Signals of *CP* Nonconservation in Hyperon Decay

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We survey the *CP*-odd asymmetries which can signal *CP* nonconservation in hyperon decay. The optimal measurement involves the β parameter in the decay distribution, i.e., $(\beta+\bar{\beta})/(\beta-\bar{\beta})$, which can be quite large in some models. Explicit calculations are provided for all the observables for several theories of *CP* nonconservation.

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Of the puzzles remaining in the low-energy weak interactions, the most fundamental is the question of the origin of *CP* nonconservation. At present *CP*-odd signals are seen only in the kaon systems,[1] and can be described by a single parameter $|\epsilon| = 2.3\times10^{-3}$. Observation of *CP* nonconservation in hyperon decays would be a major advance. Recent papers[2] have discussed some aspects of hyperon decay signals, but as these experiments are being considered more seriously, a more detailed analysis is needed. In this paper we analyze the *CP*-odd signals in hyperon decay using all the major theories of *CP* nonconservation. It turns out that the largest signal in all models is one which has not been previously discussed, i.e., $(\beta+\bar{\beta})/(\beta-\bar{\beta})$ (see definition below), which can reach 10^{-2} in some models.

Hyperon decays measure $\Delta S=1$ *CP* nonconservation while in kaon decays both $\Delta S=2$ and $\Delta S=1$ effects can occur. In fact, in many models the $\Delta S=2$ effects in the K^0-$\bar{K}^0$ mass matrix are the most important and $\Delta S=1$ *CP* nonconservation is absent or suppressed. For example, the superweak model[3,4] and models in which heavy neutral Higgs bosons[5] are responsible for *CP* nonconservation will have no $\Delta S=1$ *CP* nonconservation, and hence no signals in hyperon decay. In the Kobayashi-Maskawa model,[6] $\Delta S=1$ *CP*-odd effects are present and arise through the penguin interaction.[7] These are predicted to be of order $20\epsilon'$ (the factor of 20 emerges because ϵ' contains an intrinsic extra $\Delta I=\frac{3}{2}$ suppression factor of that magnitude). They could be expected to be more reliably calculable in hyperon decay than in kaon decay because of a more trustworthy evaluation of the hadronic matrix elements. There are also models where $\Delta S=1$ *CP* nonconservation is dominant. For example, in models like the Weinberg model,[8,9] where charged Higgs exchange generates *CP* nonconservation, it is $\Delta S=1$ dispersive effects[9,10] which produce ϵ. This model will have the largest results in hyperon decays where the signal will turn out to be $O(\epsilon)$. The left-right symmetric model[11] also has $\Delta S=1$ *CP* nonconservation. In kaons it is somewhat enhanced[12] by a relatively large matrix element for mixture of left-handed and right-handed currents compared to ones with only left-handed currents (i.e., by a factor $\mathcal{M}_{LR}/\mathcal{M}_{LL}\sim 3$–$10$) and by a large numerical coefficient. In hyperon decays the enhancement is not present, but one can still find significant effects. In summary, if we were to provide a rough characterization of $\Delta S=1$ *CP* nonconservation by introducing a parameter χ, we would find in the various models the following values:

Model	χ (approx.)
Superweak	0
Heavy neutral Higgs	0
Kobayashi-Maskawa	$20\epsilon'$
Charged Higgs (Weinberg)	ϵ
Left-right	$(\mathcal{M}_{LL}/\mathcal{M}_{RL})\epsilon$

These estimates will be justified by explicit calculations below.

