Cosmic Strings and the Correlation of Abell Clusters

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The two-point spatial correlation function for cosmic string loops in an expanding universe is determined and shown to match closely the observed two-point correlation function for clusters of galaxies. Further implications for the string theory of galaxy formation are discussed.

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One of the most important recent developments in our understanding of the large-scale structure of the Universe has been the determination of the two-point spatial correlation function of Abell's rich clusters of galaxies.[1] Abell himself pointed out[2] that the two-dimensional distribution of clusters on the sky is highly nonrandom, but only recently have enough red-shift data become available for a reliable determination of the correlation function itself. Bahcall and Soneira[3] and Klypin and Kopylov[4] found the interesting result that the correlation function $\xi_{cc}(r)$ for Abell clusters takes a similar form to that for galaxies $\xi_{gg}(r)$, both being roughly consistent with the power law $\xi(r)=\alpha r^{-1.8}$ but the coefficient α_{cc} being 18 times larger than α_{gg}. $\xi_{cc}(r)$ was also observed to be positive out to distances of more than $100h^{-1}$ Mpc (where Mpc stands for megaparsecs and h is the horizon distance).

In this Letter I will explain how the string theory of galaxy formation[5] predicts $\xi_{cc}(r)$ equal to that observed in a remarkable way.[6]

Correlations over such large distances as these are very hard to understand in most theories of galaxy formation where small-scale structure forms first (and there is by now substantial evidence against the opposite, "pancake" picture[7]). The simple problem is that in most scenarios the mass perturbation spectrum decreases monotonically with scale, so that correlations of matter on very large scales are even smaller than the correlations of galaxies. The strength of $\xi_{cc}(r)$ therefore poses an outstanding theoretical problem.

One attempt to explain the strength of $\xi_{cc}(r)$ has been to suppose that clusters are "biased" tracers of the matter distribution, forming only in regions where the primordial density perturbation lies above some threshold. As Kaiser has shown,[8] such regions are indeed more correlated than the matter as a whole as long as the probability distribution for the perturbations is Gaussian. This mechanism has been invoked in the context of cold-dark-matter scenarios where, unfortunately, it has been necessary to suppose that the galaxies themselves are "biased" ($>2\sigma$) fluctuations.[9] Fitting both the galaxy and cluster correlations then requires that the threshold for clusters be so high ($>6\sigma$) that there would be less than one in our entire observable Universe!

The strength of $\xi_{cc}(r)$ is rendered slightly more comprehensible if one writes $\xi(r)$ in a scale-invariant way,[10]

$$\xi(r)=\beta(r/d)^{-1.8}, \tag{1}$$

where d is the mean separation of the objects being considered. For galaxies,[11] $d\sim 5h^{-1}$ Mpc whereas for clusters[3] $d\sim 55h^{-1}$ Mpc. Now one finds that for galaxies $\beta_{gg}\sim 1.1$ while for clusters $\beta_{cc}\sim 0.27$. Viewed this way, galaxies appear to be 4 times more strongly correlated than clusters, consistent with the notion that small-scale structure formed first and the galaxy correlation function has grown by gravitational clustering. It is essential to this point of view, however, that the perturbations are produced in a scale-invariant pattern.

The evolution of cosmic strings in a radiation-dominated universe has been studied in some detail by Albrecht and myself.[12,13] We found strong evidence that if strings always exchange partners when they cross, the resulting distribution evolves in a self-similar manner. That is, the network of strings larger than the horizon straightens out on scales of order of the horizon scale, with a fixed number of lengths crossing each horizon volume. The self-intersection of these long strings continually produces loops with radii of order of the horizon scale. These "parent" loops then chop themselves up within a few expansion times into several "daughter" loops. The daughter loops occupy a small and decreasing fraction of space as the Universe expands and the probability of them colliding with another loop or length is negligible. The number density of loops and the mass density in string are both dominated by the smallest loops, which have radii equal to a fixed small fraction of the horizon scale and are continually disappearing as a result of gravitational radiation.

