Observation of Aharonov-Bohm Electron Interference Effects with Periods h/e and $h/2e$ in Individual Micron-Size, Normal-Metal Rings

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We report resistance oscillations as a function of magnetic field for individual aluminum and silver thin-film rings, 1–2 μm in diameter, between 1.2 and 10 K. This is the first observation in single rings of oscillations with a flux period of $h/2e$ predicted by Al'tshuler *et al.* Oscillations periodic in h/e are also seen in the 1-μm Ag rings at higher fields. These electron interference effects in metal rings are the solid-state analog of the Aharonov-Bohm effect for electrons in vacuum.

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In recent years there has been major interest, and a dramatic advance, in the understanding of electron transport in random systems, especially those of reduced dimensionality.[1] The early studies of one- and two-dimensional systems verified the picture, originally proposed by Thouless, of localization effects which reduce the conductivity at $T \rightarrow 0$. It is now clear that localization phenomena result from electron interference: Coherent backscattering adds quantum corrections to the classical electron-diffusion results.[1] Localization experiments confirm the striking result that the length over which electrons can retain phase memory and interfere is the inelastic diffusion length l_i, which can be > 1 μm at low temperatures. ($1\ \mu\text{m} = 10^4$ Å.) In contrast, the mean free path l in typical metal films is of order 10 to 100 Å.

Recently there have been predictions[2–5] that explicit, oscillatory electron interference effects should be seen in the macroscopic electrical properties of small metal rings and cylinders in a perpendicular (axial) magnetic field $\mathbf{B}$ $(= \nabla \times \mathbf{A})$. The vector potential $\mathbf{A}$ adds to the quantum phase along a path from a to b by an amount (in mks units)

$$\Delta\phi = 2\pi (e/h) \int_a^b \mathbf{A} \cdot d\mathbf{l}.$$

For electrons in vacuum this leads to the well-known Aharonov-Bohm interference effect[6] for the electron intensity, where the electron travels on either of two paths which enclose a magnetic flux Φ [see Fig. 1(a)]. The flux is given by $\Phi = B \times \text{area} = \oint \mathbf{A} \cdot d\mathbf{l}$. The flux periodicity of the resulting intensity is h/e. Aharonov-Bohm resistance oscillations have been reported in very pure single-crystal cylinders with long mean free paths,[3] with a period of h/e. Observation of such interference effects in a ring of *disordered* metal (with significant elastic scattering) would demonstrate elegantly the connection between macroscopic transport and the essentially quantum-mechanical behavior at the microscopic level.

The first prediction of interference effects in *disordered* normal-metal rings and cylinders was by Al'tshuler *et al.*[2] They predicted that the low-temperature resistance would be a periodic function of the enclosed flux, with a period of $h/2e$. This periodicity results from the interference paths shown in Fig. 1(b).[7] Different theoretical arguments, which start with a model of a closed chain of atoms, also predict a flux periodicity of $h/2e$ for various physical quantities.[5]

Interference effects with a flux periodicity of $h/2e$ have been seen in the resistance of metal cylinders[2,8] and arrays.[9] For single rings, however, numerous experiments to date have failed to find the interference effect predicted by Al'tshuler *et al.,* despite significant experimental effort.[10] A single ring is presumably the simplest theoretical case, and the absence of this interference effect in single rings has led to questions about the theoretical understanding of such phenomena.

A qualitatively different prediction for electron interference effects is given in Ref. 4. The theoretical framework employed is the formalism developed by Landauer. In the work by Büttiker *et al.* the ring is treated as an assemblage of quantum channels and scatterers, with coupling between channels. A quantum interference effect for the resistance is predicted, with a fundamental period of h/e [Fig. 1(a)]. Harmonics of this period may also be possible, but these are *not* equivalent to the effect predicted by Al'tshuler

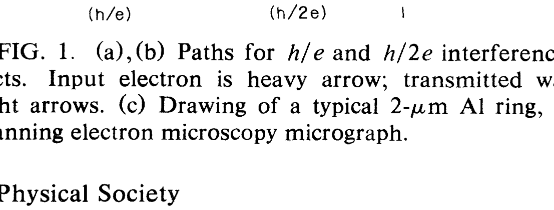

FIG. 1. (a),(b) Paths for h/e and $h/2e$ interference effects. Input electron is heavy arrow; transmitted waves, light arrows. (c) Drawing of a typical 2-μm Al ring, from scanning electron microscopy micrograph.

et al. Very recently, interference effects with a period of h/e have been observed in a single ring below 1 K.[11] h/e interference effects are not expected,[4] and have not been observed, in cylinders or arrays.

