Study of the Mixed-Symmetry $J^\pi = 2^+$ States in ^{156}Gd with Inelastic Electron Scattering

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The study of electric quadrupole transitions from the ground state into symmetric states and into states of mixed symmetry in the deformed nucleus ^{156}Gd by inelastic electron scattering has been used to derive effective neutron and proton boson charges for the $E2$ operator in the framework of the interacting-boson model. The values for e_π and e_ν differ at most by 30% in the rotational limit.

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After the discovery[1] of a new, magnetic dipole mode, Iachello pointed out[2] that this $J^\pi = 1^+$ mode was only a particular example of a new class of collective excitations at rather low excitation energy. The underlying assumptions for suggesting such excitations result from the explicit treatment of proton and neutron degrees of freedom in the interacting-boson model (IBM-2).[3,4] If the proton and neutron bosons are characterized by their F-spin quantum numbers,[3,4] $F=\frac{1}{2}$, $F_z=+\frac{1}{2}$ and $F=\frac{1}{2}$, $F_z=-\frac{1}{2}$, respectively, a system of N_π proton bosons and N_ν neutron bosons can be classified according to its total F spin. The totally symmetric orbital (*sd*-space) states have maximal F spin, i.e., $F_{\max}=\frac{1}{2}(N_\pi+N_\nu)$. The other mixed-symmetry states are classified by decreasing F spin, down to a value of $F_{\min}=\frac{1}{2}|N_\pi-N_\nu|$. The class of mixed-symmetry states are characterized by $F=F_{\max}-1$ are the lowest lying and can be excited, starting from the 0^+ ground state, in even-even nuclei, by a $\Delta F=1$ transition.

In rotational nuclei, the lowest mixed-symmetry state is the 1^+ state, now observed in a number of strongly deformed rare-earth nuclei.[5,6] In vibrational nuclei, however, a $J^\pi=2^+$ state is expected to be the lowest member of the mixed-symmetry $F=F_{\max}-1$ levels.[2] Recent experiments[7] have indicated a possible candidate (the 2_3^+ level in some $N=84$ nuclei) for such a 2^+ state.

The aim of this Letter is to provide experimental evidence from high-resolution inelastic electron scattering for the existence of a mixed-symmetry $J^\pi=2^+$ state (henceforth denoted as the 2_M^+ state) in the nucleus ^{156}Gd (in rotational nuclei the 2_M^+ state is the $J^\pi=2^+$ member of the $K^\pi=1^+$ band) and to extract information on the effective electric quadrupole operator. Electric quadrupole transitions are described within the IBM-2 in terms of the operator

$$T(E2)=e_\nu Q_\nu+e_\pi Q_\pi, \quad (1)$$

where e_ν (e_π) is the $E2$ effective boson charge for neutrons (protons) and Q_ν (Q_π) is the neutron (proton) quadrupole operator. Of prime importance for the observation of mixed-symmetry 2_M^+ states is the $0_1^+ \to 2_M^+$ transition strength and its magnitude compared with, e.g., the strength to the first excited symmetric 2_1^+ state. Analytic expressions for the corresponding $B(E2)$ values can be obtained in the F-spin–symmetric SU(3) limit[8] of the IBM-2,

$$B(E2;0_1^+\to 2_1^+) = (e_\nu N_\nu+e_\pi N_\pi)^2(2N+3)/N, \quad (2)$$

$$B(E2;0_1^+\to 2_M^+) = (e_\nu-e_\pi)^2\frac{3(N-1)}{N(2N-1)}N_\nu N_\pi, \quad (3)$$

where N is the total number of bosons and $N_\pi(N_\nu)$ is the number of proton (neutron) bosons counted from the nearest closed shell. When analyzing the experimental data, one is especially interested in the ratio of the expressions given in Eqs. (2) and (3).

For an equal number of proton and neutron bosons present, this SU(3) limit ratio is smaller by a factor of $3(N-1)/(2N-1)(2N+3)$ than the one derived[2,8] for the SU(5) limit.

