Observation of Second Sound in Superfluid ^{3}He-B

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The speed and attenuation of second sound in the B phase of superfluid ^{3}He near the transition temperature were measured at pressures of 18.2, 21.3, and 24.4 bars under zero applied magnetic field. The second sound was detected by use of a resonance method (frequency ~ 1 Hz) and Peshkov transducers. The measured speed of second sound is in good agreement with the expected value. The measured quality factor is compared with a recent theory on the second viscosity coefficient.

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In 1940 Tisza[1] predicted the existence of a new propagating temperature wave called second sound in superfluid ^{4}He. On the basis of the phenomenological two-fluid hydrodynamics of He II that Landau[2] developed, he derived the expression for the speed of second sound. Using a heater to excite and a sensitive thermometer to detect small temperature changes, Peshkov[3] observed the propagation of second sound in ^{4}He for the first time and verified the predictions of Tisza. Peshkov's experiment provided early convincing evidence in support of the two-fluid hydrodynamics. The two-fluid model has also been applied to describe various flow experiments in the superfluid phases of ^{3}He.[4] However, observation of purely thermal second-sound wave propagation in ^{3}He has not yet been reported.[5] In this paper we describe the first observation, using a resonance method, of second-sound propagation in the B phase of superfluid ^{3}He. The measured speed of second sound is compared with the measurements of the superfluid fraction and the specific heat in other laboratories. The measured resonance width is compared with theoretically expected values from the second viscosity and the thermal conductivity.

The second-sound wave is an undamped temperature wave in bulk superfluid ^{4}He in which the superfluid component (velocity $\mathbf{v}_s$ and density ρ_s) and the normal component (velocity $\mathbf{v}_n$ and density ρ_n) move out of phase such that to first order there is no net mass flow (i.e., $\rho_s\mathbf{v}_s + \rho_n\mathbf{v}_n = 0$). The speed of sound is related to the superfluid density by[6]

$$C_2^2 = (\rho_s/\rho_n)S^2T/C_V, \tag{1}$$

where S is the entropy per unit mass, T is the temperature, and C_V is the specific heat at constant volume. If dissipations are included in the hydrodynamics equations, the attenuation of second sound is given by[6]

$$\alpha = \frac{\omega^2}{2\rho C_2^3}\left[\frac{\rho_s}{\rho_n}\left(\frac{4}{3}\eta_n + \zeta_2 + \rho^2\zeta_3 - 2\rho\zeta_1\right) + \frac{K}{C_V}\right], \tag{2}$$

where η_n is the shear viscosity of the normal component, ζ_1, ζ_2, and ζ_3 are second viscosity coefficients, and K is the thermal conductivity. In the isotropic ^{3}He-B phase, the speed of second sound and its attenuation are expected to be given by the same equations as above. In the ^{3}He-A phase, anisotropy effects must be taken into account. In ^{3}He, the contributions to attenuation from ζ_1 and ζ_2 are expected to be very small compared to the other terms in Eq. (2).[7]

The expected speed of second sound from Eq. (1) in ^{3}He-B at a pressure of 20 bars and at a temperature $1 - T/T_c = 0.05$ below the superfluid transition temperature (T_c) is of order 1 cm/sec. This value is smaller than that in He II by 3 orders of magnitude. Since the expected speed of second sound in ^{3}He-B is so tiny and the viscosities and the thermal conductivity are relatively large, the attenuation would be expected to be large, particularly in the range of frequency greater than 10 Hz.[8] Thus we chose to work at frequencies near 1 Hz. As for the detector of second sound, it would be difficult to measure the temperature oscillation with a thermometer immersed in ^{3}He because of the large thermal boundary resistance at the interface. In our experiment, the mechanical transducer developed by Peshkov[9] and recently reanalyzed by Liu[10] was used to excite and detect second sound in a resonator cavity.

