Period-Doubling Lasers as Small-Signal Detectors

B. Derighetti, M. Ravani, R. Stoop, P. F. Meier, and E. Brun
Physics Institute, University of Zürich, 8001 Zürich, Switzerland

and

R. Badii
Institute of Theoretical Physics, University of Zürich, 8001 Zürich, Switzerland
(Received 30 July 1985)

Near the onset of a period-doubling bifurcation a parametrically modulated NMR laser has been used as a detector of weak input signals with a strongly peaked response curve centered near half the modulation frequency. The maximum sensitivity increases the closer the bifurcation point. A Bloch-Kirchhoff model with realistic parameters has been used for computer simulations. Experiments and theory are in fair agreement. The results suggest that any parametrically modulated laser device could be used as a small-signal detector for selected frequencies.

PACS numbers: 42.60.−v, 07.58.+g, 42.50.+q

In a recent Letter[1] Wiesenfeld and McNamara put forward the idea of a general amplification scheme for nonlinear systems near the onset of a period-doubling instability. They pointed out that any dynamical system which oscillates at a frequency f and has a control parameter A set close to the bifurcation value, where a limit cycle of frequency $f/2$ is born, couples strongly to a small signal with a frequency close to $f/2$. Thus such a system could in principle be used as a small-signal amplifier in a defined frequency range.

To test this proposition experimentally, we adapted our NMR laser,[2] together with its theoretical description[3] by model equations, to this situation. From previous work[4–6] we know that this laser exhibits period-building cascades to chaos if one of its physical parameters is modulated externally at a frequency near f_{rel}, the relaxation oscillation frequency of the laser ($\approx$30–80 Hz). Both the regular and chaotic response can be modeled fairly well by the Bloch-Kirchhoff equations which describe the nonlinear dynamics of the system. Hence with the NMR laser we have a unique bifurcating system on hand where experimental observations may be compared with numerical solutions of a realistic model.

The first qualitative demonstration of the weak-signal amplification near a period-doubling instability is shown as an artifact in the bifurcation diagram of Fig. 1. We modulated the Q of the cavity [Eq. (2)] with $f=102.7$ Hz and strobed the laser output at equal time intervals $T=1/f$. The discrete output values were then plotted versus the swept modulation strength A. The depicted bifurcation diagram differs from the usual ones by a strong low-frequency oscillation. The beat frequency of 1.35 Hz stems from the nonlinear coupling of the first subhar-

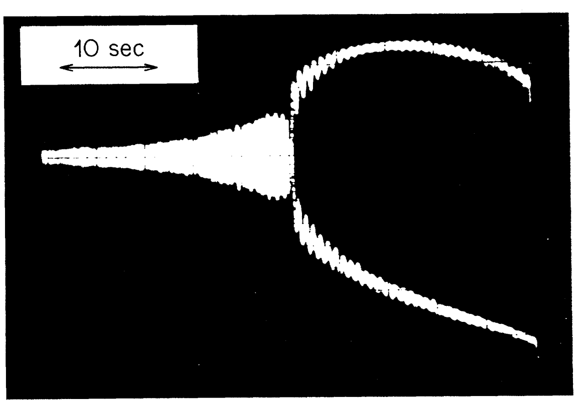

FIG. 1. Observed period-doubling bifurcation for the NMR laser with Q modulation of frequency $f=102.7$ Hz. The beat of frequency 1.35 Hz indicates an interference with the power-line of 50 Hz.

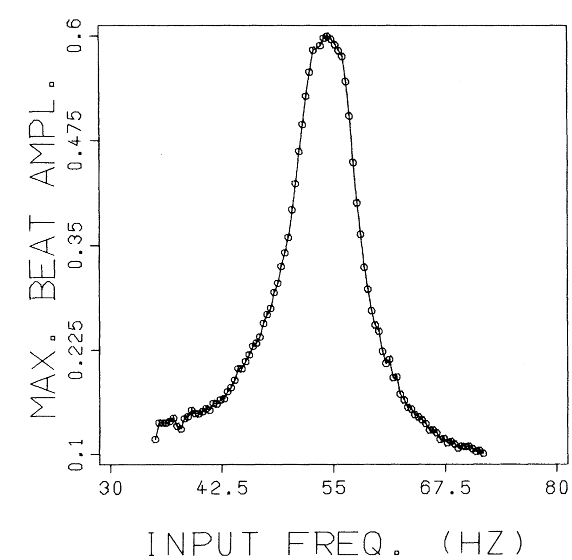

FIG. 2. Experimental response curve for the NMR laser with a parametric pump signal of frequency $f=110$ Hz and an input signal of constant amplitude. The input frequency f_i is swept, the laser output is strobed with the frequency f, and the maximal values of the beat are plotted vs f_i.

