Fluctuation Effects in Hot Quark Matter: Precursors of Chiral Transition at Finite Temperature

T. Hatsuda
Department of Physics, Kyoto University, Kyoto 606, Japan

and

T. Kunihiro
Department of Natural Sciences, Ryukoku University, Kyoto 612, Japan
(Received 2 May 1985)

Fluctuations of the order parameter of chiral transition in a hot and dense quark gas are examined in the random-phase approximation with the use of a QCD-motivated effective Lagrangean. We show that there arise soft modes having a large strength and a narrow width above the critical temperature, which are analogous to the fluctuations of the order parameter in a superconductor above the critical point. It is argued that the modes contribute to the cooling of the quark-gluon plasma.

PACS numbers: 12.35.Cn, 05.40.+j, 11.30.Rd

In recent years, much effort has been devoted to the study of thermodynamic properties and the phase transitions of the quark-gluon system at finite temperature T and chemical potential μ.[1] Such investigations enable us to understand the structure of the QCD vacuum and the nature of the quark-gluon plasma (QGP) which is expected to be realized in an intermediate stage of ultrarelativistic nucleus-nucleus collisions, the interior of neutron stars, and the early universe.

This Letter deals with the dynamical phenomena related to chiral transition of hot and dense quark matter; we examine fluctuations of the order parameter in the Wigner phase and discuss how they affect physical quantities.

Although it is now controversial as to what is the most essential mechanism of chiral transition in QCD,[1] we adopt the following Nambu-Jona-Lasinio (NJL) type of Lagrangean[2] as an effective one which incorporates the essential feature of chiral transition and works well in the intermediate scale between chiral symmetry breaking and confinement;

$$\mathscr{L}=\bar{\psi}(i\partial-\hat{m})\psi+K[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\tau\psi)^2-(\bar{\psi}\tau\psi)^2-(\bar{\psi}i\gamma_5\psi)^2], \tag{1}$$

where we have confined ourselves to $N_f=2$ and $N_c=3$, and $\hat{m}$ denotes an averaged value of the current mass of u and d quarks. Note that the interaction part of (1) has a form of a local version of the determinational interaction induced by instantons in the $T=\mu=0$ world.[3] Using (1), we have already examined the vacuum and collective excitations in the $T=\mu=0$ system and shown that (1) reproduces the relations among the physical quantities such as the Goldberger-Treiman relation, one of the current algebra relations $f_\pi^2 m_\pi^2=-\hat{m}\langle\bar{\psi}\psi\rangle$ and so on.[4] All free parameters included in (1), the coupling strength K and the momentum cutoff Λ, were determined so that the pion decay constant f_π and the pion mass m_π are reproduced within the lowest order of the chiral perturbation; the results are $K=0.13\ \mathrm{fm}^2$ ($0.092\ \mathrm{fm}^2$) and $\Lambda_4=1007$ MeV ($\Lambda_3=825$ MeV) for the four- (three-) momentum cutoff scheme. The dynamical quark mass M_D and the vacuum condensate of u and d quarks ($\langle\bar{u}u\rangle=\langle\bar{d}d\rangle$) generated by (1) are 240 MeV and (-249 MeV)[3] respectively, for these parameters. Here it should be noted that Λ_4 characterizes the scale of chiral transition[4] and the value of it is larger than the scale of confinement ($\Lambda_{\mathrm{QCD}}\simeq 200$ MeV); this fact coincides with the picture of the two-scale model at $T=0$.[1,5] In the following, we explore the system at finite T and μ using the parameters thus fixed. (We adopt the three-momentum cutoff scheme in this paper for calculational conveniences.)

The dynamical quark mass $M_D(T,\mu)$ depending both on T and μ can be determined by the self-consistency condition in the Hartree-Fock theory at finite temperature[4,6];

$$M_D(T,\mu)=-2g\langle\langle\bar{\psi}\psi\rangle\rangle, \tag{2}$$

where g denotes the effective coupling strength including the contribution from the Fock term, $g=K(1+1/2N_c)$, and the double bracket denotes a statistical average. From now on, we neglect the current quark mass ($\hat{m}$); the effect of finite $\hat{m}$ will be examined later. The right-hand side of (2) may be evaluated with the temperature (Matsubara) Green's function[7] $\mathscr{G}^0(\tau,\mathbf{x})$,

$$\langle\langle\bar{\psi}\psi\rangle\rangle=\lim_{\tau\to 0}\mathrm{Tr}\,\mathscr{G}^0(\tau,\mathbf{0}). \tag{3}$$