In the context of the string theory of galaxy formation all large condensed objects such as galaxies and clusters formed around oscillating loops of string. Larger loops are more massive and rarer—eventually causing more massive and rarer objects to form about them. Abell clusters (with richness $R\geq 1$) are defined observationally as regions containing more than fifty bright galaxies within an Abell radius $1.5h^{-1}$ Mpc. One can therefore identify Abell clusters with loops of all radii greater than some cutoff r_c chosen so that the mean separation of the resulting loops equals the mean separation of Abell clusters. These loops initially had radii of a kiloparsec or so and were produced in the radiation-dominated era (see Turok and Brandenberger[14]). Of course loops too large to capture Abell clusters should be excluded, but since the number density of loops decreases rapidly with radius, the contribution of loops with radii $\gg r_c$ to the correlation function is negligible.

In the string theory, the correlation function of loops with radii greater than r_c is indeed of the form of Eq. (1). This is because at the time loops of a given size are formed there is only one relevant scale, the horizon scale. All loops are formed in the same way, so that at the time they are formed their correlations at any fraction of the horizon scale (or of their separation) are identical. The effect of the subsequent expansion of the Universe is simply to stretch all separations by the same amount. Therefore the dimensionless function $\xi(r/d)$ does not change as the Universe expands. Furthermore there are no free parameters in $\xi(r/d)$ at all (it is independent of μ, the mass per unit length, which is the only free parameter in the string theory, and independent of the cosmological parameters h or Ω). It thus provides a very good test of the string theory.

I have calculated $\xi(r/d)$ numerically in simulations identical to those performed by Albrecht and myself, choosing the cutoff radius r_c to be larger than the radius of the smallest loops initially present in the simulation to ensure that the correlations really represent those of loops formed by the self-intersection process. I used a total of eighteen runs, each involving an average of 83 loops. For comparison, Bahcall and Soneira used a total of 104 Abell clusters with known red shifts and a larger sample of 1547 clusters whose red shifts are only approximately known in various comparison tests to confirm their results.

The correlation function $\xi(r/d)$ is defined as the excess probability over random of finding a pair of loops separated by a distance r. I calculated it from the formula

$$\xi(r/d)=n(r)/n^R(r)-1, \tag{2}$$

where $n(r)$ is the observed number of pairs of loops whose centers of mass are separated by $r\pm\delta$, 2δ being the bin size, and $n^R(r)$ is the number expected in the bin if no correlations were present. ξ is plotted in Fig. 1 and compared to the function $\xi_{cc}(r/d)$ determined in a similar way for Abell clusters by Bahcall and Soneira. The agreement is remarkably good.

The range of the simulations is very similar to the range of the observations—the cutoff radius r_c in the simulation obeys $r_c/d\sim 0.05$ whereas an Abell radius divided by the mean separation of Abell clusters is ~ 0.03. The simulations were performed in a box of size $4.4d$ whereas the observations for Abell clusters go out to a radius $\sim 5d$. The apparent disagreement at small r/d is not significant, since the observations only involve a small number of clusters at these separations.[3]

What causes these correlations among loops? On large

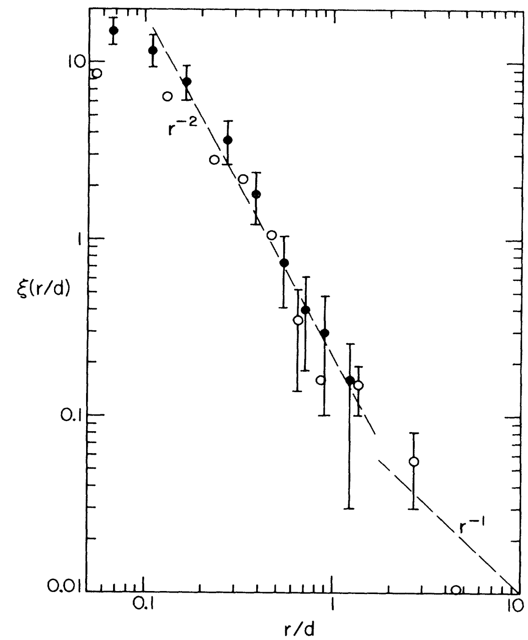

FIG. 1. Two-point spatial correlation function ξ for Abell clusters (open circles) and for cosmic string loops (filled circles) as a function of r/d, where r is the separation and d is the mean separation of the objects considered. Error bars in the data for Abell clusters correspond to the square root of the number of pairs in each separation bin (assuming Poisson statistics). Error bars in string simulations correspond to the standard deviation of the mean for eighteen independent simulations.