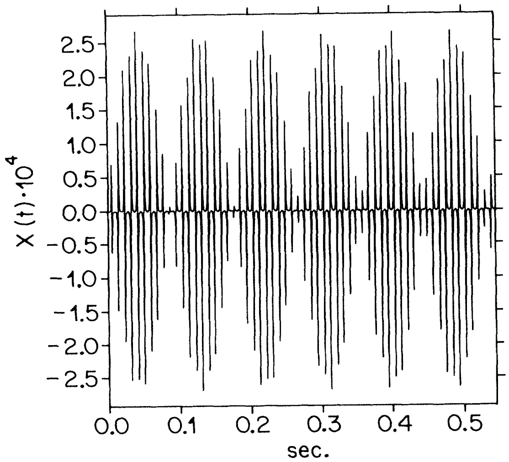

FIG. 3. Computed time plot $x(t)=\tilde{M}(t)-M(t)$ from the NMR laser model equations with a parametric pump signal of amplitude $A=0.02$ and frequency $f=109.82$ Hz, and an input signal of amplitude $a=10^{-5}$ and frequency $f_i=49.34$ Hz. Note that the beat frequency of 5.67 Hz equals $f/2-f_i$.

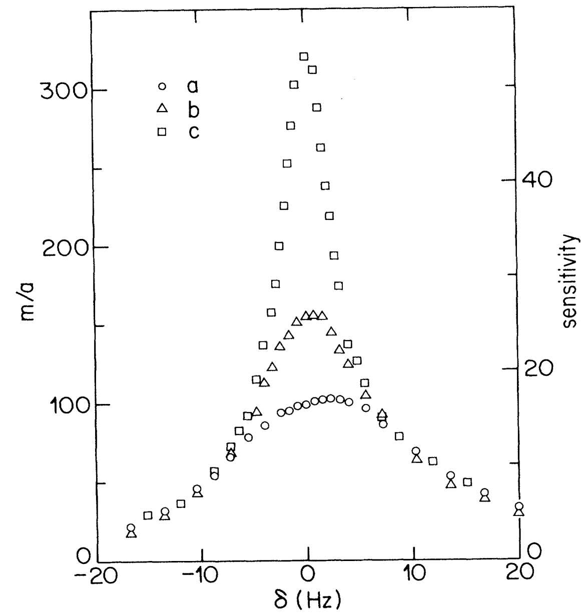

FIG. 4. Computed response curves for the NMR laser model with three different values of the parametric pump amplitude (a) $A=0.027$, (b) $A=0.028$, and (c) $A=0.029$. The parametric pump frequency is $f=110$ Hz, the signal amplitude $a=10^{-5}$ is constant, and the signal frequency $f_i=f/2+\delta$ is swept. The critical value for period doubling is $A_c=0.0301$.

monic of the modulation signal with a 50-Hz pickup from the powerline. The pickup signal is usually hidden in the thermal noise of the NMR laser. However, near the onset of the bifurcation it is strongly amplified.

To investigate this nonlinear phenomenon in more detail, we applied a double-modulation technique. In addition to the Q modulation at $f=110$ Hz, the NMR linewidth was modulated [see Eq. (3)] with a frequency close to $f/2$. We call the two modulation signals parametric pump and input signal, respectively. The laser output was strobed with the rate f, and the discrete values were then digitized and stored in a DATA 6000 acquisition system. The maximum peak-to-peak value of the strobed output was determined. This procedure was repeated in computer-controlled increments of the input frequency $f_i=f/2+\delta$. A typical response is shown in Fig. 2 for a small[7] input amplitude. A similar result was observed with $f_i\approx f/4$ for a period-doubling bifurcation of period 2 period 4.

To substantiate these experimental observations the dynamical behavior of the system has also been investigated theoretically. We started from the nonlinear Bloch-Kirchhoff equations[3,8]

$$\dot{M}_v=-\gamma_\perp M_v-9(CQM_v-D)M_v\ ,$$
$$\dot{M}_z=-\gamma_{\parallel}(M_z-M_e)+(CQM_v-D)M_v\ , \qquad (1)$$

which provide an excellent description for a variety of experiments with the unmodulated NMR laser. Here M_z denotes the nuclear magnetization along the direction of the static external field, and M_v is the perpendicular magnetization in the rotating frame. The corresponding relaxation rates are $\gamma_\parallel$ and $\gamma_\perp$. M_e is the pump magnetization, and Q is the quality of the coil. The parameters C and D are proportional to the gyromagnetic ratio of the laser-active ^{27}Al nuclei. C contains the filling factor of the coil, and D contains an adjustable driving field to provide the proper damping of the relaxation oscillations.

