Magnetic Field Dependence of Activation Energies in the Fractional Quantum Hall Effect

G. S. Boebinger
Department of Physics and Francis Bitter National Magnet Laboratory, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139

A. M. Chang[a]
AT&T Bell Laboratories, Holmdel, New Jersey 07733

H. L. Stormer[a]
AT&T Bell Laboratories, Murray Hill, New Jersey 07974

and

D. C. Tsui[a]
Department of Electrical Engineering and Computer Science, Princeton University, Princeton, New Jersey 08544
(Received 19 July 1985)

We have studied the temperature dependence of the fractional quantum Hall effect at Landau-level filling factors $\nu=\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, $\frac{2}{5}$, and $\frac{3}{5}$ in magnetic fields up to 28 T to determine the magnitude of the associated energy gaps. The data suggest a single activation energy for $\nu=\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, and $\frac{5}{3}$. Its magnitude, much smaller than predicted by current theories, vanishes for $B \lesssim 6$ T and saturates at $B \gtrsim 18$ T. The data also suggest a single activation energy for $\nu=\frac{2}{5}$ and $\frac{3}{5}$ which is smaller than predicted.

PACS numbers: 72.20.My, 73.40.Lq, 73.60.Fw

The fractional quantum Hall effect, FQHE, is observed in high-mobility ($\mu \gtrsim 100\,000$ cm^2/V-s) two-dimensional electron systems at low temperatures ($T \lesssim 4$ K) and high magnetic fields ($B \gtrsim 5$ T).[1–7] The FQHE is phenomenologically similar to the integral quantum Hall effect, IQHE: Plateaus are observed in the Hall resistivity, ρ_{xy}, concomitant with minima in the diagonal resistivity, ρ_{xx}. While the IQHE exists at magnetic fields corresponding to integral Landau-level filling, ν, the FQHE is observed at fractional Landau-level filling $\nu=p/q$ where q is always odd ($\nu=nh/eB$, $n=$area density, and eB/h =Landau-level degeneracy). In both cases, observation of a zero-resistance state implies the existence of a gap in the excitation spectrum of the system. Measurements of the activation energy in the IQHE reproduce closely the Landau-level splitting representing the gaps in the single-particle density of states of the system.[8] The FQHE is of many-particle origin and, hence, activation-energy data on ρ_{xx} in a given fractional state is expected to provide a measure of the size of the gap in the excitation spectrum of the correlated electronic ground state.

Theories have been developed assessing the nature of the underlying electronic state. In particular, a model due to Laughlin interprets the FQHE as the signature of an incompressible quantum fluid at fractional filling factors $\nu=p/q$ with q odd.[9–12] At finite temperatures, it predicts thermal excitation of fractionally charged quasielectrons and quasiholes across an energy gap above the ground state. The magnitude of the gap depends on ν and is expected to scale with magnetic field as e^2/l_0, where $l_0=(\hbar c/eB)^{1/2}$ is the magnetic length, the only relevant length scale in the system. Although many interesting properties of the electron liquid can be deduced from the existing model, presently the only experimentally accessible quantity is the size of the energy gap associated with a given fractional state.

In this Letter we report the determination of the magnetic field dependence and the relative magnitude of the activation energies of the FQHE at $\nu=\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{5}{3}$, and $\nu=\frac{2}{5}$, $\frac{3}{5}$. The four specimens are modulation-doped GaAs/AlGaAs heterostructures with typical mobilities $\mu=5000\,000$–$850\,000$ cm^2/V-s and electron densities $n=(1.5$–$2.3)\times10^{11}$ cm^{-2}. The electron densities are tunable by a backside gate bias which also affects the sample mobility.[13] Standard Hall bridge specimens were used to measure ρ_{xx} in samples A and B, while quasi Corbino-geometry specimens were used to measure the diagonal conductance σ_{xx} in samples C and D. Near the ρ_{xx} minima, $\rho_{xx} \ll \rho_{xy}$ and thus σ_{xx} and ρ_{xx} differ only by a constant factor: $\sigma_{xx}=\rho_{xx}/(\rho_{xx}^2+\rho_{xy}^2)\sim\rho_{xx}/\rho_{xy}^2$. A specifically designed dilution refrigerator was used which reached 64 mK in a hybrid magnet at 28.6 T. The specimens were immersed in the dilute ^{3}He-^{4}He liquid near a carbon resistor thermometer. No significant temperature hysteresis was observed during temperature sweeps and the slight magnetoresistance of the carbon resistor was determined as in Naughton *et al.*[14]

