Comment on "Relativistic Plasma-Wave Excitation by Collinear Optical Mixing"

The measurement of plasma waves with phase velocity close to c (the velocity of light) was recently described.[1] The waves were produced by beating nominally collinear ($f/7.5$ optics) light at wavelengths of 9.6 and 10.6 μm (from a CO_2-laser system) in a plasma of density 10^{17} e/cm^3. The plasma density waves were diagnosed by small-angle scattering, with use of a ruby-laser system with its axis at right angles to the CO_2 optical axis. It is important to be sure that the plasma-wave phase velocity is close to c because only then can one hope to accelerate electrons to relativistic energy[2] by using the beating of laser light. While collinear plasma waves with velocities close to c may well have existed in the system, we assert that the diagnostic system measured plasma waves at 45° to the CO_2-laser axis at a phase velocity of $c/\sqrt{2}$.

The point is that the commonly used formula for the scattering wave vector k_s, for a scattering frequency ω_s and a scattering angle θ_s between the two light waves with wave vectors k_H (high) and k_L (low), usually both nearly equal to the mean light wave vector $\langle k \rangle$, namely,[3]

$$k_s = 2\langle k \rangle \sin(\theta_s/2), \tag{1}$$

is an approximation valid only for $\omega_s^2/k_s^2 \ll c^2$, which is not the case here. The correct formula is

$$k_s^2 = (k_H - k_L)^2 \cos^2(\theta_s/2) + (k_H + k_L)^2 \sin^2(\theta_s/2). \tag{2}$$

For the diagnostic laser scattering system, defining $k_H + k_L = 2\langle k_d \rangle$ and noting that $k_H - k_L = \omega_s/c$ for light waves close to the vacuum velocity, this is more transparently related to (1) as

$$k_s^2 = (\omega_s^2/c^2)\cos^2(\theta_s/2) + 4\langle k_d \rangle^2 \sin^2(\theta_s/2). \tag{3}$$

One can usually neglect the term $(\omega_s^2/c^2) \times \cos^2(\theta_s/2)$, but not here, since k_s^2 is not much greater than ω_s^2/c^2. In the case considered here θ_s is small, so that (3) then becomes for $\theta_s^2 \ll 1$,

$$k_s^2 \approx \omega_s^2/c^2 + \langle k_d \rangle^2 \theta_s^2. \tag{4}$$

The parallel component ω_s/c was ignored[1] by Clayton *et al.* When it is restored, since $\langle k_d \rangle \theta_s$ was, in fact, measured[1] to be ω_s/c, one infers (i) that the plasma waves (at ω_s) measured had $k_s \approx \sqrt{2}\omega_s/c$ and so a phase velocity of $c/\sqrt{2}$, and (ii) that k_s was at an angle of 45° to the ruby-laser direction. (This implies that the angle between the beating 9.6- and 10.6-μm light waves was about 0.1 rad, which is consistent with the $f/7.5$ optics.)

It may seem remarkable that a very clear angular selection was observed since other plasma waves are presumably present, but, in fact, only those 45° $c/\sqrt{2}$ beat plasma waves can be detected if the angle between the CO_2-laser-light wave vectors is small. More generally, applying Eq. (4) to both the small-angle driver beat-wave system and the small-angle scattering diagnostic systems, with axes at an arbitrary angle of θ_{db}, one can quickly see that the only beat waves that are detected are those whose wave vectors are either along the bisector of the angle between the two axes or at right angles to it (in the scattering plane). These will be detected at scattering angles of

$$\theta_s = (\omega_s/\langle k_d \rangle c)[\tan(\theta_{db}/2) \text{ or } -\cot(\theta_{db}/2)]. \tag{5}$$

The respective phase velocities are $c\cos(\theta_{db}/2)$ and $c\sin(\theta_{db}/2)$. For the work discussed, θ_{db} was 90°, and so one was measuring plasmons that were at 45° to the mean CO_2-laser axis. In this case only, $\langle k_d \rangle \theta_s = \omega_s/c$ as measured by Clayton *et al.*[1] [and predicted by Eq. (1)]. The scattering angles are the same for blue- and red-shifted light; this would not be so in general.

In connection with the sideband measurements, it should be pointed out that these have also been reported by Lavigne *et al.*[4]

In view of the above, perhaps some of the discussion of the results needs revision. The experimental result is important and encouraging but, while plasmons at $c/\sqrt{2}$ have been seen, those near c await measurement and confirmation still.

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