where λ is an adjustable parameter. The displacing fluid is inviscid, and we assume that surface tension stabilizes the interface between the fluids in each individual tube. However, the interfacial pressure drop between the fluids is neglected. [Note that this is not mathematically possible in the Hele-Shaw cell—the pressure difference (2) is needed to make the problem well posed. In the porous medium the pore size provides the required short-distance cutoff on the finger size.] If the tubes are considered long compared to their width, then we may neglect the pressure drop associated with the nodes, and estimate the pressure drop between nodes i and j by assuming Poiseuile flow,

$$p_i - p_j = \frac{8}{\pi}\frac{\mu(L - x_{ij})}{r_{ij}^4} Q_{ij} = g_{ij}^{-1} Q_{ij}, \tag{3}$$

where μ is the viscosity of the displaced phase, x_{ij} is the length of bond ij which contains the displacing phase, and Q_{ij} is the flux flowing from node i to node

j. The quantity g_{ij} is the "flow conductance" of bond ij, and changes as a function of time. At each node we have the conservation equation

$$\sum_j Q_{ij} = 0, \tag{4}$$

so that Eqs. (3) and (4) are analogous to Kirchoff's laws for an electrical network. The model is a simplified version of recent computer simulations of viscous-capillary flow due to Koplik and Lasseter[6] and Payatakes and Dias.[7] It is also similar to models of viscous fingering due to Simon and Kelsey,[8] though our emphasis here is rather different.

The boundary condition that we use is injection of the inviscid fluid into the center of a circular region occupied by the viscous fluid, with the interfaces between the fluids initially placed at random positions in the four tubes connected to the origin. Constant-pressure boundary conditions are imposed in the inviscid-fluid region, and on the circular boundary. In the absence of surface tension the equations are linear, so that the actual value of the pressure difference is irrelevant. For simplicity, we solve Eqs. (3) and (4) for the pressure field by Gauss-Seidel iteration,

$$p_i \rightarrow \alpha \frac{\sum_j g_{ij} p_j}{\sum_j g_{ij}} + (1-\alpha) p_i, \tag{5}$$

with the overrelaxation parameter α around 1.6. Given the solutions p_i we may obtain the Q_{ij} from (3). In a time Δt, an interface in tube ij will move a distance

$$\Delta x_{ij} = (Q_{ij}/\pi r_{ij}^2)\Delta t. \tag{6}$$

In principle we should choose the time step Δt in accordance with some convergence criterion, but in practice we choose it to be the time that it takes for exactly one interface to reach a node. We then move all the interfaces according to (6), and introduce new interfaces in the tubes adjacent to the node reached. The pressure field is solved again, and so on. (Note that the pressure field is such that the interfaces always move outward—this is a feature due to the assumed zero viscosity of the displacing fluid.) The simulation ends when the displacing fluid reaches the circular boundary.

In Fig. 1 we show results of the computer simulation for three different values of the randomness parameter λ. The dots mark the nodes reached by the displacing fluid. Figure 1(a), $\lambda = 0$, is the case of identical tubes. It is seen that the fingering is dendritic in character with growth along the coordinate axes. Note that solving Kirchoff's laws on a square grid of equal resistors is equivalent to solving the continuum Laplace equation with a five-point Laplacian. In the latter case, this kind of fingering along the axes would be purely a grid artifact, but here we predict that it is a real effect for a network of tubes. In Fig. 1(b), $\lambda = \frac{1}{3}$, we still see the dendritic growth, but now there are 90° side branches in evidence in addition to the growth along the axes. Finally in Fig. 1(c), $\lambda = 1$, we see that the randomness has overcome the anisotropy due to the square grid, and the displacement is chaotic. An estimate of the fractal dimension of the cluster (based on three simulations on a system of radius 80) gives a value around 1.72.

In order to check these predictions, we have performed viscous-fingering experiments in etched-glass networks.[9] The samples are prepared by etching a pat-

