Calculations for Cosmic Axion Detection

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We present calculations, using properly normalized couplings and masses for Dine-Fischler-Srednicki axions, of power rates and signal temperatures for axion-photon conversion in microwave cavities. The importance of the galactic-halo axion line shape is emphasized. We mention spin-coupled detection as an alternative to magnetic-field—coupled detection.

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There has been a great deal of interest recently in the possible detection of a galactic-halo density of cosmic axions[1] which is likely to exist for a range of axion parameters. Since a wild range of possible experiments have been proposed,[2] it is very important to ascertain exactly what signals may be expected as a function of axion mass and galactic density. It has also been pointed out recently that several numerical errors exist in the standard literature on axions.[3] Here we summarize the properly normalized axion couplings and mass relations for the Dine-Fischler-Srednicki (DFS) invisible-axion model[4] along with explicit conventions, and present in some detail calculations of power and signal temperatures for axion-photon conversion in the presence of a background magnetic field in a microwave cavity. Our results can be used to obtain powers and signal temperatures for experiments which use other couplings of axions.

We define the axion decay constant F_a by $\langle 0 | j_\nu^x | a \rangle = F_a q_\nu$, where a is the axion field, with momentum q_ν. Here j_ν^x is the total current associated with the Peccei-Quinn X symmetry spontaneously broken by the vacuum expectation value of the complex scalar field, ϕ, which contains the axion. (For the definitions below one finds $\langle \phi \rangle = 2^{1/2} F_a$). This current is normalized so that in the quark and lepton sector it has the following form:

$$j_\nu^x = (X_u/2)(\bar{u}\gamma_\nu\gamma_5 u + \dots) + (X_d/2)(\bar{d}\gamma_\nu\gamma_5 d + \dots) + (X_d/2)(\bar{e}\gamma_\nu\gamma_5 e + \dots).$$

The sums are over all quarks and leptons. Any change in the definition of the current will change the normalization of the relevant charges and the relationship between F_a and scalar vacuum expectation values. X_u and X_d are related to the vacuum expectation values of the Higgs bosons which give mass to the quarks and leptons under the U(1) symmetry broken by the ϕ field, and they are constrained by the relation $X_u + X_d = 1$.

With these conventions, DFS axions have the following couplings to electrons and photons,[3] in addition to their required coupling to quarks:

$$L_{\rm int} = \dots + i(a/F_a) X_d m_e \bar{e}\gamma_5 e - (\alpha/4\pi)(a/F_a)(N/3)F\tilde{F}\xi \,, \tag{1}$$

where this Lagrangean is expressed in rational units ($\alpha = e^2/4\pi$). The mass of the axion is given by

$$m_a^2 = (F_\pi/F_a)^2 m_\pi^2 N^2 Z(1+Z)^{-2}. \tag{2}$$

In (1) and (2), N is the number of families, $F_\pi = 93$ MeV, $Z = m_u/m_d \approx \frac{1}{2}$, and $\xi = 4-(4+Z)/(1+Z) \approx 1$.

If axions provide a closure density for the Universe, standard axion cosmology[1] suggests that the preferred axion mass is roughly 10^{-5} eV. This corresponds to a frequency $f_0 = 2.4$ GHz and a Compton wavelength $\lambda_C = 2$ cm.

A search experiment should focus on the range of frequencies around 2.4 GHz first. However, as there is much uncertainty in early-Universe physics, a thorough search would ideally cover the range of axion masses from 10^{-12} eV (corresponding to an axion decay constant F_a equal to the Planck mass) to 10^{-2} eV (as limited by stellar-evolution constraints[5]).

If axions provide the closure density, then they most likely also constitute the dark halo of our galaxy. If they possess the galactic virial velocity $v \sim 10^{-3}c$, then the galactic axion field can be treated as spatially constant on laboratory scales. Specifically, for $m_a \approx 10^{-5}$ eV, the axion de Broglie wavelength $\lambda_{\rm dB} = (c/v)\lambda_C \approx 10$ m is the minimum coherence length for the galactic field.