The observation of mixed-symmetry 2^+ levels will depend in a critical way on the difference $e_\pi - e_\nu$. From a shell-model approach,[9] one can expect to find the largest differences when one type of particles, i.e., protons, is near a closed-shell configuration and the other type of particles, i.e., neutrons, is near a midshell configuration. From this point of view, the nucleus ^{156}Gd is not the best case but it still provides a valuable test ground for the search for mixed-symmetry $J^\pi=2_M^+$ levels, especially since the 1_M^+ level is known experimentally. [We note here, in passing, that it has been pointed out[10–12] that the dependence of $E2$ strength on the effective charges in IBM-2 is

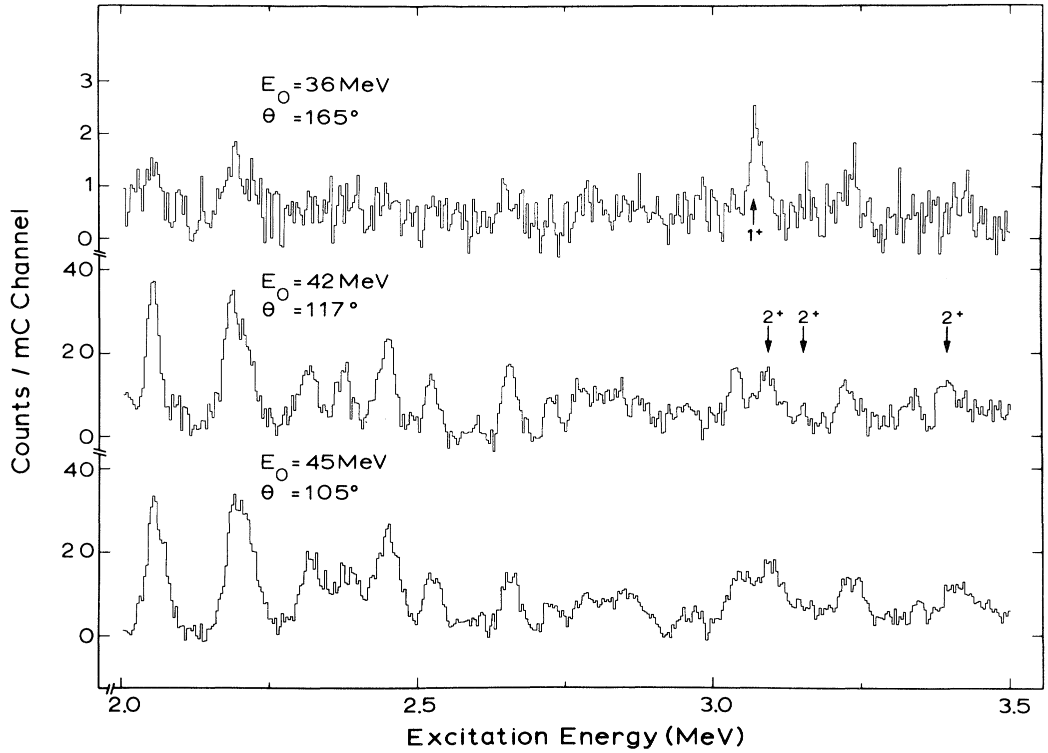

FIG. 1. Three ^{156}Gd(e,e') sample spectra taken at the same momentum transfer, $q=0.36$ fm^{-1}. The $J^\pi=1^+$ state at $E_x=3.075$ MeV is indicated as an arrow in the backward-angle spectrum. The three candidates for mixed-symmetry $J^\pi=2^+$ states at energies above the $J^\pi=1^+$ states are also marked, but in the forward-angle spectrum at $\theta=117°$.

quite similar to the dependence found in the single-j-shell model, where for like signatures $B(E2) \sim (e_p-e_n)^2$ holds, p standing for proton, n for neutron.]

The experiment has been performed at the high-resolution facility ($\Delta E \approx 30$ keV FWHM) of the Darmstadt electron linear accelerator DALINAC. Experimental details are given in Refs. 1 and 5. In Fig. 1, samples of three inelastic electron scattering spectra on ^{156}Gd, taken at different electron energies E_0 and scattering angles θ but at constant momentum transfer $q=0.36$ fm^{-1}, are shown. The strongest signal in the backward-angle spectrum corresponds to the $J^\pi=1^+$ state at $E_x=3.075$ MeV discussed in detail in Ref. 1. The spectra at foward angles ($\theta=117°$ and 105°) reveal much more structure than the $\theta=165°$ spectrum, the reason being a wealth of longitudinal electric excitations with multipolarity $\lambda \lesssim 3$ at the fairly small measured q.