A schematic of our second-sound resonator is shown in Fig. 1. The resonator is a cylindrical cavity whose diameter is 1.3 cm and length is 1.9 cm. The ends of the cavity are closed by identically constructed Peshkov transducers (PT). The PT consists of (1) a 2-mm-thick glass capillary array,[11] containing parallel

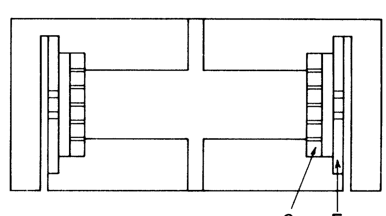

FIG. 1. Second-sound resonator.

holes of uniform diameter equal to 25 μm with a porosity equal to 50%, and (2) a 1-μm-thick nickel diaphragm. The glass capillary array (G) acts as a stationary superleak. The nickel diaphragm (not shown) and the fixed plate (F) behind it form two plates of a capacitor (~ 7 pF) and act as a sensitive microphone. We chose the PT rather than the more commonly used oscillating superleak transducer (OST) since the signal strength would be limited by the thermal-diffusion nuisance currents in the OST.[12] We did not attempt to use the OST in our search for second sound in superfluid ^{3}He. The measured effective tension at 20 mK of the nickel diaphragm was 1.0×10^5 and 7×10^3 dyn/cm for the drive and detector transducers, respectively. On the drive side we typically applied a 200-V dc bias and a 10-V p.-p. oscillating voltage between the diaphragm and the fixed plate. The microphone response was measured by detection of the capacitance changes using a capacitance bridge.

The resonator is contained in a cerium-magnesium-nitrate (CMN) adiabatic demagnetization cell and is immersed in liquid ^{3}He. The design of the thermal contact through a column of superfluid ^{3}He is very similar to those that we have used in the past and it provides an excellent thermal link. The thermal contact is made via 2-mm-diam holes at the center of the resonator. A lanthanum-diluted CMN thermometer pill whose magnetic susceptibility was measured was thermally linked to the interior of the resonator via a 2.5-cm-length, 3-mm-diam column of liquid ^{3}He. The magnetic susceptibility of the La-diluted CMN powder measured by a SQUID-based mutual inductance bridge was calibrated against a calibrated germanium resistance thermometer in the range between 0.3 and 1 K. Our magnetic temperature was converted to "absolute" temperature by comparison of the measurement of our T_c^* to the phase diagram measured by Greywall[13] in the pressure range from 18.2 to 24.4 bars. The comparison showed that the relationship $T=T_c^*+137$ μK gave a good fit.

In a typical run, the demagnetization cell was precooled to about 17 mK in a field of 1000 G, and then it was demagnetized to the Earth's field over 3 h. From the minimum temperature of about 2.1 mK, the cell warmed at a rate of about 15 μK/h. Measurements presented here were obtained as the temperature drifted up by the residual heat leak.

In Fig. 2 we show examples of the detector response (proportional to the rms displacement of the diaphragm) as a function of the driving frequency at a pressure of 21.3 bars and at the temperatures given in the figure caption. We found that the second-sound resonances (peaks shown by arrows) were superimposed on the low-frequency shoulder of a relatively large reproducible resonance (shown partly by dashed lines in Fig. 2; see below for discussion). The smaller

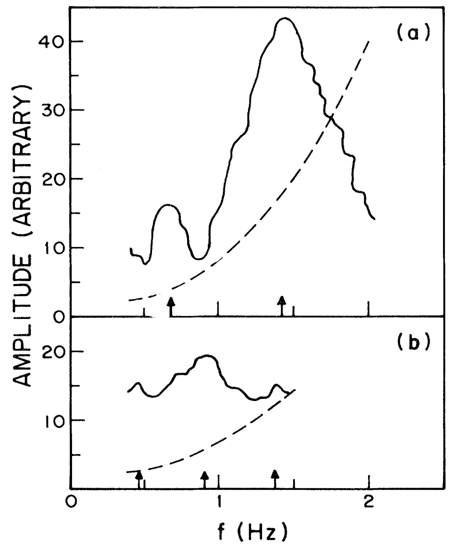

FIG. 2. Received second-sound signals as a function of frequency at 21.3 bars. Measurement (a) at $1-T/T_c=0.0878$ and (b) at $1-T/T_c=0.0231$.

irreproducible "wiggles" seen in the response could easily be distinguished from the reproducible second-sound resonance peaks. The observed signal level of the second-sound resonance peak was linearly proportional to the ac drive at sufficiently low levels. The responses shown by the solid lines in Fig. 2 are obtained by subtraction of the "background" from the total response.