To describe a galactic-halo axion field, we shall use the following Fourier representation (where T is a large reference time):

$$a(\omega)=(2T)^{-1/2}\int_{-T}^{T}a(t)e^{i\omega t}dt\ ,$$
$$a(t)=(2T)^{1/2}\int_{-\infty}^{\infty}a(\omega)e^{-i\omega t}(d\omega/2\pi)\ . \quad (3)$$

From these definitions follows the relation

$$\langle a^2\rangle \equiv (2T)^{-1}\int_{-T}^{T}a^2(t)dt = \int_{-\infty}^{\infty}|a(\omega)|^2 d\omega/2\pi\ . \quad (4)$$

We model an axion galactic halo with a Maxwellian velocity distribution. This distribution is expected for a self-gravitating isothermal sphere of particles which has been thermalized via violent relaxation.[6] The Maxwellian distribution $P(\omega,\langle v^2\rangle)$ determines the axion power spectrum:

$$|a(\omega)|^2\,d\omega \sim P(\omega,\langle v^2\rangle)d\omega\ . \quad (5)$$

Here $\langle v^2\rangle$ is the mean squared velocity of the axion gas. The magnitude of the axion power spectrum is determined by the requirement that

$$\langle\rho\rangle_{\text{halo}}=m^2\langle a^2\rangle\ . \quad (6)$$

With these constraints, the axion power spectrum is given by

$$|a(\omega)|^2=\begin{cases}\frac{1}{2}CR(|\omega|), & -\infty<\omega<\infty\\ CR(\omega), & 0<\omega\ .\end{cases} \quad (7)$$

Here, the Maxwellian form $R(\omega)$ and the magnitude C are given by

$$R(\omega)\equiv\exp[-(\omega-m)/\tfrac{1}{3}m\langle v^2\rangle](\omega-m)^{1/2}\theta(\omega-m)\ ,$$
$$C\equiv 4\sqrt{\pi}\langle\rho\rangle/m^2(\tfrac{1}{3}m\langle v^2\rangle)^{3/2}\ . \quad (8)$$

The peak of the axion power spectrum is at

$$\omega_{\max}=m+\tfrac{1}{6}m\langle v^2\rangle\ , \quad (9)$$

with $(\omega>0)$

$$|a(\omega_{\max})|^2=\left[\frac{2\pi}{e}\right]^{1/2}\frac{6\langle\rho\rangle}{m^2\langle v^2\rangle}\ . \quad (10)$$

We can define an axion bandwidth $\Delta\omega$ by the condition

$$\langle\rho\rangle=m^2|a(\omega_{\max})|^2\Delta\omega\ . \quad (11)$$

From $\Delta\omega$, one can define Q for the halo axion field:

$$Q_a\equiv\frac{m}{\Delta\omega}\approx\frac{6(2\pi/e)^{1/2}}{\langle v^2\rangle}\ . \quad (12)$$

The local halo density for our galaxy[7] is believed to be about 0.3 GeV/cm^3 while the virial velocity[8] is roughly 300 km/sec or 10^{-3} in natural units. The corresponding Q_a is 9×10^6.

Note that our choice of a Maxwellian distribution of frequencies is only a logical first guess; the actual distribution will be different because of our motion with respect to the axion background, if the axion virial velocity is different from that of stars, or if the axion halo did not undergo violent relaxation. Furthermore the axion velocity distribution may well be anisotropic; axions moving in eccentric, radial galactic orbits may have a larger velocity dispersion than those moving in only mildly eccentric orbits. All these "complications" could provide a gold mine of information about the formation of the galaxy if an axion background is found.

The couplings of the axion given in (1) allow the conversion of axions into photons. For example, the Lagrangean for electromagnetism including the direct two-photon coupling can be written (in Gaussian units where $\alpha=e^2$)

$$L=(1/8\pi)[\epsilon E^2-(1/\mu)B^2]-(\kappa/4\pi)\mathbf{E}\cdot\mathbf{B}a, \quad (13a)$$

where

$$\kappa\equiv(N/3)(e^2/\pi)\xi/F_a\ . \quad (13b)$$

The coupling allows conversion into photons in the presence of a static background magnetic field, as originally proposed by Sikivie and co-workers,[2] and refined by Morris and co-workers.[2] In a background field B_0, Maxwell's equations as derived from Eqs. (13) lead to the equation of motion describing production of electromagnetic waves in the presence of a spatially uniform oscillating background axion field,

$$\nabla^2E-\mu\epsilon\,\partial^2E/\partial t^2=\mu\kappa B_0\,\partial^2a/\partial t^2\ . \quad (14)$$