scales, loops show the same correlations as the network of strings that they were chopped off from. For a network of Brownian walks with density ρ, mass per unit length μ, and step length λ, it is not hard to show that the density-density correlation function is

$$\xi(r)=3\mu/\pi\rho\lambda r. \tag{3}$$

Parent loops will show these same correlations. From the simulations,[12,13] one has that $\rho\sim 2.3\mu/t^2$, $\lambda\sim 2t$, and the mean separation of parent loops when they were formed $d_{\text{parent}}\sim 5t$. This yields for the correlation of the parent loops $\xi(r)\sim 0.04d_{\text{parent}}/r$. The parent loops then split into an average of about ten daughters each. They too inherit the above correlations on scales $\geq d_{\text{parent}}$ but their mean separation d is $d_{\text{parent}}/10^{1/3}\sim 0.5d_{\text{parent}}$, and so one expects for the daughter loops

$$\xi(r/d)\sim 0.1d/r, \quad r\geq 2d. \tag{4}$$

On smaller scales the correlations of the daughter loops are determined by the detailed manner in which the parent loops split up to produce them. The parent loops are more or less straightened out when they form, so that if one is sitting on a daughter loop and goes out a distance r, it is reasonable to suppose that the excess number of daughter loops that one finds (those formed from the same parent loop) is proportional to r. That is, the number of loops between r and $r+dr$ is

$$n[1+\xi(r/d)]\sim n\,4\pi r^2\,dr+N\,dr/D, \qquad (5)$$

where n is the mean number density of loops, N is the number of daughter loops from each parent, and D is the size of a parent loop. Thus on scales less than the size of a parent loop,

$$\xi(r/d)\sim(N/4\pi)(d_0/D)(d/r)^2, \qquad (6)$$

where d_0 is the separation of the daughter loops when they are formed. Using $N\sim 10$ and $d_0/D\sim 0.2$, we find

$$\xi(r/d)\sim 0.2(d/r)^2, \quad r\leq d. \qquad (7)$$

The models described above explain in a qualitative way at least the form of $\xi(r/d)$. Their prediction is marked by the dashed lines in Fig. 1.

I want to return now to the question of the peculiar velocities of the loops. After a loop is formed, its initial peculiar velocity v_i is shifted to the red, $v=v_i(t_i/t)^{1/2}$ (in a radiation-dominated universe). Relative to a comoving frame it moves a total distance $2v_it_i[(t/t_i)^{1/2}-1]$. This is to be compared to the mean separation of loops after they are produced, $d\sim 0.9(t_it)^{1/2}$. With a mean velocity $v_i\sim 0.1$, this means that the correlations are washed out to some extent by the initial peculiar velocities on scales $r<0.2d$ or so. This may explain the fact that $\xi(r/d)$ appears to rise less steeply for small r/d. To understand this effect better, I calculated $\xi(r/d)$ for all loops with peculiar velocities less than 0.2 (about 80% of all loops), less than 0.1 (about 50% of all loops), and less than 0.05 (about 20% of all loops). These results are plotted in Fig. 2. In the first two cases ξ was not significantly different from that calculated with all loops except at the smallest separations where ξ increased for the slower loops, as one would expect. For the last case, the error bars are larger but ξ does seem significantly different from that calculated with all loops at intermediate r/d. If the above picture of clustering of daughter loops is correct, this may be because this selection of velocities significantly reduces the number of loops in a cluster.