The temperature dependences of ρ_{xx} and σ_{xx} have been previously interpreted as activation energies in the FQHE.[7,13,15] The value of ρ_{xx} or σ_{xx} at the minimum corresponding to a given fractional factor is determined as a function of temperature from 120 mK to 1.4 K. Figure 1 shows such graphs for $\nu=\frac{2}{3}$ at two different magnetic fields.

The data of Fig. 1(a) follow a straight line, indicating activated conduction. The activation energy, Δ, is determined from $\rho_{xx}=\rho_0\exp(-\Delta/2T)$. As defined here, Δ represents the quasiparticle pair-creation energy. (Note that this definition of Δ differs from the activation energies defined in Refs. 7, 13, and 15 by a factor of 2.) At higher T, ρ_{xx} deviates from a simple activated dependence as a result of the weak minimum riding on a slightly temperature-dependent background. All data taken at magnetic fields between 6 and ~ 10 T indicate simple activated behavior. Data taken at $B \gtrsim 10$ T deviate from simple activated dependence also at the lowest temperatures.

Figures 1(b) and 1(c) show the two qualitatively different types of low-temperature behavior. The majority (eleven out of fourteen) of our high-field data resemble Fig. 1(b) in which the deviation is smooth and curved. Data like those in Fig. 1(b) fit very well over the entire temperature range to a sum of activated conduction at higher T and hopping conduction at lower T [solid curve in Fig. 1(b)]. This dependence suggests that the quasiparticles in the FQHE become localized at low temperatures, in analogy to the localization of electrons in the IQHE.[16,17]

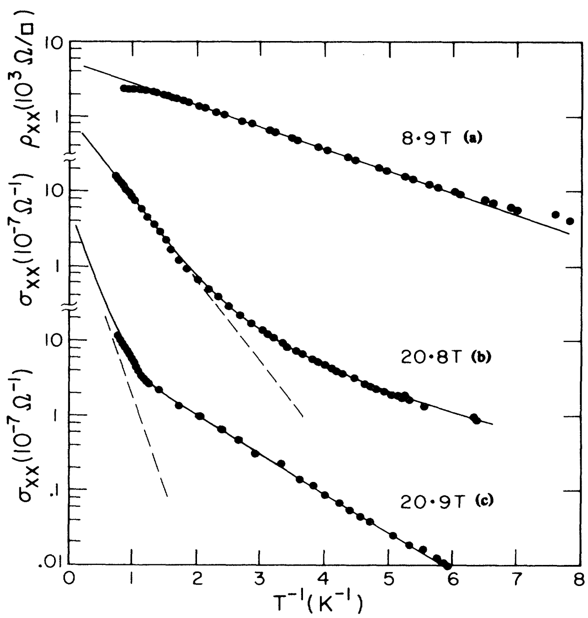

FIG. 1. Temperature dependence of the minimum at $\nu=\frac{2}{3}$. (a) at $B=8.9$ T, showing simple activated behavior; (b) at $B=20.8$ T, showing the smooth, curved deviation from activated behavior at lower temperatures; (c) at $B=20.9$ T, showing the sharp, linear deviation.

The formula used to model the hopping conduction in a magnetic field is from Ono,[18] $\sigma=\sigma_0(T)\times\exp[-(T_0/T)^{1/2}]$, although the two-dimensional Mott variable-range hopping formula,[19] $\sigma=\sigma_0(T)\times\exp[-(T_0/T)^{1/3}]$, fits the data as well. The data could not be fitted if we assumed only hopping conduction over the entire temperature range. Ihm and Phillips[20] have suggested the existence of a second activation energy in the FQHE, due to excitations of electrons which, in the presence of potential fluctuations, have not condensed into the Laughlin quantum liquid. Attempts to fit the data by use of two activation energies are equally successful. In any case, the activation energy at higher T is only slightly dependent on the formula chosen to fit the lower-T data. This dependence is within the error bars given in Fig. 2.