The experiments we report here are designed to test the two different predictions for quantum interference. *We observe both predicted quantum interference effects in a single ring, with periods of $h/2e$ and h/e.* These results constitute the first observation in a single ring of the $h/2e$ interference effect predicted by Al'tshuler *et al.* h/e oscillations are observed in the same ring, but with a different field and temperature dependence. This demonstrates clearly that the h/e and $h/2e$ interference effects can occur in the same system, but that these two effects arise from different interference mechanisms.

The systems we have studied are single thin-film rings of aluminum and silver, of diameter 1 or 2 μm, with thickness ~ 200 Å and linewidth between 1000 and 2300 Å. A drawing of a typical ring is given in Fig. 1(c). Samples are patterned with electron-beam lithography and liftoff,[12] by use of an adapted scanning electron microscope. The sample resistance is measured with a four-terminal ac bridge between 1.2 and 10 K, with measuring currents 0.5–10 μA, small enough to avoid self-heating. Sample parameters are given in Table I. Aluminum is a superconductor at temperatures below ~ 1.2 K, so that here it is a normal metal.

The theoretical prediction[2] for the resistance change due to localization effects in a large ring ($2\pi r > l_\phi$) is[2]

$$\frac{\delta R}{R} = -(\beta+\tfrac{1}{2})\frac{l_\phi R_\Box}{W(h/2e^2)}\left[1+\cos\left(\frac{2\pi\Phi}{h/2e}\right)e^{-2\pi r/l_\phi}\right] + \frac{3}{2}\frac{\tilde{l}_\phi R_\Box}{W(h/2e^2)}\left[1+\cos\left(\frac{2\pi\Phi}{h/2e}\right)e^{-2\pi r/\tilde{l}_\phi}\right]. \quad (1)$$

r is the radius of the ring and β is the Larkin electron-electron interaction parameter,[13] related to Maki-Thompson superconducting fluctuations; β diverges as $T \to T_c$. The second term in each bracket is oscillatory, with a flux period of $h/2e$. l_ϕ is the phase-breaking length for the "singlet" term,

$$l_\phi^{-2} = l_i(B)^{-2} + 2l_s^{-2}, \quad (2a)$$

with

$$l_i(B)^{-2} = l_i(0)^{-2} + \tfrac{1}{3}(WeB/\hbar)^2. \quad (2b)$$

The phase-breaking length for the "triplet" term[14] is given by

$$\tilde{l}_\phi^{-2} = l_i(B)^{-2} + \tfrac{4}{3}l_{\mathrm{s.o.}}^{-2} + \tfrac{2}{3}l_s^{-2}.$$

l_s is the diffusion length for magnetic scattering $=(D\tau_s)^{1/2}$, $l_{\mathrm{s.o.}}$ the diffusion length for spin-orbit scattering, D the diffusion constant $=\frac{1}{3}v_F l$, and W the linewidth. The zero-field inelastic length is $l_i(0) = (D\tau_i)^{1/2}$. Inelastic and magnetic scattering processes destroy the phase coherence of the two electron waves, as does application of a magnetic field.

Equation (1) correctly reduces to the one-dimensional localization result[14] when $2\pi r \gg l_\phi$. An expression exactly analogous to Eq. (1) has been shown to provide a quantitative description of the low-field experimental data for cylinders of Li[2] and Al and Mg.[8]

The oscillatory component of Eq. (1) may be rewritten, after some manipulation, as

$$\left[\frac{\delta R}{R}\right]^{h/2e} = -(\beta+\tfrac{1}{2})\frac{0.98}{N}\frac{l_\phi}{l}\left[\cos\left(\frac{2\pi\Phi}{h/2e}\right)e^{-2\pi r/l_\phi}\right] \quad (3)$$

for the case that $l_{\mathrm{s.o.}} \ll l_\phi$. N is the number of atoms in the wire cross section.[4] l is 50–160 Å in the films studied.