Standard Fourier methods are used to solve (14). Define, in analogy to Eqs. (3) and (4),

$$E(\omega)=(2T)^{-1/2}\int_{-T}^{T}E(t)e^{i\omega t}dt\ ,$$
$$E(t)=(2T)^{1/2}\int_{-T}^{T}E(\omega)e^{-i\omega t}d\omega/2\pi\ , \quad (15)$$

so that

$$\langle E^2\rangle\equiv(2T)^{-1}\int_{-T}^{T}E(t)^2dt = \int|E(\omega)|^2d\omega/2\pi\ . \quad (16)$$

We solve by expanding $E(x,\omega)$ in cavity modes:

$$E(x,\omega)=\sum_j E_j(\omega,x)=\sum_j\lambda_j(\omega)\mathbf{e}_j(x)\ . \quad (17)$$

Here, the normal modes $\mathbf{e}_j$ are normalized such that $\int(\mathbf{e}_i\cdot\mathbf{e}_j)d^3x=V\delta_{ij}$, and they satisfy the equation for a perfect, lossless cavity,

$$(\mu\epsilon\omega_j^2-\nabla^2)\mathbf{e}_j(x)=0\ . \quad (18)$$

Similarly, we expand the background magnetic field in modes:

$$\mathbf{B}_0(x)=\sum_j\frac{\int\mathbf{B}_0(x)\cdot\mathbf{e}_j(x)}{\int\mathbf{e}_j(x)^2}\mathbf{e}_j(x)\equiv\sum_j\eta_j\mathbf{e}_j(x)\ . \quad (19)$$

The spatial dependence thus drops out of (14) yielding a cavity response equation

$$\lambda_j(\omega)=(\kappa/\epsilon)\eta_j[\omega^2/(\omega^2-\omega_j^2)]a(\omega)\ . \quad (20)$$

The steady-state energy in mode j is

$$U_j=(\epsilon/4\pi)\int |E_j(\omega,x)|^2(d\omega/2\pi)d^3x \ ,$$
$$=(\epsilon V/4\pi)\int \lambda_j^2(\omega)d\omega/2\pi \ . \quad (21)$$

To express the mode energy in quantities of greater practical use, we use (20) and define a filling factor G_j by

$$G_j^2\langle B^2\rangle \equiv \eta_j^2 \ , \quad (22)$$

with

$$\langle B^2\rangle \equiv V^{-1}\int |B(x)|^2 d^3x \ . \quad (23)$$

Furthermore, we include the effects of dissipation in the standard way by substituting $(\omega^2-\omega_j^2)^2 \to (\omega^2-\omega_j^2)^2+\omega^4/Q^2$ where Q is the quality factor of the loaded cavity (see below). The mode energy is now given by

$$U_j=\frac{\kappa^2}{4\pi\epsilon}G_j^2 V\langle B^2\rangle \int \frac{|a(\omega)|^2\omega^4}{(\omega^2-\omega_j^2)^2+\omega^4/Q^2}\frac{d\omega}{2\pi} \ . \quad (24)$$

When $Q<Q_a$ and $\omega_j=\omega_{\max}$, (24) is easily evaluated, yielding

$$U_j \approx (\kappa^2/4\pi\epsilon)G_j^2\langle B^2\rangle VQ^2\langle a^2\rangle$$
$$=(\kappa^2/4\pi\epsilon m^2)G_j^2\langle B^2\rangle VQ^2\langle \rho\rangle \ . \quad (25)$$

An experimental strategy to search for the axion-photon conversion is to load a resonant cavity (of quality Q_c) with a receiver (quality $Q_r \approx Q_c$) and vary the mode frequency, seeking resonance with the background oscillations. The overall, loaded Q is $(1/Q_c+1/Q_r)^{-1}$. Since typically $Q \ll Q_a$, it will be necessary to divide the receiver response into Q_a/Q frequency channels of bandwidth $\Delta f=\omega/2\pi Q_a$. Near resonance ($\omega_j \approx m$), the power absorbed *by the receiver* is

$$P=U\omega/Q_r \ . \quad (26a)$$

The signal temperature, or maximum power extracted by the receiver per frequency band Δf, is given by

$$T_s=P/\Delta f=2\pi U Q_a/Q_r \ . \quad (26b)$$

To achieve a given signal-to-noise ratio s/n in a bandwidth Δf requires an integration time as given by Dicke's radiometer equation of

$$t=\left[\frac{s}{n}\right]^2\left[\frac{T_n}{T_s}\right]^2\frac{Q_a}{2f} \ , \quad (27)$$

where T_n is the noise temperature of the receiver. In this time $N=Q_a/Q_L$ channels, corresponding to the frequency interval $N\Delta f$, are observed. To scan an octave in frequency (from f_0 to $2f_0$) requires M separate measurements [with M given by $2=(1+N\Delta f/f)^M$] which take total time

$$t_{\rm oct}=\left[\frac{s}{n}\right]^2\left[\frac{T_n}{T_s}\right]^2\frac{Q_a Q}{4f_0} \ . \quad (28)$$