Three sets of data, all from sample D, resemble Fig. 1(c) in which the deviation from simple activated behavior is a sharp break to a second linear region. This behavior is similar to that reported by Kawaji *et al.*[22] The existence of two linear regions does suggest two separate activation energies, Δ_1 and Δ_2. The linear regions of the data are best described by $\Delta_1=6.4$ K and $\Delta_2=1.4$ K. However, a plot of their sum,

$$\sigma_{xx}=\sigma_1\exp(-\Delta_1/2T)+\sigma_2\exp(-\Delta_2/2T),$$

does not adequately fit the sharp break in the data. A

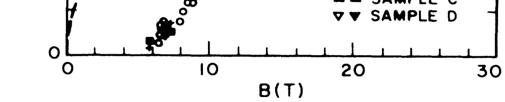

FIG. 2. Activation energies for the minima of $\nu=\frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, $\frac{5}{3}$ vs magnetic field. Open symbols indicate data from $\nu=\frac{2}{3}$. Filled symbols at $B=5.9$ and 7.3 T are from $\nu=\frac{5}{3}$ and $\frac{4}{3}$, respectively. All other filled symbols are from $\nu=\frac{1}{3}$. The dashed line is given by $0.03e^2/\epsilon l_0$. Data from sample A are from Ref. 13. The four data points shown by plusses are from Refs. 7 and 22.

successful least-squares fit using the above equation yields $\Delta_1 = 11.4$ K and $\Delta_2 = 2.4$ K [solid curve in Fig. 1(c)]. However, Δ_1 and Δ_2 exhibit wide variations apparently uncorrelated with B. (At $B = 19.3$ T, $\Delta_1 = 4.2$ K and $\Delta_2 = 1.4$ K. At $B = 26.8$ T, $\Delta_1 = 6.0$ K and $\Delta_2 = 1.8$ K.) In view of the fact that data sets containing a sharp break were the exception and are uncorrelated with magnetic field, we regard them as an artifact probably caused by a nonequilibrium configuration of the electronic state within the sample. An additional observation is consistent with this speculation: A second set of data taken immediately after the data in Fig. 1(c) but with a slower cooling rate follows a smooth curve similar to Fig. 1(b).

Figure 2 presents the activation energies from the data on $\nu = \frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, and $\frac{5}{3}$. The three atypical sets of data resembling Fig. 1(c) have not been included. The data from Kawaji *et al.*[22] are not included because they also resemble Fig. 1(c). Four features of Fig. 2 should be stressed:

(1) There is no apparent sample dependence among these samples of similar mobility.

(2) The data for $\nu = \frac{1}{3}$ and $\frac{2}{3}$ overlap at $B \sim 20$ T. Also, the data for $\nu = \frac{4}{3}$ and $\frac{5}{3}$ are consistent with the data for $\nu = \frac{2}{3}$ at similar magnetic fields. This suggests a single activation energy, ${}^3\Delta$, for all of the filling factors: $\nu = \frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, and $\frac{5}{3}$.

(3) The observed activation energies are much smaller than theoretically predicted. These theories all yield quasiparticle pair-creation energies for $\nu = \frac{1}{3}$ and $\frac{2}{3}$ of the form $\Delta = Ce^2/\epsilon l_0$, where $\epsilon \sim 12.8$ is the dielectric constant of GaAs. The constant of proportionality, C, is model dependent. From hypernetted-chain calculations, Laughlin determines[12] $C = 0.056$ and Chakraborty determines $C = 0.053$.[23] Monte Carlo calculations by Morf and Halperin give $C = 0.094$.[24] From calculations on finite numbers of electrons, Haldane has extrapolated to $N \to \infty$ to yield $C = 0.105$.[25] A single-mode approximation, in analogy with Feynman's theory for ^{4}He by Girvin, MacDonald, and Platzman yields $C = 0.106$.[26] To compare these results with our experimental data, Fig. 2 contains a curve of ${}^3\Delta$ vs B for $C = 0.030$, almost a factor of 2 smaller than the lowest theoretical value.