Hyperon decays proceed into both *S*-wave (parity-nonconserving) and *P*-wave (parity-conserving) final states, and can be decomposed into $\Delta I=\frac{1}{2}$ and $\Delta I=\frac{3}{2}$ matrix elements. We will use Ξ decays, which are characterized by particularly simple final states, as our explicit example; although similar formulas can be easily found for any of the other hyperon decays. In an obvious notation, the decay amplitudes are

$$\mathrm{Amp}(\Xi^-\to\Lambda\pi^-) = (S_1+\tfrac{1}{2}S_3)e^{i\delta_S}+(P_1+\tfrac{1}{2}P_3)e^{i\delta_P}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}, \tag{1a}$$

$$\sqrt{2}\,\mathrm{Amp}(\Xi^0\to\Lambda\pi^0) = (S_1-S_3)e^{i\delta_S}+(P_1-P_3)e^{i\delta_P}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}, \tag{1b}$$

$$\mathrm{Amp}(\bar{\Xi}^+\to\bar{\Lambda}\pi^+) = -(S_1^*+\tfrac{1}{2}S_3^*)e^{i\delta_S}+(P_1^*+\tfrac{1}{2}P_3^*)e^{i\delta_P}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}, \tag{1c}$$

$$\sqrt{2}\,\mathrm{Amp}(\bar{\Xi}^0\to\bar{\Lambda}\pi^0) = -(S_1^*-S_3^*)e^{i\delta_3}+(P_1^*-P_3^*)e^{i\delta_P}\boldsymbol{\sigma}\cdot\hat{\mathbf{q}}, \tag{1d}$$

where δs and δp are the (strong) final-state interaction phases for $\Lambda\pi$ scattering at $E=m_\Xi$. In addition S_1, S_3, P_1, and P_3 can have (weak) *CP*-nonconserving phases which we will denote by ϕ_s^1, ϕ_s^3, ϕ_p^1, and ϕ_p^3, respectively. The observables in hyperon decay[13] are the total decay rate, Γ, and the parameters α and β which govern the decay angular distribution,

$$\alpha = 2\,\mathrm{Re}S^* P e^{i(\delta p - \delta s)}/(|S|^2+|P|^2), \tag{2a}$$

$$\beta = 2\,\mathrm{Im}S^* P e^{i(\delta p - \delta s)}/(|S|^2+|P|^2). \tag{2b}$$

With these we can form three *CP*-odd quantities in Ξ decay. To lowest order in the $\Delta I=\frac{3}{2}$ amplitudes and in the *CP*-nonconserving phases, we find

$$\frac{\alpha+\bar\alpha}{\alpha-\bar\alpha} = \tan(\delta_s-\delta_p)\tan(\phi_s^1-\phi_p^1)\left[1-\frac{S_3}{S_1}\frac{\sin(\phi_s^3-\phi_p^1)}{\sin(\phi_s^1-\phi_p^1)}-\frac{P_3}{P_1}\frac{\sin(\phi_s^1-\phi_p^3)}{\sin(\phi_s^1-\phi_p^1)}\right], \tag{3a}$$

$$\frac{\beta+\bar\beta}{\beta-\bar\beta} = \frac{\tan(\phi_s^1-\phi_p^1)}{\tan(\delta_s-\delta_p)}\left[1-\frac{S_3}{S_1}\frac{\sin(\phi_s^3-\phi_p^1)}{\sin(\phi_s^1-\phi_p^1)}-\frac{P_3}{P_1}\frac{\sin(\phi_s^1-\phi_p^3)}{\sin(\phi_s^1-\phi_p^1)}\right], \tag{3b}$$

$$\frac{\beta_{\Xi^0}}{\alpha_{\Xi^0}}-\frac{\beta_{\Xi^-}}{\alpha_{\Xi^-}} = \frac{3}{2}[1-\tan^2(\delta_s-\delta_p)]\left[\frac{S_3}{S_1}\sin(\phi_p^3-\phi_p^1)+\frac{P_3}{P_1}\sin(\phi_s^1-\phi_s^3)\right]. \tag{3c}$$