The prediction for the h/e interference effect at $T=0$ is[4]

$$\left[\frac{\delta R}{R}\right]^{h/e} \propto \frac{a}{N_{\mathrm{eff}}}\cos\left[\frac{2\pi\Phi}{h/e}+\gamma_0\right]. \quad (4)$$

a is a constant of order unity and γ_0 is a phase factor

TABLE I. Sample parameters. Al films are 250 Å thick, Ag films 150 Å thick. $T_c = 1.2$–1.3 K for the Al rings. $R_\Box$ is at 4.5 K, $l_\phi(0)$ at ~ 1.7 K except for sample E, for which l_ϕ is at 4.5 K. h/e interference was not searched for in samples A and B. $l_{\mathrm{s.o.}}$ is ~ 0.5 μm for all the rings.

Sample	Film	$R_\Box$ (Ω)	Diam (μm)	W (μm)	$l_\phi(B=0)$ (μm)	h/e effect
A	Al	1.1	2.3	0.19	1.7	. . .
B	Al	1.6	2.3	0.23	2.0	. . .
C	Al	3.0	1.1	0.19	1.0	?
D	Ag	2.5	1.0	0.14	0.9	yes
E	Ag	1.6	1.0	0.14	0.7	yes

which may be different for each ring. Equation (4) may represent a lower bound.[4] The relation of $N_{\rm eff}$ to N is discussed in Ref. 4. For our experimental regime, significant averaging (reduction) of the result of Eq. (4) occurs,[4] though a full theoretical prediction is not yet available. When $l_i < \pi r$ we anticipate that Eq. (4) must at least be multiplied by $\exp(-\pi r/l_i)$. Unlike the case of Eq. (3), here l_i is apparently the *zero-field* inelastic length, as the experimentally observed h/e interference effects are not significantly attenuated in large magnetic fields. For the h/e interference effect, the restrictions of **k**-space scattering paths are apparently quite different from the restrictions on the $h/2e$ interference term of Al'tshuler *et al.*[1] The effects of spin-orbit and magnetic scattering for h/e interference are not yet known.

Experimental results for the magnetoresistance of a 2.3-μm-diam aluminum ring at 1.7 K are shown in Fig. 2. The length $l_\phi(0)$ determined from fitting of the oscillatory curve in Fig. 2 is 1.7 μm. This fit uses the full theoretical expression, without the restriction applicable to Eq. (1) that $l_\phi < 2\pi r$. High-field magnetoresistance data for a similar ring are shown in the inset. For sample B, data are available at 4.5 K which allow extraction of l_ϕ by fitting of either the low-field oscillations or the high-field data. These values of l_ϕ at 4.5 K, ~ 1.2 μm, agree to 20%. Values of $l_\phi(0)$ derived from fitting of the low-field oscillations are also in good agreement with results from previous experiments on aluminum films[15] and wires.[14] The high-field magnetoresistance, at low temperatures only, cannot be satisfactorily fitted. This appears to be an unresolved problem with the one-dimensional theory as applied to short samples ($l_\phi \sim \pi r$). This issue deserves further study. At 4.5 K, $h/2e$ oscillations are still clearly observed. Here $\beta = 0.7$, so that at this temperature the *localization* contribution to $\delta R/R$ is significant. Above 4.5 K, interference effects are too small to be resolved in 2-μm rings. In all eleven rings studied, interference effects with a period of $h/2e$ are observed at low fields.

Experimental results for a 1-μm Ag ring are shown in Fig. 3. Oscillations of period $h/2e$ are seen at low fields (see below); they are observable to 8 K. At higher fields ($B > 250$ G) we observe clear oscillations with a period of h/e ($\Delta B = 52$ G), with a magnitude 6×10^{-5} at 1.7 K. These oscillations persist to a field of at least 1 kG. The h/e oscillations are observed up to 10 K, dying out with an increasing temperature somewhat faster than T^{-1}. The weakening or beating of the oscillations at $B \sim 650$ G is also seen at higher temperatures. Such beating is also seen in the experiments of Ref. 11 and in simulations by Stone and Imry.[4]

The low-field data for the Ag ring in Fig. 3 cannot be fully fitted with the $h/2e$ theory of Al'tshuler *et al.* alone. However, if one assumes a reasonable h/e contribution at low fields (shown by the dotted line) the predicted total $\delta R/R$ fits the data rather well. (The h/e contribution is assumed to be damped after a few periods, consistent with the "beating" pattern seen at high fields.) This fitting is not taken to prove definitively the existence of h/e interference at low fields. What this fitting does show is that the Al'tshuler *et al.* $h/2e$ oscillations *are* clearly evident in the data. The fitted $h/2e$ period is 24 G, corresponding to $r = 0.53$ μm. A Fourier analysis of the high-field data shows a clear peak at a field period nearly, but not exactly, twice this value: $\Delta B = 52$ G, for $r_{\rm eff} = 0.50$ μm. This