It has been suggested that the quasiparticle-quasihole interactions can result in bound states. Laughlin[27] discusses a quasiexciton state with a minimum energy equivalent to $C = 0.014$ at $kl_0 = 0$, where k is the wave vector of the quasiexciton. However, this calculation is unreliable for $k \to 0$. Haldan and Rezayi[25] and Girvin, MacDonald, and Platzman[26] find a roton minimum, analogous to the roton minimum in superfluid ^{4}He, at $kl_0 \sim 1.4$, where $C \sim 0.075$ for both. These activation energies, with the exception of Laughlin's quasiexciton, lie well above the observed values. It is not clear, however, that the electrically neutral bound states would be observed in magnetotransport measurements.

(4) ${}^3\Delta$ vs B does not follow the predicted $B^{1/2}$ magnetic field dependence. Rather, the phenomenon has a finite threshold at $B \sim 5.5$ T. For higher magnetic fields, there is a roughly linear increase in ${}^3\Delta$ up to $B \sim 18$ T, followed by an apparent saturation of ${}^3\Delta \sim 5.2$ K for $B \gtrsim 18$ T.

For completeness, we have also studied the temperature dependence of ρ_{xx} at $\nu = \frac{2}{5}$ and $\frac{3}{5}$ for 14 T $\lesssim B \lesssim 28$ T. Within our temperature range, ρ_{xx} changes by less than an order of magnitude. The data deviate from simple activated behavior but can be fitted with activated conduction at higher T and any of the discussed models at lower T. Attempts to fit the data over the entire temperature range with a hopping conduction formula were unsuccessful. The fifteen sets of experimental data again suggest a single activation energy, ${}^5\Delta$, for $\nu = \frac{2}{5}$ and $\frac{3}{5}$, which varies monotonically from ${}^5\Delta \sim 1.4$ K at $B = 14$ T to ${}^5\Delta \sim 2.5$ K at $B = 28$ T. Halperin[11] estimates that the pair-creation energies at $\nu = p/q$ should scale as $q^{-2.5}$, which yields ${}^5\Delta \sim 0.28\,{}^3\Delta$, and $C \sim 0.015$–0.030 for $\nu = \frac{2}{5}$ and $\frac{3}{5}$. This corresponds to ${}^5\Delta \gtrsim 2.9$ K and $\gtrsim 4.0$ K at $B = 14$ and 28 T, respectively. A more extensive description of these observations at $\nu = \frac{2}{5}$ and $\frac{3}{5}$ will be given in a forthcoming article.

There exists a startling discrepancy between the experimental results and the theoretical calculations of C and the magnetic field dependence of ${}^3\Delta$ which remains to be explained. A reduction of the many-particle gap due to disorder and subsequent thermal excitation to a mobility edge provides a qualitative explanation for the reduced values of C as well as for the finite threshold field.[13] Recent theoretical work is attempting to assess quantitatively the effects of disorder and finite thickness of the two-dimensional electron system on the energy gaps in the FQHE.[28] Initial results from these calculations arrive at an average Δ which approaches the experimental results; however, the observed B dependence is not well reproduced.

In conclusion, we find a single activation energy, ${}^3\Delta$, for $\nu = \frac{1}{3}$, $\frac{2}{3}$, $\frac{4}{3}$, and $\frac{5}{3}$ in magnetic fields up to 28 T. ${}^3\Delta$ is much smaller than expected and does not exhibit the expected $B^{1/2}$ magnetic field dependence. Instead, ${}^3\Delta$ has a finite magnetic field threshold above which it has a roughly linear increase with magnetic field. ${}^3\Delta$ apparently saturates at the highest magnetic fields. We also find a single activation energy, ${}^5\Delta$, for $\nu = \frac{2}{5}$ and $\frac{3}{5}$. ${}^5\Delta$ is also much smaller than predicted. The discrepancies between these experimental results and the theoretical values remain to be explained.