The first two of these were written for Ξ^0 decay. To obtain the asymmetries of Ξ^-, change $S_3\to -\frac{1}{2}S_3$, $P_3\to -\frac{1}{2}P_3$. For some other hyperon decays there is also a total rate asymmetry, which has been extensively discussed in Ref. 2. For completeness we will also include an example of the rate asymmetry. In the case of Λ decay, in the limit of *s*-wave dominance,[2]

$$\sqrt{2}A(\Lambda\to n\pi^0) = |A_1|\exp(i\phi_s^1)\exp(i\delta_1) - |A_3|\exp(i\phi_s^3)\exp(i\delta_3), \tag{4a}$$

$$\sqrt{2}A(\bar\Lambda\to \bar n\pi^0) = |A_1|\exp(-i\phi_s^1)\exp(i\delta_1) - |A_3|\exp(-i\phi_s^3)\exp(i\delta_3), \tag{4b}$$

$$(\Gamma-\bar\Gamma)/(\Gamma+\bar\Gamma) = -2|A_3/A_1|\sin(\delta_3-\delta_1)\sin(\phi_s^1-\phi_s^3), \tag{5}$$

where δ_1 and δ_3 are the final-state shifts for *S*-wave $N\pi$ scattering in isospin $\frac{1}{2}$ and $\frac{3}{2}$, respectively, at $E=m_\Lambda$. The rate asymmetry vanishes in Ξ decay because there is only one isospin channel in the final state. We can see that all signals except $(\beta+\bar\beta)/(\beta-\bar\beta)$ are suppressed by either the small size of $\Delta I=\frac{3}{2}$ amplitude (generally $A_3/A_1\approx\frac{1}{20}$) and/or by final-state interaction phases [for Ξ, $\delta_s-\delta_p<10°$ from β/α measurements and we take $\sin(\delta_s-\delta_p)\approx 0.1$; for Λ, $\delta_3-\delta_1$ is about 10°]. Thus, within a given model there is a hierarchy in strength of these observables. Since $\sin(\phi_s-\phi_p)\sim\chi$, we expect

$$(\Gamma-\bar\Gamma)/(\Gamma+\bar\Gamma) \approx \sin(\delta_3-\delta_1)(A_3/A_1)\chi \approx 10^{-5}(\chi/\epsilon), \tag{5a}$$

$$(\alpha+\bar\alpha)/(\alpha-\bar\alpha) \approx \tan(\delta_s-\delta_p)\chi \approx 10^{-4}(\chi/\epsilon), \tag{5b}$$

$$(\beta+\bar\beta)/(\beta-\bar\beta) \approx \chi/\tan(\delta_s-\delta_p) \approx 10^{-2}(\chi/\epsilon), \tag{5c}$$

$$\beta_{\Xi^0}/\alpha_{\Xi^0}-\beta_{\Xi^-}/\alpha_{\Xi^-} \approx (A_3/A_1)\chi \sim 10^{-4}(\chi/\epsilon). \tag{5d}$$

By far the largest signal is in $(\beta+\bar\beta)/(\beta-\bar\beta)$. To some extent, the way the observable is formed is somewhat misleading because $\beta-\bar\beta$, which enters the denominator, is itself a small quantity. Perhaps a better way to present this would be

$$(\beta+\bar\beta)/(\alpha-\bar\alpha) \sim \chi \sim 10^{-3}(\chi/\epsilon), \tag{6}$$

as this more accurately indicates the level of difficulty of the experiment. However, the basic message is unchanged: the measurement of $(\beta+\bar\beta)/(\beta-\bar\beta)$ is at least an order of magnitude more favorable than any other signal.