The second-sound resonances are expected to occur at frequencies

$$f_m=mC_2/2L, \tag{3}$$

where m is an integer and L is the length of the cavity. The resonances of modes up to $m=3$ were observed in the frequency range between 0.4 and 2 Hz. The measured resonant frequencies were proportional to m within 5%. It is not understood why the signal for the $m=1$ mode is weaker than that for the $m=2$ mode, contrary to what might be expected from Eq. (2). We measured the resonance frequency of the modes as a function of temperature by following the temperature dependence of the resonance spectrum. The resonance frequency was converted[14] to the speed of second sound by means of Eq. (3). The background-subtraction procedure did not affect the measured speed of second sound.

The peak frequency f_H of the background resonance was 6.3 Hz at $1=T/T_c=0.09$ and $P=21.3$ bars, and its temperature dependence could be accurately described by $f_H^2\propto 1-T/T_c$ in the whole temperature range of our measurement. We searched for but failed to find any higher harmonics of this mode. The linear dependence on the reduced temperature is the same as that of the square of fourth-sound speed or the bulk superfluid fraction of ^{3}He near T_c. Estimates of normal fourth-sound modes in any part of our demagnetization cell will give much higher frequencies than the measured value. These observations suggest to us that the observed background resonance is a fourth-sound Helmholtz resonance[15] in which the nickel diaphragm provides a restoring force. If we take the connecting hole at the center of the resonator as the neck and the second-sound resonator cavity as the volume, we estimate that at $1-T/T_c=0.1$ and at 21.3 bars the Helmholtz resonance is 14 Hz, which is fairly close to the measured value. Since we did not find a simple analytical frequency dependence of the background resonance shape,[16] we smoothly extended (as shown by dashed lines in Fig. 2) the portion of the background resonance where the contribution from the second sound is small. The uncertainty in the background contributes to the error in our measurement of Q described below.

The values of the speed of second sound measured by following the mode $m=2$ at pressures of 18.2, 21.3, and 24.4 bars are shown by dots as a function of reduced temperature in Figs. 3(a)–3(c), respectively. At $\rho=21.3$ bars, where the measurement was repeated over three runs, the measured speed of sound was reproducible within 5%. In the measurements at pressures of 18.2 and 21.3 bars, the signal disappears as the temperature approaches T_c. In the measurement at 24.4 bars, the signal disappears at a much lower reduced temperature. We believe that the temperature where the signal disappears marks the $B \rightarrow A$ transition temperature T_{AB} shown by the arrow in Fig. 3(c). Interpolation of the table of T_{AB} given by Paulson *et al.*[17] gives for $\rho=24.4$ bars a value of $1-T_{AB}/T_c=0.056$ which is rather close to our observation. A small (~ 3 G) external magnetic field applied to the resonator cavity region did not help to produce second-sound resonance in the A phase. The nature of the disappearance of the second-sound signal at T_{AB} is not known. The fourth-sound Helmholtz resonance could still be observed in the A phase.

The lines in Fig. 3 are the expected speed of second sound from Eq. (1). The temperature dependence of S was obtained from the theory by Serene and Rainer.[18] The jump in specific heat, $\Delta C/C$, at T_c is an adjustable parameter in their theory and was taken from the measurements by Alvesalo *et al.*[19] The normal-fluid specific heat at T_c was taken from the measurement by Greywall and Busch.[20] We used the measurements of normal-component density by Archie *et al.*[21] to evaluate ρ_s/ρ_n. The qualitative agreement between the expected and the measured speed of second sound does not change if we use the temperature scale either by Alvesalo *et al.*[19] or by Paulson *et al.*[17] If we take $\Delta C/C=1.44$ instead of 1.72 at $P=18.2$ bars,[17] the expected C_2 increases by 8%. If the specific heat at T_c is taken from Wheatley's[4] tabulation, the expected C_2 increases by an additional 4%.