Now T_s is proportional to Q^2Q_a/Q_r. Optimizing Q_r to minimize the octave scan time (28) gives

$$Q_r=\tfrac{1}{2}Q_c \ , \qquad Q=\tfrac{1}{3}Q_c \ . \quad (29)$$

(This differs from the configuration $Q_r=Q_c$, which yields the largest signal temperature.)

With $\epsilon=1$, $G^2=\frac{1}{4}$, $\rho=0.3$ GeV/cm^3, $Z=\frac{1}{2}$, we find (expressing the result in terms of Q_c)

$$T_s=(0.75\ \mathrm{K})\left[\left[\frac{B}{10\ T}\right]^2\left[\frac{V}{10^6\ \mathrm{cm}^3}\right]\left[\frac{Q_c}{10^6}\right]\right]. \quad (30)$$

Let us translate this into the scanning time. As indicated by (28), obtaining a low noise temperature of the system is critical. A state of the art high—electron-mobility field-effect-transistor amplifier has been found to have $T_{\rm amp}<3$ K at 2.3 GHz,[9] suggesting that a system noise temperature $T_n \sim 4$ K is obtainable. To achieve $s/n=3$ over an octave starting at $f_0=2.3\times 10^9$ Hz, given $T_n=4$ K and the values used in (30), requires, according to (28),

$$t_{\rm oct}=(8.3\times 10^4\ \mathrm{sec})\left[\left[\frac{T_n}{4\ \mathrm{K}}\right]^2\left[\frac{2.3\ \mathrm{GHz}}{f_0}\right]\right]. \quad (31)$$

This time is a mere 24 hours. A smaller scale experiment envisioned by Lubin, Morris, and Pennypacker[2] with $V=2\times 10^5$ cm^3 and $B=8$ T would require an octave scan of two months, which is still very doable. $s/n=3$ is already adequate since false alarms can be checked by repeated measurement at the same frequency.

It appears that a search experiment of this kind is feasible, but very demanding, with existing technology. If the frequency could be narrowed down theoretically or determined by another experiment, a microwave experiment of this type would be able to determine the line shape, which encodes important cosmological information.

The signal-to-noise analysis presented here is intended only as a rough tool to gauge the feasibility of axion-search experiments. The actual signal-to-noise ratio attainable for a given experiment could be *better* than that suggested here. This results from two simplifications which we have made. First, we did not take full advantage of the Maxwellian line shape; an optimal experiment would use an optimal filtering strategy based on this line shape. Second, a measurement strategy based upon optimal filtration would eliminate the need for arbitrary, albeit conservative, definitions like (11) and (12).

It should be remarked that the DFS axion's coupling to two photons is suppressed as a result of a near cancellation of intrinsic and π-mixing couplings.[3] In other models the coupling can be larger. For instance if the axion couples only to the gluon anomaly, $\xi \approx 3$, this decreases the scan time by $(\frac{1}{3})^4 \approx \frac{1}{80}$.

It was recently proposed by Krauss *et al.*[2] that it might be possible to substitute aligned electron spins for the magnetic field as a catalyst of axion-photon conversion. For free electrons, one derives a coupling

$$\mathscr{L}=\left(\frac{a}{F_a}\right)\left(\frac{eX_d}{2m_e}\right)sE\ , \tag{32}$$

where s is the net number density of spin-aligned electrons (i.e., twice the spin density). For $X_d=\frac{1}{2}$ and $s=10^{23}/\text{cm}^3$ we find that this coupling of axions to photons is equivalent to that induced by a magnetic field

$$B_{\text{eq}}=\frac{s\pi^2}{em_e}\simeq 270\ \text{T}\ . \tag{33}$$

This large value suggests the use of magnetized plates instead of pure B fields. Unfortunately, the free-electron coupling will be quenched in insulators, while conductors appear to entail large losses or unfortunate shielding effects.[10] There may well be some practical way to exploit this large equivalent field, but so far it has eluded us.

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Note added.—After undertaking this work, it came to our attention that Sikivie has recalculated detection rates for invisible-axion searches.[11] Our techniques and results differ somewhat from his.

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