Concentration Scaling for Spin-Glasses with Multiple Magnetic Impurities

Recently, Vier and Schultz[1] presented a study of the concentration dependence of the freezing temperature T_g in metallic spin-glasses with multiple impurities. For two magnetic impurity species in Au, their data could be described by

$$T_g(C_1,C_2,\rho)=T_g(C_1,0,\rho)+T_g(0,C_2,\rho), \qquad (1)$$

where C_α is the concentration of species α and ρ is the resistivity, which they relate to the damping of the Ruderman-Kittel-Kasuya-Yosida (RKKY) exchange interaction by mean free path effects. Equation (1) is a generalization of the well-known concentration scaling law[2] for a single species, $T_g(C_\alpha)\propto C_\alpha$.

In this Comment, I note that (1) is the exact result of a simple mean-field[3] calculation, *with properly formulated cutoffs*, for *undamped* RKKY interaction ($\rho\to 0$). I neglect "replica symmetry breaking"; in the infinite-range model[4] this is exact for $T\geqslant T_g$ and always gives the correct T_g.

In the dilute limit, the system is modeled by impurities placed at random positions $\mathbf{x}_i$, where $\mathbf{x}_i$ takes on a *continuum* of values. The RKKY exchange interaction between unit Heisenberg spins $\mathbf{s}_i$ and $\mathbf{s}_j$, of species $\sigma(i)=\alpha$ and $\sigma(j)=\beta$, is given by

$$J_{ij}=\cos(2\phi_{ij})\bar{J}_{\alpha\beta}(|\mathbf{x}_i-\mathbf{x}_j|), \qquad (2)$$

$$\bar{J}_{\alpha\beta}(r)=A_{\alpha\beta}/r^3, \qquad (3)$$

where the $\{\phi_{ij}\}$ are independent, *random* phases.

At $T\lesssim T_g$ each spin has a frozen thermal average $\langle\mathbf{s}_i\rangle_T$ which is parallel to the average local field $\mathbf{h}_i$ and depends on it by a Brillouin function, $|\langle\mathbf{s}_i\rangle_T|=B(|\mathbf{h}_i|/T)$, where $B(x)=\coth x-x^{-1}\cong x/3$. Also, $\mathbf{h}_i=\sum_j J_{ij}\langle\mathbf{s}_j\rangle_T$, which gives a set of equations to be solved self-consistently.

I now define Edwards-Anderson order parameters for each species, $q_\alpha=[\langle\mathbf{s}_i\rangle_T^2]_{\sigma(i)=\alpha}$, averaging over all spins and configurations but keeping the different species α distinct; in the same spirit an average local field $\bar{h}_\alpha$ is defined for each species, $\bar{h}_\alpha^2=[|\mathbf{h}_i|^2]_{\sigma(i)=\alpha}$. Taking the approximation $\langle\mathbf{s}_i\rangle_T^2\to q_{\sigma(i)}$ [depending only on $\sigma(i)$], we get

$$\bar{h}_\alpha^2=\Big[\sum\nolimits_j [J_{ij}^2]_\phi q_{\sigma(j)}\Big]_{\sigma(i)=\alpha}, \qquad (4)$$

averaging over the random phases first and then over positions. Collecting the q_α and performing the averages, we have

$$\bar{h}_\alpha^2=\sum\nolimits_\beta K_{\alpha\beta}q_\beta, \qquad (5)$$

where

$$K_{\alpha\beta}=\int_{\xi_{\alpha\beta}}^\infty 4\pi r^2\,dr\,C_\beta[\tfrac{1}{2}\bar{J}_{\alpha\beta}(r)^2]$$
$$=\tfrac{2}{3}\pi C_\beta A_{\alpha\beta}^2\xi_{\alpha\beta}^{-3}. \qquad (6)$$

Note that a cutoff $\xi_{\alpha\beta}$ is needed to prevent a divergence. Mathematically, this is due to the continuum distribution of impurity positions $\mathbf{x}_i$ which allows rare, arbitrarily close pairs. Actually, such close spins lock together (ferromagnetically or antiferromagnetically) at $T\gg T_g$ and do not contribute to the fluctuations which determine T_g. Therefore, I argue that the cutoff should be chosen so that no one term in the summation inside (4) is counted if it exceeds $(\epsilon\bar{h}_\alpha)^2$, where ϵ is a parameter of order unity,[5] i.e.,

$$\tfrac{1}{2}\bar{J}_{\alpha\beta}(\xi_{\alpha\beta})^2q_\beta=\epsilon^2\bar{h}_\alpha^2. \qquad (7)$$

This choice[6] is the essential step of the derivation.

Substituting from (3), (6), and (7) into (5), we get

$$\bar{h}_\alpha=\sum\nolimits_\beta(2\sqrt{2}\pi/3)\epsilon C_\beta A_{\alpha\beta}q_\beta^{1/2}. \qquad (8)$$

A solution is $q_\alpha=0$; as T decreases, this goes unstable when Eq. (8) (linearized in $\{q_\alpha^{1/2}\}$) first has a nontrivial solution, which defines T_g. Using $q_\alpha^{1/2}\cong\bar{h}_\alpha/3T$ (from the Brillouin form for $\langle\mathbf{s}_i\rangle_T$), we find $Tq_\alpha^{1/2}=\sum_\beta M_{\alpha\beta}q_\beta^{1/2}$, where $M_{\alpha\beta}=(2\sqrt{2}\pi/9)\epsilon C_\beta A_{\alpha\beta}$, so that T_g *is given by the largest eigenvalue* of the matrix $(M_{\alpha\beta})$. Now, for the RKKY interaction, $A_{\alpha\beta}\propto V_\alpha V_\beta$, where V_α is the local-moment–conduction-spin coupling of species α, so that $|A_{\alpha\beta}|=(A_{\alpha\alpha}A_{\beta\beta})^{1/2}$; then $(M_{\alpha\beta})$ is of rank 1 and its largest eigenvalue is

$$T_g=\sum\nolimits_\alpha(2\sqrt{2}\pi/9)\epsilon A_{\alpha\alpha}C_\alpha, \qquad (9)$$

which implies (1) as claimed. The derivation works for any number of impurity species.

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[5]Susceptibility measurements on *Au*Fe in the dilute limit (Ref. 2) give $T_g/C\cong 2.34\times10^{-36}$ erg cm^3, while $A\cong 2.80\times10^{-36}$ erg cm^3 from the low-temperature susceptibility [L. R. Walker and R. E. Walstedt, Phys. Rev. B **22**, 3816 (1980)]. Thus [Eq. (9)] $\epsilon\cong 0.84$ experimentally.

[6]For one species Eq. (7) leads to $\xi=(\frac{4}{3}\pi\epsilon^2C)^{-1/3}$; if $\epsilon=1$, this is the cutoff of U. Larsen, Solid State Commun. **22**, 311 (1977), Eq. (3).