Relativistic Description of Baryon Magnetic Moments

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(Received 26 July 1985)

A new relativistic baryon–wave-function model is obtained by the requirement that it describe three valence quarks with the usual constituent masses and the universal hadronic scale (≈ 1 fm) and contract in the nonrelativistic limit into the usual harmonic-oscillator model. It provides a parameter-free prediction for baryon magnetic moments, which gives very good agreement for the hyperon data but reveals a significant discrepancy for the nucleon. This result contradicts the usual static-quark–model calculations and may suggest the relative importance of the nonvalence part of the nucleon wave function.

PACS numbers: 13.40.Fn, 12.35.Ht

The simple quark model which describes the baryon octet as three quarks slowly orbiting in relatively similar states has been challenged by recent precise hyperon–magnetic-moment measurements.[1–4] The model, completed with the assumption that the baryon moment is obtained by summation of the quark Dirac moments, gives typically excellent agreement with the moments for the nucleon [i.e., reproduces the famous value[5] $\mu(p)/\mu(n) = -1.5$] and the Λ, but disagrees with the other hyperon moments at the level of 20%.

At this time it seems to be appropriate to look at the implications of the measurements for the basic physics of the quark model. There are already model-independent analyses of the disagreement, due to Franklin[6] and Lipkin.[7] They come to the conclusion that a good understanding of baryon magnetic moments will require a model with quark-moment contributions which are nonstatic and/or baryon dependent. An alternative approach, guided to some extent by the corresponding discrepancies in nuclear physics (i.e., tritium and ^{3}He nuclear moments), is also discussed. There are attempts to explain the disagreement by the existence of other relevant degrees of freedom within the baryon. Pion contributions, as suggested by several authors,[8–10] seem to be of particular importance.

In this Letter we present a simple relativistic model of baryon magnetic moments with the aim of shedding some light on the above-mentioned controversy. The analysis reveals substantial nonstatic corrections to the calculation based on simple additivity, in particular for the nucleon.

In spite of all of the problems with the baryon magnetic moments the constituent quark model (CQM), especially when supplemented with ideas of QCD, has proved successful and has accounted for an enormous amount of experimental data on hadron structure.[11] Among the fundamentals of the model the following three concepts seem to be the most essential:

(a) *Constituent quark.*—Hadrons are built of the constituent quarks with effective masses which define the scale of the hadron masses.

(b) *Universal hadronic scale.*—There is a universal scale of ≈ 1 fm relevant to all static hadron properties.

(c) *Valence-quark dominance.*—The hadron spectrum is described by the states of a $q\bar{q}$ pair for mesons and of qqq for baryons. Contributions from other degrees of freedom appear to be irrelevant.

With these assumptions a determination of the light-quark effective-mass parameters from the baryon masses allows a prediction of many hadronic properties, among others, the baryon magnetic moments. Let us quote as a typical example of the static-model results the prediction given by Rosner.[12] Table I shows a comparison with the recent experimental data.

Looking for a departure point for model building let us be very conservative and keep all three of the above-mentioned dogmas of the CQM but give up the static approach only. This will give us the possibility of a parameter-free prediction. According to our knowledge Lipkin[14] was the first to point out that the universal hadronic scale invalidates the nonrelativistic

TABLE I. Baryon magnetic moments (in nuclear magnetons) in the static CQM and the relativistic CQM.

Baryon moments	Experiment[a]	Static CQM[b]	Relativistic CQM
$\mu(p)$	2.793 ± 0.000	2.79	2.60
$\mu(n)$	-1.913 ± 0.000	-1.86	-1.55
$\mu(\Lambda)$	-0.613 ± 0.005	-0.58	-0.61
$\mu(\Sigma^+)$	2.379 ± 0.020	2.68	2.42
$\mu(\Sigma^-)$	-1.00 ± 0.12	-1.05	-0.99
$\mu(\Xi^0)$	-1.250 ± 0.014	-1.40	-1.25
$\mu(\Xi^-)$	-0.69 ± 0.04	-0.47	-0.60

[a]Reference 13

[b]Reference 12.

approximation used in the CQM. Relativistic effects in the description of baryon magnetic moments have also been emphasized by others.[15]