The quality factor, $Q=f_m/\Delta f_m$, was determined from background-subtracted responses such as shown in Fig. 2 by reading the full width Δf_m at half power

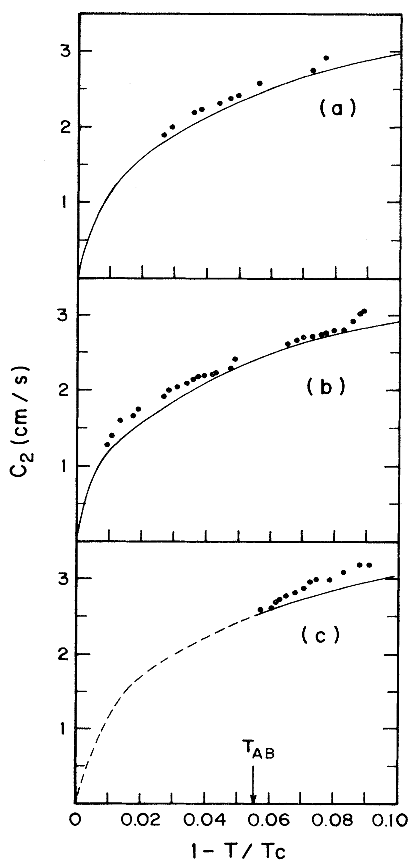

FIG. 3. Speed of second sound as a function of reduced temperature at (a) $P=18.2$, (b) 21.3, and (c) 24.4 bars.

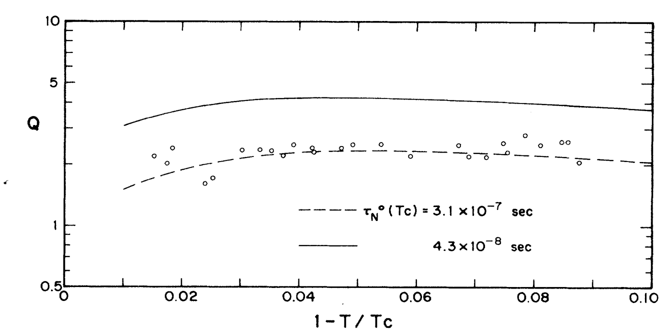

FIG. 4. Quality factor of the second mode at $P=21.3$ bars as a function of reduced temperature. The lines are the expected value from theory.

points. The measured quality factor of the $m=2$ mode at $P=21.3$ bars is shown in Fig. 4 as open circles. The quality factor depends on the bulk attenuation, Eq. (2), in addition to the viscous losses at the wall of the cavity.[22] The wall losses in the present experiment can be calculated to be small (less than 14%) compared to the bulk losses. The quality factor for the mode $m=2$ expected from Eq. (2) and the wall losses is shown as a solid line in Fig. 4. The shear viscosity measured by Parpia *et al.*[23] was used. The thermal conductivity is expected to be relatively independent of temperature[24] just below T_c, so that we took the thermal conductivity[4] to be a constant, evaluated at T_c. The second viscosity ζ_3 was evaluated by use of Einzel's theory.[24] The quasiparticle relaxation time τ_N^0 at T_c and the density of states N_F were taken to be 4.3×10^{-8} sec[21] and 2.04×10^{38} (ergs cm^3)$^{-1}$,[4] respectively. If the shear viscosity term in Eq. (2) is neglected, the expected Q in Fig. 4 will increase by about 14%. The second viscosity and the thermal conductivity contribute to the attenuation about equally in Eq. (2). A fit between the expected and the measured Q can be obtained if we adjust the quasiparticle relaxation time at T_c to be 3.1×10^{-7} sec. This is shown by the dashed line in Fig. 4. It should be stated that the frequency dependence expected from Eq. (2) has not been verified in our experiment. We cannot rule out a possibly large cell-model-dependent contribution to the measured Q of the $m=2$ mode. For these reasons, the comparison of measured Q to attenuation theory and the adjusted value of the quasiparticle relaxation time must be viewed as tentative. The pressure dependence of the Q value was not measured.

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