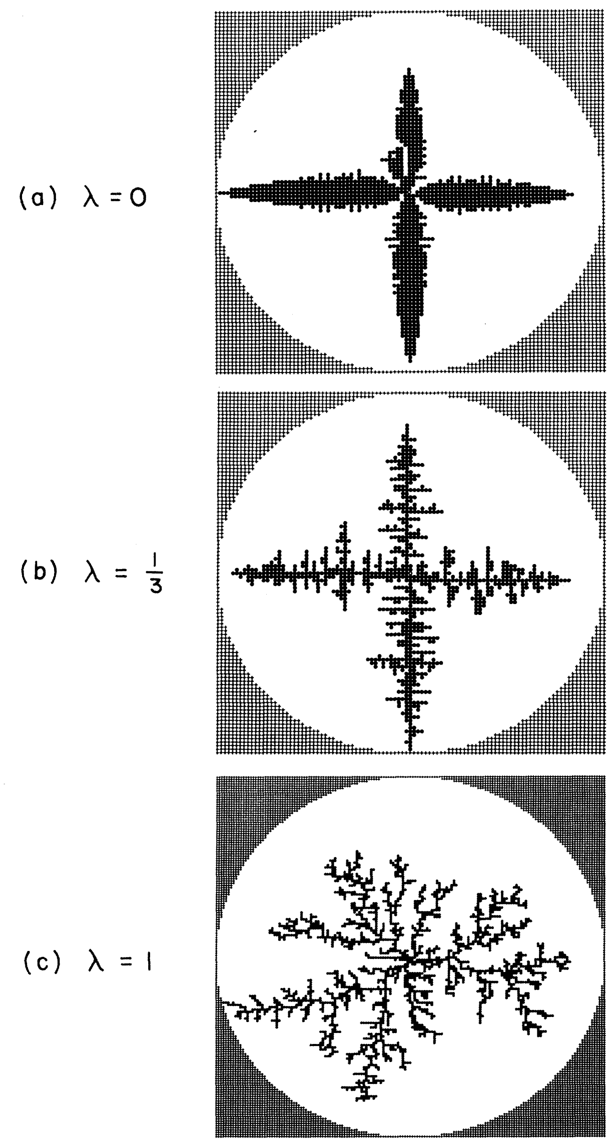

FIG. 1. Three computer simulations of viscous fingering for different values of the randomness parameter λ. In (a) and (b) the radius of the circular region is 50 lattice spacings, and in (c) it is 80.

tern in a glass plate, and then clamping, or in some cases fusing, a flat glass plate on top. The upper plate has a small hole drilled in the center, and the edges of the sample are open to the atmosphere. Note that, unlike some other recent experiments,[10] there is no gap between the plates—the flow takes place only in the etched channels. Initially the network contains air, and then glycerine (viscosity 1200 cP) is injected through the hole in the upper plate. Since this is a stable displacement, the front is approximately circular. The displacement is stopped just before the glycerine reaches the edge of the network. The experiment itself consists of injecting oil (viscosity 1 cP) through the central hole. Thus the viscosity ratio is high (around 1200), and the circular boundary of the glycerine (which is open to the atmosphere) is at constant pressure, as in the simulation. Glycerine is the wetting phase and the interfacial tension is 20 dyn/cm. The displacement is carried out at a constant flow rate of 1.4×10^{-4} cm^3/sec.

The experiment was repeated for three different samples. The first, shown in Fig. 2(a), was a 120×120 square network of interconnected channels of length 500 μm, etch depth 35 μm, and uniform width 250 μm. It is seen that the fingering takes place along the coordinate axes in a manner very similar to the simulation with constant tube radius, Fig. 1(a). The second sample, shown in Fig. 2(b), was a 90×90 square lattice, with channel length 500 μm, etch depth 44 μm, but variable width between 80 and 730 μm. Again, the fingering takes place along the axes, but with side branches, in a manner similar to the simulation with intermediate randomness, Fig. 1(b). In the final sample, shown in Fig. 2(c), the flow channels were formed by placing approximately circular "grains" at random in the plane and etching the region between them. The diameter of the grains was 250 μm, the etch depth was 26 μm, and the average gap between the grains around 250 μm. In this case the fingering pattern is chaotic, and of a character similar to the simulation on the most random square lattice, Fig. 1(c).

Needless to say, the geometry of our model does not precisely correspond to the experimental geometry. In addition, simple estimates suggest that, although the flow rate in the experiment is considerably higher than in most oil-field situations, the pressure drops due to surface tension are not completely negligible. Nevertheless, we see that the simulations are in good qualitative agreement with the experiments. In particular, the fingering passes from a dendritic to a chaotic character. We believe that this feature of the model, embodied in the randomness parameter λ, makes it of interest in the general context of the dynamics of moving interfaces and pattern formation.[11] It remains an interesting unanswered question whether the transition occurs at some sharp value of λ, or whether the nature of the displacement depends also on the size of the system.