To simulate our experimental observations, we introduced the parameteric pump by

$$Q(t)=Q[1+A\sin(2\pi ft)]\ , \qquad (2)$$

and the input signal by

$$\gamma_\perp(t)=\gamma_\perp[1+a\sin(\pi ft+2\pi\delta t)]\ . \qquad (3)$$

For the parameters $C=24.09$ mA^{-1} s^{-1}, $D=-6.9\times10^{-3}$ s^{-1}, $M_e=-1.6$ A/m, $Q=250$, $\gamma_\perp=3\times10^4$ s^{-1}, $\gamma_\parallel=10$ s^{-1}, and $f=110$ Hz, the first period-doubling bifurcation from a limit cycle of period 1 to period 2 occurs at $A_c\cong0.0301$, if $a=0$. With A chosen to be slightly below the bifurcation point A_c, the M_v of the period-1 limit cycle is changed by the input signal (3) to $\tilde{M}_v(t)$. In Fig. 3 the calculated difference $x(t)=\tilde{M}_v(t)-M_v(t)$ is shown for $a=10^{-5}$, which displays a beating between f_i and $f/2$. To relate this result to our strobed-data values, the maximum deviation $m=\max[\tilde{M}(t)-M(t)]$ was calculated as a function of the input parameter δ for a constant value of a. In general, it was found that m is proportional to a for $10^{-6}<a<10^{-4}$. In Fig. 4 the calculated values m/a are plotted versus the detuning frequency δ for three different values of $A<A_c$. The resonance

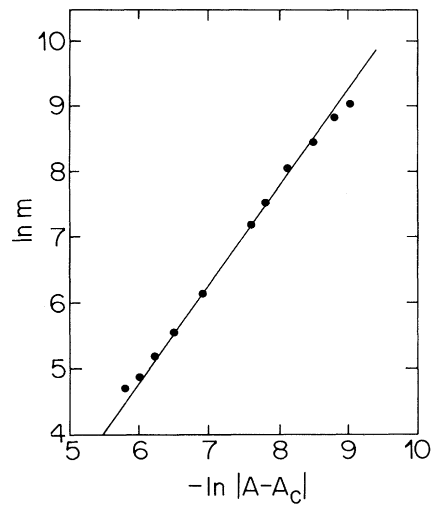

FIG. 5. Computed values of the response at resonance $m(\delta=0)$ vs the distance of A from the critical value A_c. The straight line suggests a power-law dependence with an exponent around 1.5. Deviations far from A_c are to be expected; deviations close to A_c are due to saturation effects.

behavior which was found experimentally and predicted in Ref. 1 is clearly seen. Note the asymmetry for larger values of $|\delta|$ and for larger $A_c - A$.

For $\delta=0$, the system exhibits a limit cycle of period 2. The spectral Fourier amplitude P at the resonance frequency $f/2$ was calculated for an input signal with and without the parametric pump present. The corresponding value $P(A\neq 0)/P(A=0)$ is a measure of the sensitivity of the system to the input signal and is indicated in Fig. 4 on the right-hand scale. This sensitivity increases roughly as $(A_c - A)^{-1.5}$ as the bifurcation point is approached, as shown in Fig. 5.

The agreement between experiment and the NMR-laser model calculations is satisfactory. Since the theoretical model[8] can generally be used to describe homogeneously broadened single-mode two-level lasers, we propose that lasers in the optical and microwave region will exhibit a similar behavior. Thus period-doubling laser devices may be used as small-signal detectors for various frequencies in the range $1-10^6$ Hz. The modulated systems offer the advantage that the amplitude and frequency of the parametric pump are controllable parameters which determine the center and the width of the response curve within certain limits.

We come to the conclusion that the propositions put forward by Wiesenfeld and McNamara have been proven experimentally with a nontrivial physical system of great potential. Further investigations are under way with lasing systems where period doubling is a generic phenomenon such as in detuned lasers with an injection field.[3,9–11]

We gratefully acknowledge financial support by the Swiss National Science Foundation.

[1]K. Wiesenfeld and B. McNamara, Phys. Rev. Lett. **55**, 13 (1985).

[2]P. Bösiger, E. Brun, and D. Meier, Phys. Rev. Lett. **38**, 602 (1977).

[3]E. Brun, B. Derighetti, D. Meier, R. Holzner, and M. Ravani, J. Opt. Soc. Am. B **2**, 156 (1985).

[4]D. Meier, R. Holzner, B. Derighetti, and E. Brun, in *Evolution of Order and Chaos,* edited by H. Haken, Springer Series in Synergetics, Vol. 17 (Springer-Verlag, New York, 1982), p. 146.

[5]E. Brun, B. Derighetti, R. Holzner, and D. Meier, Helv. Phys. Acta **56**, 825 (1983).

[6]E. Brun, B. Derighetti, R. Holzner, and M. Ravani, Acta Phys. Austriaca **56**, 81 (1984).

[7]Since the beat-signal amplitude scales with the amplitude of the input, one has to avoid effects near $\delta=0$. Moreover, a has to be small such that no bifurcation by the input signal is induced.

[8]For detailed derivation of these equations see Ref. 3, where also the equivalence with the Maxwell-Bloch equations used in optical laser systems is shown.

[9]L. A. Lugiato, L. M. Narducci, D. K. Bandy, and C. A. Pennise, Opt. Commun. **46**, 64 (1984).

[10]D. K. Bandy, L. M. Narducci, and L. A. Lugiato, J. Opt. Soc. Am. B **2**, 148 (1985).

[11]J. R. Tredicce, F. T. Arecchi, G. L. Lugiato, and G. M. Puccioni, J. Opt. Soc. Am. B **2**, 173 (1985).