It is becoming widely accepted that the most convenient and intuitive relativistic formalism for the bound-state problem is provided by a momentum-space Fock-state basis defined at equal $\tau = t + z$ on the light cone,[16] rather than the more familiar equal-t wave functions. In the valence sector, any baryon state with momentum $p^\mu = (p^+, p^-, \mathbf{p}_\perp) = (p^0 + p^2, (M_B^2 + \mathbf{p}_\perp^2)/p^+, \mathbf{p}_\perp)$ is determined by the light-cone wave function

$$\psi_{q_1q_2q_3}(x_i, \mathbf{k}_{\perp i}, \lambda_i), \quad \sum_{i=1}^{3} x_i = 1, \quad \sum_{i=1}^{3} \mathbf{k}_{\perp i} = 0,$$

where $x_i = k_i^+/p^+$, $\mathbf{k}_{\perp i}$, and λ_i specify the longitudinal and transverse momenta and spin projection S_z of each on-mass-shell constituent quark. It turns out that electromagnetic and weak form factors have exact expressions in terms of ψ_n ($n \geqslant 3$ for baryons). What is interesting to us is that the anomalous magnetic moment $a = F_2(0)$ of any spin-$\frac{1}{2}$ system can be written[17] (in the valence-dominance approximation) as

$$\frac{a}{M_B} = -\sum_j e_j \int [dx][dk_\perp] \psi_{B\uparrow}^* \sum_{i=j} \left(\frac{\partial}{\partial k_i^1} + i \frac{\partial}{\partial k_i^2} \right) \psi_{B\downarrow}, \tag{1}$$

where e_j is the charge of each constituent quark q_j. It should be emphasized that the relativistic constituent quark model (RCQM) of Eq. (1) offers a different explanation of the origin of anomalous baryon magnetic moments than the static model ($\boldsymbol{\mu} = \sum_j \boldsymbol{\mu}_j$) does. Moreover, it shows why the anomalous moments are large [i.e., $O(1)$ on the scale of the nuclear magneton]. Namely, the analysis of Brodsky and Drell[17] shows that in the case where all of the mass-scale parameters of the composite system are of the same order of magnitude, one obtains $a = O(M_B R)$, where $R = \langle k_\perp^2 \rangle^{-1/2}$ is the characteristic size of the three-quark state. The universal hadronic scale leads to a size $R \approx M_B^{-1}$.

In order to perform the calculations of the baryon magnetic moments of Eq. (1) we assume a simplified model of the basic wave functions $\psi_{q_1q_2q_3}$. We shall make the following prescriptions:

(i) We assume that quarks in baryons have typical constituent masses. To be specific we use the values $m_u = m_d = 363$ MeV and $m_s = 538$ MeV given by Rosner's[12] fit to baryon masses. For the universal hadronic scale we take $R = 250$ MeV which is fixed by the proton charge radius and is equal to the baryon Gaussian parameter of the Isgur-Karl wave function.[18] We have checked that our conclusions are not very sensitive to the detailed values of the parameters.

(ii) With these values in mind, we can convert, via the correspondence principle, any Galileo-invariant wave function into a Lorentz-invariant one.[19] For that purpose, we take the simplest possible model, viz., the harmonic-oscillator model. The momentum wave function is symmetric to an exchange of the individual momenta $\mathbf{k}_i$ and can be written (in the c.m. frame) as

$$\Phi(\mathbf{k}_i) = A \exp\left(-\frac{1}{2R^2} \sum_{i=1}^{3} \mathbf{k}_i^2 \right). \tag{2}$$

The baryon states of interest have two identical quarks (except those of the Λ) which we shall label with $i = 1$ and 2. The overall symmetry of the wave function in momentum, spin, and flavor spaces then implies that the spin-flavor wave functions for $B = p$, n, Σ^+, Σ^-, Ξ^0, and Ξ^- are symmetric under exchange of 1 and 2, but for the Λ it is antisymmetric. They have the form

$$\chi_{B\uparrow} = 6^{-1/2} |q_1q_2q_3\rangle [2I_\uparrow^v(12,3) - I_\uparrow^v(13,2) - I_\uparrow^v(23,1)], \tag{3a}$$

where

$$I_\uparrow^v(12,3) = \chi_\uparrow^\dagger \boldsymbol{\sigma} \chi_{\lambda_3} \cdot \chi_{\lambda_1}^T \sigma_2 \boldsymbol{\sigma} \chi_{\lambda_2},$$

and for the Λ

$$\chi_{\Lambda\uparrow} = 2^{-1/2} |uds\rangle [I_\uparrow^s(12,3) - I_\uparrow^s(21,3)], \tag{3b}$$

where

$$I_\uparrow^s(12,3) = \chi_\uparrow^\dagger \chi_{\lambda_3} \chi_{\lambda_1}^T \sigma_2 \chi_{\lambda_2}.$$

In the above expressions χ_λ are two-component Pauli spinors.