Among these results, it is clear that those for random systems, Figs. 1(c) and 2(c), are most representative of fingering in a disordered system such as a porous rock. These pictures are also qualitatively similar to those seen in various other kinetic growth phenomena considered recently in the literature. In particular, they are very similar to the viscous fingering observed in a Hele-Shaw cell by Nittmann, Daccord, and Stanley.[12] Their experiment is different from traditional Hele-Shaw experiments in that the fluids are miscible and the displaced phase is a polysacharide solution. Exactly why this leads to fingers so different from the usual Saffman-Taylor fingers is not

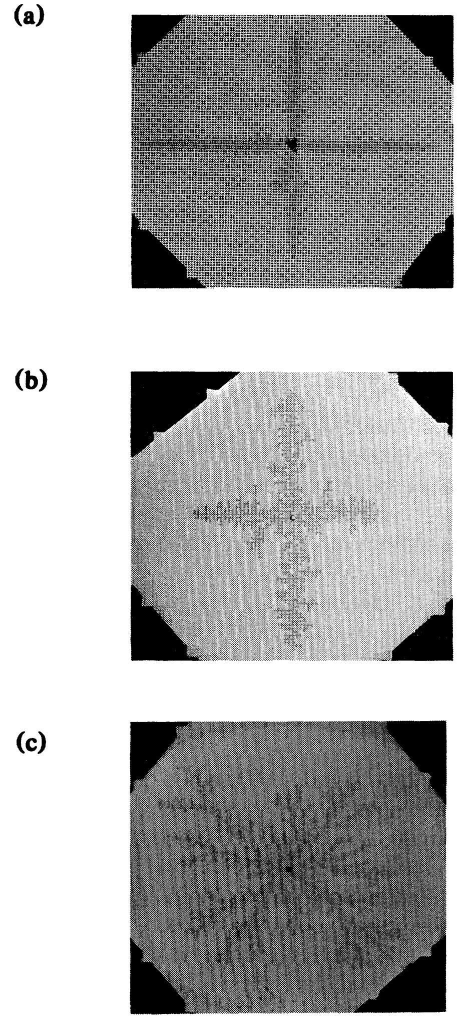

FIG. 2. Experimental results for viscous fingering of oil into glycerine in three different etched-glass networks. The characteristics of the samples are described in the text.

clear, but presumably the lack of the stabilizing surface-tension forces and the non-Newtonian nature of the displaced fluid are important features. Following Paterson,[13] these authors model their experiment by means of the diffusion-limited aggregation (DLA) process introduced by Witten and Sander.[14] While the connection between DLA and viscous fingering is not, in the present authors' opinion, a rigorous one, it is undoubtedly true that DLA clusters are visually very similar to both the simulations and experiments presented here. In addition, our estimate of 1.72 for the fractal dimension is close to that of DLA in two dimensions.[14] It is interesting to note that in DLA the randomness comes not from the underlying medium, but rather from the stochastic nature of random walks.

Our simulations are also similar to simulations of dielectric breakdown by Niemeyer, Pietronero, and Wiesmann,[15] and of viscous fingering by Sherwood and Nittmann.[16] In our language, these authors also solve for the pressure field (in their case on a uniform square lattice), but instead of moving all the interfaces as in (6), they move the interface by a whole lattice spacing at a single location which is chosen with a probability weighted by the local velocity. Although it might seem that on the average the interfaces are moved in the same way as in our model, it is clear that had these authors simply moved the interface in accordance with our rule (6), they would have obtained a dendritic pattern similar to our Fig. 1(a), since their lattice structure is assumed to be uniform. As in the case of DLA, it is interesting that the stochastic rule considered by these authors produces the same kind of fractal growth as is caused by the randomness of the medium in our purely deterministic model.

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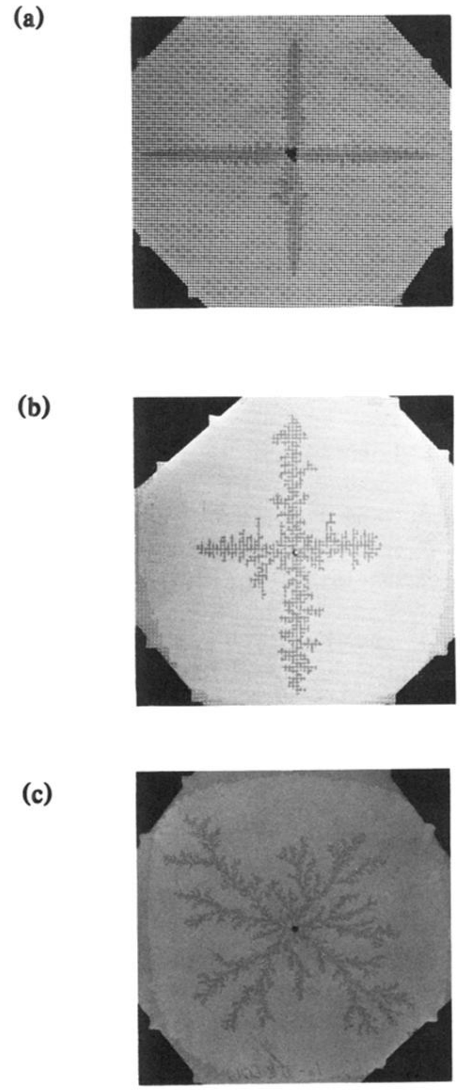

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