Those spectra were decomposed into various lines with use of the line shape of the elastic line (not shown) and a consistency criterion for the excitation energies of the levels observed in the spectra at different E_0 and θ values. Concerning the search for the $J^\pi=2_M^+$ candidates we concentrate first on the region $E_x \gtrsim 3$ MeV. As indicated in the middle part of Fig. 2, three $J^\pi=2^+$ states have been identified at excitation energies $E_x=3.096$, 3.150, and 3.400 MeV through their respective $E2$ form factors. All three states lie above the $K^\pi=1^+$ state at $E_x=3.075$ MeV and hence could be mixed-symmetry states. Using the known

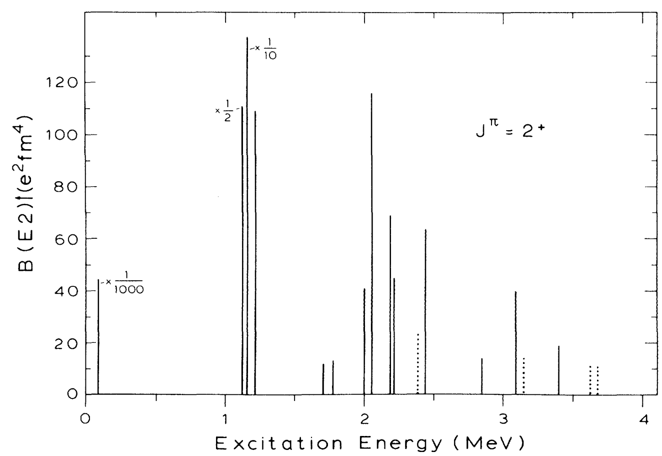

FIG. 2. The $E2$ strength distribution determined in the present ^{156}Gd(e,e') experiment. The dotted lines indicate candidates for which a 2^+ assignment is not completely unambiguous.

moment of inertia of the ground-state rotational band $[(\hbar^2/2\tau) = 14.7$ keV] and making the assumption that the moments of inertia of the ground-state (g.s.) and the $K^\pi = 1^+$ bands are the same [as would be the case in the SU(3) limit[13] of the IBM], one would associate the state at $E_x = 3.150$ MeV with the 2_M^+ member of the $K^\pi = 1^+$ band. However, we provide arguments below for the fact that the strongest of the three 2^+ states at $E_x = 3.096$ MeV is a much better candidate.

If we assume that one of the three observed (see Fig. 2) $J^\pi = 2^+$ states above $E_x = 3.075$ MeV is due to a $\Delta F = -1$ excitation, insert their respective transition strengths and the transition strength of the 2_1^+ state at $E_x = 0.089$ MeV into Eqs. (2) and (3), and take the ratio

$$R = B(E2;0_1^+ \to 2_M^+)/B(E2;0_1^+ \to 2_1^+),$$

we obtain $1\times 10^{-4} \leqslant R \leqslant 9\times 10^{-4}$, resulting in the following limits for the effective quadrupole boson charges for ^{156}Gd:

$$1.1 \leqslant e_\pi/e_\nu \leqslant 1.3.$$

Although the expressions (2)–(4) are valid only in the SU(3) limit we use them together with the experimental $B(E2)\uparrow$ values to determine values for the effective boson charges e_ν and e_π in ^{156}Gd [the parameters needed for a description of the excited states in ^{156}Gd, i.e., $\chi_\nu = -1.0$ and $\chi_\pi = -1.1$, are reasonably close to the pure SU(3) value, and with use of the expressions (2) and (3) will not affect the estimate of the $E2$ boson charges drastically]. In order to reproduce the measured transition strength $B(E2;0_1^+ \to 2_1^+) = 4.48 \pm 0.52\ e^2\cdot\mathrm{b}^2$ we finally have

$$10.8 \pm 0.3\ e\cdot\mathrm{fm}^2 \geqslant e_\nu \geqslant 10.0 \pm 0.3\ e\cdot\mathrm{fm}^2 \quad (4)$$

and

$$12.4 \pm 1.4\ e\cdot\mathrm{fm}^2 \leqslant e_\pi \leqslant 13.0 \pm 1.4\ e\cdot\mathrm{fm}^2, \quad (5)$$

i.e., near equality of e_π and e_ν in case of the rotational nucleus ^{156}Gd.

From the above results, the following conclusions can be drawn: (i) The values for e_π, e_ν differ drastically—but not unexpectedly—from the result obtained for the $N = 84$ isotones ^{140}Ba, ^{142}Ce, and ^{144}Nd, where (in the vibrational limit of IBM-2) the ratio $e_\pi/e_\nu = 0.5$ is obtained.[7] The result that $e_\pi > e_\nu$ is also supported by the finding[14] of a deformation for protons that is larger than for neutrons for ^{154}Sm and by the reproducing of various $E2$ transitions in ^{148,154}Sm in terms of the interacting-boson model.[15] (ii) With respect to the location and strength of the mixed-symmetry 2_M^+ states we have assumed that the $J^\pi = 1^+$ state at $E_x = 3.075$ MeV is the only level excited through the convection-current part of the $M1$ operator. Up to now this is the only state in ^{156}Gd that we could uniquely identify[5] as due to the orbital motion. If this state is split, however, some of the $E2$ strength connected with the 2_M^+ member of the $K^\pi = 1^+$ band will reside in other bands. The identification of other mixed-symmetry $K^\pi = 1^+$ bands is therefore highly desirable. A general conclusion of our analysis is that the 2_M^+ levels in ^{156}Gd are only weakly excited. This might well be a general feature characteristic for the SU(3) [as well as the O(6)] limit since it predicts the ratio R to be small.