(iii) Finally, in order to get the Lorentz-invariant light-cone wave function

$$\psi_{B\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \Phi(x_i, \mathbf{k}_{\perp i}) \chi_\uparrow(x_i, \mathbf{k}_{\perp i}, \lambda_i),$$

we first use the Brodsky-Huang-Lepage[19] prescription for the harmonic-oscillator wave function (2) which leads to the expression

$$\Phi(x_i, \mathbf{k}_{\perp_i}) = A \exp\left[\frac{1}{6R^2} \left(M_B^2 - \sum_{i=1}^{3} \frac{k_{\perp i}^2 + m_i^2}{x_i} \right) \right]. \tag{4}$$

Then, we make a simple relativistic generalization[20] of the vector and scalar spin wave functions (3),

$$I_\uparrow^v(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \bar{u}_\uparrow \gamma_\mu \gamma_5 u_{\lambda_3} u_{\lambda_1}^T C \gamma^\mu u_{\lambda_2} \tag{5a}$$

and

$$I_\uparrow^s(x_i, \mathbf{k}_{\perp i}, \lambda_i) = \bar{u}_\uparrow u_{\lambda_3} u_{\lambda_1}^T C \gamma_5 u_{\lambda_2}, \tag{5b}$$

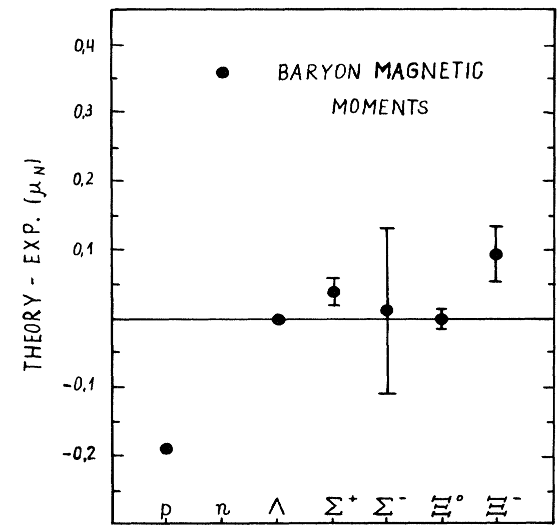

FIG. 1. A plot of the difference between the theoretical (RCQM) and experimental values of the magnetic moments given in Table I.

where C is the charge-conjugation matrix ($C = -i\gamma^2\gamma^0$) and u_λ are the light-cone spinors of Ref. 16. Note the dependence of Φ in (4) on the quark masses m_i and of u_λ in (5) on the baryon mass M_B. This causes the contribution of a given constituent to the moment of a bound state to be bound-state dependent.

Now, the specification of the baryon wave function of RCQM is completed. With the help of the Fierz identities, the Weyl representation of bispinors, and the methods of Farrar and Neri,[21] one can perform the multiple (eventually double) integrations in (1). This leads to the parameter-free predictions of the RCQM which are given in Table I and shown in Fig. 1. Our model gives a very good description of all of the hyperon moments (to an accuracy of 2σ). However, there is a serious disagreement for the nucleon [$\Delta\mu(p) = -0.19\mu_N$, $\Delta_\mu(n) = +0.36\mu_N$, where μ_N is the nuclear magneton].

In conclusion, we would like to note that this simple model should not be taken as an ultimate description, but rather as an illustration of the role of the relativistic corrections when discussing magnetic moments. One can see in Table I that the relativistic effects considerably improve the CQM predictions for the hyperon magnetic moments. There is the salient discrepancy for the nucleon. This might suggest that there is a missing ingredient in the nucleon wave function. From this point of view, the pion-contribution effects should be seriously taken into account (see Ref. 10). They are known to be particularly relevant for the nucleon.

We have had the benefit of conversations with J. Namysłowski on the subject of the light-cone formalism. This work is supported in part by the Research Program MR-1-7.

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