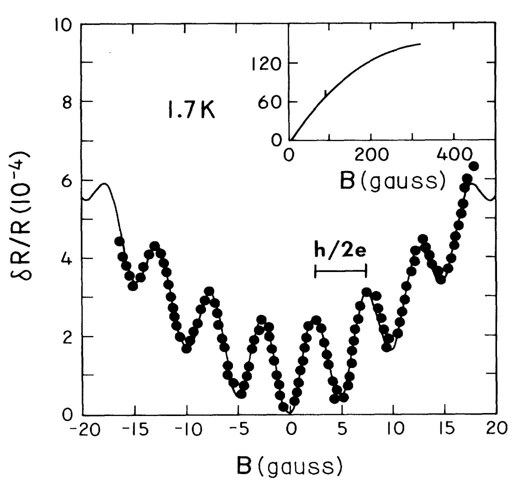

FIG. 2. Magnetoresistance of 2.3-μm aluminum ring, sample A. Solid curve is theoretical fit with the parameters listed in Table I and $\beta = 5.3$. Inset: High-field magnetoresistance of sample B.

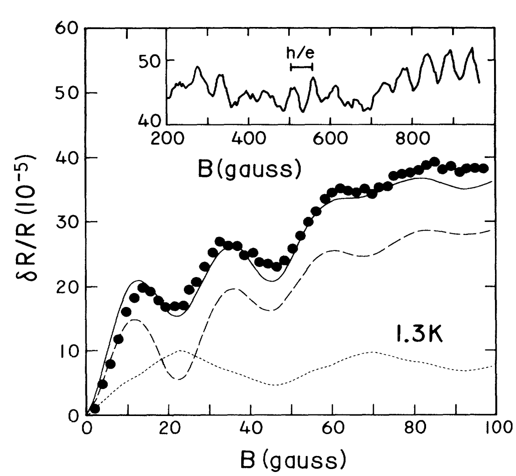

FIG. 3. Magnetoresistance of a 1-μm-diam Ag ring, sample D. Heavy dots show the experimental low-field oscillations. Dashed line, Al'tshuler $h/2e$ contribution; dotted line, assumed h/e term; solid line, sum of these two terms. $h/2e$ period is ~ 24 G. Inset: High-field magnetoresistance, showing h/e oscillations, period ~ 52 G (see text).

~ 5% difference in effective radius may indicate a different mode of averaging of the flux in the metal for the h/e interference, and should be addressed in future studies.

In the 1-μm Al ring, sample C, h/e oscillations cannot be resolved with certainty. The effect may be larger in the silver rings because the Ag film in the region of the ring is somewhat granular, as in a strongly conducting percolation system. The effective linewidth may thus be smaller than the lithographically defined linewidth. In any case, the h/e interference effects we observe in Ag rings must arise from a path encircling the ring itself, as no other path encloses a sufficiently large flux.

The Al'tshuler *et al.* theory for $h/2e$ oscillations in single disordered metal rings is thus clearly confirmed at low fields in our experiments. The *absence* of this effect for the rings studied in Refs. 10 and 11 may be due to magnetic scattering, for example, or may have resulted from insufficient measurement sensitivity.

Our work points toward a number of issues which need to be resolved to better understand the physical mechanisms for the two observed interference effects. It is clearly desirable to produce a theoretical prediction for *both* types of interference effects within a *single* theoretical framework. This would illuminate more clearly the **k**-space restrictions on h/e scattering. The length scale for damping of the h/e effect needs to be established more rigorously, and justified physically. Experimentally it is clear that the h/e and the $h/2e$ effects have different dependences on the transport scattering processes. Finally, the case of strong quantum interference, when $l_\phi > \pi r$, needs to be explored. Samples in this size regime in fact constitute lithographically produced, quasiatomic systems, with the possibility of even more novel behavior.[16]

In conclusion, our work demonstrates unambiguously that single rings do exhibit electron quantum interference effects, *with a flux period of h/2e at low fields, and with a flux period of h/e clearly visible at higher fields.* Our results provide strong evidence that these two interference effects, which are seen in the same ring, arise from *different* interference mechanisms. Further studies are desirable to advance the physical understanding of the theoretical predictions.

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