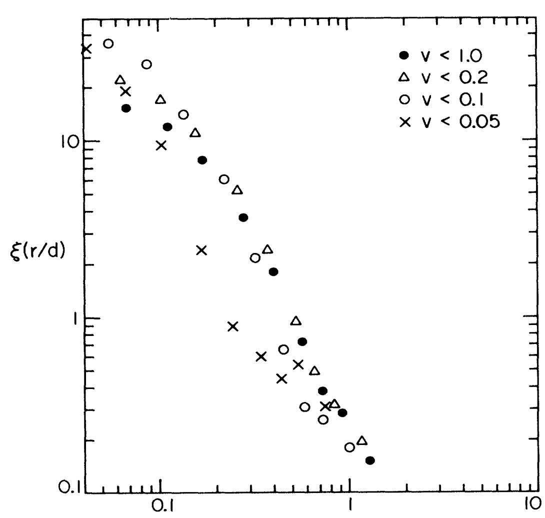

FIG. 2. The function $\xi(r/d)$ when selection for center-of-mass velocity v is imposed on the loops.

Finally I would like to point out that the identification of loops with Abell clusters proposed here permits a much cleaner determination of the single free parameter in the string theory, μ. The accretion of matter onto a loop is well described by the spherical collapse model.[15] In this model, the overdensity of an object just after it has collapsed is ~ 150 and thereafter grows as t^2. Abell clusters have an overdensity of ~ 170 and thus formed very recently. One can then require that the loops of the separation of Abell clusters be massive enough to have accreted an Abell overdensity within an Abell radius by this time. The final result is that $G\mu\sim 2\times 10^{-6}$, where G is Newton's constant. One can also describe the evolution of the correlation function for galaxies in terms of a spherical model—the result is that $\xi_{gg}\sim r^{-2}$ also; requiring the amplitude to be ~ 4 times that of ξ_{cc} provides a completely independent determination of $G\mu$, which gives essentially the same answer. Details of these and other calculations will be published elsewhere.[14]

In conclusion, the cosmic string theory has passed a test that no other viable theory of galaxy formation yet has. In this test, it has to fit not just a number but a function, and there were no adjustable parameters at all in the theory. If more red-shift data and more detailed simulations of strings confirm the result reported here, the cosmic string theory will become very difficult to surpass.

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[1]G. O. Abell, Astrophys. J. Suppl., Ser. 3, 211 (1958).

[2]G. O. Abell, Astrophys. J. **66**, 607 (1961).

[3]N. A. Bahcall and R. M. Soneira, Astrophys. J. **270**, 20 (1983).

[4]A. A. Klypin and A. I. Kopylov, Pis'ma Astron. Zh. **9**, 75 (1983) [Sov. Astron. Lett. **9**, 41 (1983)].

[5]For reviews see T. W. B. Kibble, in *Quantum Structure of Spacetime*, edited by M. J. Duff and C. J. Isham (Cambridge Univ. Press, Cambridge, England, 1982), and Nucl. Phys. **B252**, 277 (1985); A. Vilenkin, Phys. Rep. **121**, 263 (1985).

[6]Speculations along these lines were made by N. Turok and D. N. Schramm, Nature (London) **312**, 598 (1984).

[7]See, for example, C. S. Frenk, S. D. M. White, and M. Davis, Astrophys. J. **271**, 417 (1983); N. Kaiser, Astrophys. J. **273**, L17 (1983).

[8]N. Kaiser, Astrophys. J. **284**, L9 (1984).

[9]M. Davis, C. Frenk, G. Efstathiou, and S. D. M. White, Astrophys. J. (to be published).

[10]This was independently pointed out by A. Szalay and D. N. Schramm, Fermilab Report No. pub 85/24-A 1985 (to be published).

[11]M. Davis and J. Huchra, Astrophys. J. **254**, 437 (1982).

[12]A. Albrecht and N. Turok, Phys. Rev. Lett. **54**, 1868 (1985).

[13]A. Albrecht and N. Turok, to be published.

[14]N. Turok and R. Brandenberger, to be published.

[15]P. J. E. Peebles, *The Large Scale Structure of the Universe,* (Princeton Univ. Press, Princeton, N. J., 1980).