We thank P. A. Wolff and L. Rubin for support at the National Magnet Laboratory; J. C. M. Hwang for growth of the molecular-beam-epitaxy crystals; and R. B. Laughlin, S. M. Girvin, and J. C. Phillips for discussions. One of us (G.S.B.) is supported by the Hertz Foundation, and one of us (D.C.T.) is supported by the National Science Foundation through Grant No. DMR-8212167. The Francis Bitter National Magnet Laboratory is supported by the National Science Foundation through its Division of Materials Research.

[(a)]Guest Scientist at the Francis Bitter National Magnet Laboratory, Cambridge, Mass. 02139.

[1]D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. Lett. **48**, 1559 (1982).

[2]H. L. Stormer, A. Chang, D. C. Tsui, J. C. M. Hwang, A.C. Gossard and W. Wiegmann, Phys. Rev. Lett. **50**, 1953 (1983).

[3]A. M. Chang, P. Berglund, D. C. Tsui, H. L. Stormer, and J. C. M. Hwang, Phys. Rev. Lett. **53**, 997 (1984).

[4]G. S. Boebinger, A. M. Chang, H. L. Stormer, and D. C. Tsui, to be published.

[5]E. E. Mendez, M. Heiblum, L. L. Chang, and L. Esaki, Phys. Rev. B **28**, 4886 (1983).

[6]E. E. Mendez, L. L. Chang, M. Heiblum, L. Esaki, M. Naughton, K. Martin, and J. Brooks, Phys. Rev. B **30**, 7310 (1984).

[7]G. Ebert, K. von Klitzing, J. C. Maan, G. Remenyi, C. Probst, G. Weimann, and W. Schlapp, J. Phys. C **17**, L775 (1984).

[8]For example, B. Tausenfreund and K. von Klitzing, Surf. Sci. **142**, 220 (1984).

[9]R. B. Laughlin, Phys. Rev. Lett. **50**, 1395 (1983).

[10]F. D. M. Haldane, Phys. Rev. Lett. **51**, 605 (1983).

[11]B. I. Halperin, Phys. Rev. Lett. **52**, 1583 (1984).

[12]R. B. Laughlin, Surf. Sci. **142**, 163 (1984).

[13]A. M. Chang, M. A. Paalanen, D. C. Tsui, H. L. Stormer, and J. C. M. Hwang, Phys. Rev. B **28**, 6133 (1983), and unpublished data.

[14]M. J. Naughton, S. Dickinson, R. C. Samaratunga, J. S. Brooks, and K. P. Martin, Rev. Sci. Instrum. **54**, 1529 (1983).

[15]D. C. Tsui, H. L. Stormer, J. C. M. Hwang, J. S. Brooks, and M. J. Naughton, Phys. Rev. B **28**, 2274 (1983).

[16]D. C. Tsui, H. L. Stormer, and A. C. Gossard, Phys. Rev. B **25**, 1405 (1982).

[17]G. Ebert, K. von Klitzing, C. Probst, E. Schuberth, K. Ploog, and G. Weimann, Solid State Commun. **45**, 625 (1983).

[18]Y. Ono, J. Phys. Soc. Jpn. **51**, 237 (1982).

[19]N. F. Mott and E. A. Davis, *Electronic Processes in Non-Crystalline Materials* (Clarendon, Oxford, 1979), 2nd ed.

[20]J. Ihm and J. C. Phillips, J. Phys. Soc. Jpn. **54**, 1506 (1985).

[21]V. M. Pudalov and S. G. Semenchinsky, Solid State Commun. **52**, 567 (1984).

[22]S. Kawaji, J. Wakabayashi, J. Yoshino, and H. Sakaki, J. Phys. Soc. Jpn. **53**, 1915 (1984).

[23]T. Chakraborty, Phys. Rev. B **31**, 4026 (1985).

[24]R. Morf and B. I. Halperin, to be published.

[25]F. D. M. Haldane and E. H. Rezayi, Phys. Rev. Lett. **54**, 237 (1985).

[26]S. M. Girvin, A. H. MacDonald, and P. M. Platzman, Phys. Rev. Lett. **54**, 581 (1985).

[27]R. B. Laughlin, Physica (Amsterdam) **126B**, 254 (1984).

[28]A. H. MacDonald, S. M. Girvin, and P. M. Platzman, Bull. Am. Phys. Soc. **30**, 301 (1985), and to be published.