In the Kobayashi-Maskawa model with three generations, the dominant *CP*-nonconserving interaction is in the "penguin" operator[7]

$$\mathscr{L}_{CP} = i(G_F/2\sqrt{2})\sin\theta_1\cos\theta_1\,\mathrm{Im}c_5\bar d t^A\gamma_\mu(+\gamma_5)s\bar q t^A\gamma^\mu(1-\gamma_5)q, \tag{7}$$

where t^A are the Gell Mann SU(3) matrices [for color SU(3)]. The coefficient $\mathrm{Im}c_5$ is reliably calculated[14] to be

$$\mathrm{Im}c_5 = -0.1\sin\theta_2\sin\theta_3\sin\delta \tag{8}$$

and we will use the matrix elements calculated by Donoghue *et al.*[15] Taking the real parts of the amplitude from experiment, we find for the S and P waves

$$S_1 = 34.3(1 - 0.29 i \,\mathrm{Im} c_5) \times 10^{-8}, \quad (9a)$$

$$P_1 = 13.1(1 + 0.92 i \,\mathrm{Im} c_5) \times 10^{-8}, \quad (9b)$$

thus the phase difference is

$$\tan(\phi_s^1 - \phi_p^1) = -1.2\,\mathrm{Im} c_5 \approx 0.12 \sin\theta_2 \sin\theta_3 \sin\delta \lesssim 2 \times 10^{-4}. \quad (10)$$

It is the smallness of the penguin coefficient which provides some extra suppression. Allowing for four generations, in general, tends to make the numbers even smaller.[16]

In the Weinberg Higgs model, the dominant CP-nonconserving interaction[17] involves the gluon field strength tensor $F^A_{\mu\nu}$:

$$\mathscr{L}_{CP} = i\bar{f}\bar{d}t^A \sigma^{\mu\nu}(1-\gamma_5) s F^A_{\mu\nu}. \quad (11)$$

We normalize this using the analysis of Ref. 9, where it is shown that

$$2m_K \,\mathrm{Im} M_{12} = 2m_K \sqrt{2} |\epsilon| \Delta M \approx 2 \times 10^{-7} \langle \pi^0 | \mathscr{L}_{CP} | K^0 \rangle \quad (12)$$

or

$$\langle \pi^0 | \mathscr{L}_{CP} | K^0 \rangle = 5.8 \times 10^{-11}\ \mathrm{GeV}^2. \quad (13)$$

Hyperon amplitudes can now be calculated relative to this by use of the tables of matrix elements of Donoghue *et al.*[18] The parameter ρ in that paper is set equal to 1. If $\rho < 1$, the signals in hyperon decay would be further enhanced. The S-wave amplitudes are calculated from baryon-baryon matrix elements by use of partial conservation of axial-vector current,

$$S(\Xi^0 \to \Lambda\pi^0) = \frac{+i}{2F_\pi} \langle \Lambda | \mathscr{L}_{CP} | \Xi^0 \rangle, \quad (14)$$

while for the P waves we use the pole model. (The formulas are given on p. 573 of Marshak, Riazuddin, and Ryan.[13]) The net result, again with use of the experimental real part for S_1, P_1, is

$$S_1 = 34.3[1 + i(0.5 \times 10^{-3})] \times 10^{-8}, \quad (15a)$$

$$P_1 = 13.1[1 - i(0.8 \times 10^{-3})] \times 10^{-8}, \quad (15b)$$

leading to a phase difference

$$\sin(\phi_s^1 - \phi_p^1) = 1.3 \times 10^{-3}. \quad (16)$$

The signal here is the largest of all the models.

There are several versions of left-right symmetric models of CP nonconservation, but the most appealing is that with the "isoconjugate[11] structure" which generates sizable $\Delta S = 1$ CP-odd interaction even though $\epsilon'/\epsilon = 0$ in kaon decay (in the limit of no $W_L - W_R$ mixing). The full $\Delta S = 1$ Hamiltonian has the form

$$Hw = \frac{G_F}{\sqrt{2}} \sin\theta_1 \cos\theta_1 (O_{LL} + \eta e^{i\beta} O_{RR}), \quad (17)$$