In order to provide arguments that the 2^+ states above $E_x \simeq 3$ MeV are indeed candidates for the mixed-symmetry 2^+ state,[16] we have used Eq. (2) and the known $B(E2;0_1^+ \to 2_1^+)$ values in neighboring strongly deformed even-even nuclei (i.e., ^{154}Gd, ^{154}Sm, ^{158}Gd, ^{160}Dy) to deduce values for e_π and e_ν in an independent way. The results of a best fit of the quantity $[NB(E2)/(2N+3)]^{1/2}$ versus the neutron boson number N_ν (see Refs. 7, 17, and 18) gives a straight line with slope $e_\nu = 9.2 \pm 0.9\ e\cdot\mathrm{fm}^2$ and intercept $e_\pi = 13.2 \pm 1.1\ e\cdot\mathrm{fm}^2$ with the ordinate at $N_\nu = 0$. With these values, one calculates $B(E2;0_1^+ \to 2_1^+) = 43\,100\ e^2\cdot\mathrm{fm}^4$ and $B(E2;0_1^+ \to 2_M^+) = 67\ e^2\cdot\mathrm{fm}^4$.

The experimental transition strengths from the present experiment are $B(E2;0_1^+ \to 2_1^+) = 44\,780 \pm 500\ e^2\cdot\mathrm{fm}^4$ and $B(E2;0_1^+ \to 2_M^+) = 40 \pm 6\ e^2\cdot\mathrm{fm}^4$ for the state at $E_x = 3.096$ MeV, which we thus favor as the $J^\pi = 2^+$ member of the $K^\pi = 1^+$ band. The effective boson charges deduced from these values ($e_\pi = 13\ e\cdot\mathrm{fm}^2$, $e_\nu = 10\ e\cdot\mathrm{fm}^2$) corresponding to the limits on the right-hand side of Eqs. (4) and (5) are in good agreement with the results of our fit. A possible splitting of the mixed-symmetry 2^+ state, however, cannot be ruled out by our approach.

The $E2$-strength distribution in ^{156}Gd is summarized in Fig. 2 up to $E_x \simeq 4$ MeV. The $0_1^+ \to 2_1^+$ transition within the g.s. band has the largest $B(E2)$ value. Slightly above $E_x \simeq 1$ MeV, one observes the excitation of 2^+ members of the β and γ band, as well as an extra $K^\pi = 0^+$ low-lying band. The $E2$ strength in the energy region $2.0 \lesssim E_x \lesssim 2.5$ MeV, however, cannot be accounted for[19] within the IBM-2. In the (n,γ) study[20] of ^{156}Gd, low-lying $K^\pi = 0^+, 1^+, 2^+$ bands have been observed in the energy region $1.2 < E_x < 2.3$ MeV. Since the typical $E2$ strength into the 2^+ members of those bands, i.e., two-quasiparticle (2qp) configurations, is of the order of 0.2 single-particle units (s.p.u.) [for $B(E2)$, 1 s.p.u. $= 0.30A^{4/3}\ e^2\cdot\mathrm{fm}^4$], this may be one possible explanation of the observed strength near $E_x \simeq 2$ MeV, especially when we note the high density of proton and neutron 2qp configurations in this energy region. This is also the region where two-phonon ($\beta\beta, \beta\gamma, \gamma\gamma$) states are expected. The Bohr-Mottelson collective model[21] thus leaves the attractive suggestion that the $E2$ strength might arise from two-phonon $E2$ excitations coupling to 2qp states and the $E2$ strength might thus be redistributed over an interval of $\Delta E_x \simeq 0.5$ MeV.

We are presently extending the study of mixed-symmetry $J^\pi = 2_M^+$ states to other well-deformed rare-earth nuclei. As yet, the mass dependence of the effective boson charges as well as the origin of $E2$ strength near $E_x \simeq 2$ MeV deserves further experimental and theoretical attention.

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