where $\eta = Mw_L^2/Mw_R^2$ and O_{LL}, O_{RR} are identical operators except that O_{LL} involves two left-handed currents and O_{RR} has two right-handed currents. Because of this structure one can easily see that all $\Delta S = 1$ parity-conserving processes have an *identical* phase factor $(1 + i\eta\beta)$, while all parity-nonconserving ones have phase $(1 - i\eta\beta)$. In hyperon decay the phase difference is then

$$\sin(\phi_s - \phi_p) = 2\eta\beta \quad (18)$$

for all channels. It remains to extract a value for $\eta\beta$ from ϵ. In the model, the box diagrams[11,12] yield

$$\epsilon = \frac{G_F^2}{12\sqrt{2}\pi^2} \frac{s_1^2 f_K^2 m_K m_C^2}{\Delta M} B \left| 60 C_{LR} \frac{\mathscr{M}_{LR}}{\mathscr{M}_{LL}} \eta\beta \right|, \quad (19)$$

where $C_{LR} \sim 3$ is the quantum chromodynamic correction factor, and

$$\frac{\mathscr{M}_{LR}}{\mathscr{M}_{LL}} = \frac{\langle K^0 | \bar{s}_R d_L \bar{s}_L d_R | \bar{K}^0 \rangle}{\langle K^0 | \bar{s}_L \gamma_\mu d_L \bar{s}_L \gamma^\mu d_L | \bar{K}^0 \rangle}. \quad (20)$$

This last ratio of matrix elements is expected to be in

TABLE I. CP-nonconserving signals as calculated in the text. In the Kobayashi-Maskawa model the maximal value of the mixing angles was assumed.

	Kobayashi-Maskawa	Weinberg Higgs	Left-right	Superweak/heavy neutral Higgs
$(\alpha_\Xi + \bar{\alpha}_{\bar{\Xi}})/(\alpha_\Xi - \bar{\alpha}_{\bar{\Xi}})$	2×10^{-5}	1.3×10^{-4}	0.8×10^{-5}	0
$(\beta_\Xi + \bar{\beta}_{\bar{\Xi}})/(\beta_\Xi - \bar{\beta}_{\bar{\Xi}})$	2×10^{-3}	1.3×10^{-2}	0.8×10^{-3}	0
$\beta_{\Xi^0}/\alpha_{\Xi^0} - \beta_{\Xi^-}/\alpha_{\Xi^-}$	1.6×10^{-5}	0.8×10^{-4}	0	0
$\frac{\Gamma(\Lambda \to n\pi^0) - \Gamma(\bar{\Lambda} \to \bar{N}\pi^0)}{\Gamma(\Lambda \to N\pi^0) + \Gamma(\Lambda \to N\pi^0)}$	4.5×10^{-7}	4×10^{-6}	0	0

the range 3–10. Note the large factor $60 C_{LR} \mathscr{M}_{LR} / \mathscr{M}_{LL}$ which enhances the left-right box diagram. This decreases the size of the signals in the hyperon sector. The value for ϵ implies

$$\eta\beta \approx (\mathscr{M}_{LL}/\mathscr{M}_{LR}) 2.2\times 10^{-4}. \tag{21}$$

In numerical estimates we will use $\mathscr{M}_{LL}/\mathscr{M}_{LR} \approx \frac{1}{5}$.

The results of our calculations are given in Table I. Many of the observables are discouragingly small, but in some cases the signal is quite possibly measurable. At present, these parameters are not known to an accuracy which makes any meaningful test of *CP* nonconservation. However, after a decade in which little new experimental work was performed, there is again interest in the use of hyperons as tests of fundamental physics.[19,20] The favorable case of $(\beta+\bar{\beta})/(\beta-\bar{\beta})$ requires hyperon polarization and therefore is challenging experimentally. The signal for hyperons can probably be adequately analyzed, but at present antihyperon polarization seems difficult to obtain except in the sequential decay of other antihyperons.[20] If this difficulty can be overcome and a sensitive experiment can be designed, the importance of understanding *CP* nonconservation suggests that these measurements should be undertaken.

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