Here, the Fourier transform of $\mathscr{G}^0(\tau, \mathbf{x})$ is written as

$$\mathscr{G}^0(\nu_n^{\mathrm{F}}, \mathbf{p}) = [p_+ - M_D(T,\mu) - \hat{m}]^{-1}, \tag{4}$$

where $p_+^\mu = (i\nu_n^{\mathrm{F}} + \mu, \mathbf{p})$ with ν_n^{F} being the Matsubara frequency for fermions, $(2n+1)\pi T$. The condition that the nontrivial solution of (2) should vanish gives a critical line $T = T(\mu)$ in the phase diagram, which is shown in Fig. 1 for $M_D(T=\mu=0) = 240$ MeV. Above (below) the critical line, the system is in the Wigner (Nambu-Goldstone) phase. One sees that the critical temperature T_χ and the critical chemical potential μ_χ are 164 and 289 MeV, respectively. It can be shown that both T_χ and μ_χ tend to increase for finite $\hat{m}$, larger numbers of flavor and/or a larger dynamical mass $M_D(T=\mu=0)$. Note that our T_χ is consistent with the value deduced from the lattice Monte Carlo simulations.[8]

We are now in a position to examine collective excitations in the system. For this purpose, let us calculate the correlation function of a specific pair operator with the quantum number $(J^P, I) = (0^+, 0)$ or the fluctuation of the order parameter in the Wigner phase;

$$D(\omega, \mathbf{q}) = \mathscr{F}(\langle\langle T\bar{\psi}(x)\psi(x)\bar{\psi}(0)\psi(0)\rangle\rangle), \tag{5}$$

where $\mathscr{F}$ denotes Fourier transform. Owing to analytic properties of $D(\omega, \mathbf{q})$ and as a manifestation of the dissipation-fluctuation theorem, $\mathrm{Im}D(\omega, \mathbf{q})$ turns out to be proportional to the so-called strength function $S(\omega, \mathbf{q})$ which is the measure of the excitation strength of collective modes

$$S(\omega, \mathbf{q}) = -(1/\pi)2g\,\mathrm{Im}D^R, \tag{6}$$

where D^R is the retarded Green's function which is obtained through an analytic continuation of the temperature Green's function $\mathscr{D}(\nu_n^B, \mathbf{q})$ $(\nu_n^B = 2n\pi T)$ on account of the Abrikosov-Gor'kov-Dzyaloshinskii-Fradkin theorem.[9] The correlation function of the pionlike pair operator $\bar{\psi} i\gamma_5 \boldsymbol{\tau}\psi$ is also proportional to $S(\omega, \mathbf{q})$ since we are considering the chirally symmetric phase (recall that we are now neglecting the current quark mass $\hat{m}$). Evaluating $\mathscr{D}(\nu_n^B, \mathbf{q})$ in the ring approximation, we get

$$2g\,\mathrm{Im}D^R = \mathrm{Im}(1 + 2g\Pi^R)^{-1}, \tag{7}$$

with

$$\Pi^R(\omega, \mathbf{q}) = \mathrm{Tr}\int \frac{d^4p}{(2\pi)^4} \tanh\frac{p^0}{2T}[G^A(p^0-\omega, \mathbf{p}-\mathbf{q})\mathrm{Im}G^R(p^0, \mathbf{p}) + G^R(p^0+\omega, \mathbf{p})\mathrm{Im}G^R(p^0, \mathbf{p}-\mathbf{q})], \tag{8}$$

where $p^\mu = (p^0, \mathbf{p})$ and G^R (G^A) is the retarded (advanced) single-particle Green's function for massless quarks.

$$G^R(p^0, \mathbf{p}) = (\tilde{p} + i\eta\,\mathrm{sgn}\tilde{p}^0)^{-1}, \tag{9}$$

with $\tilde{p}^\mu = (\tilde{p}^0, \mathbf{p}) = (p^0 + \mu, \mathbf{p}), \eta > 0$, and $G^A = (G^R)^*$. $S(\omega, \mathbf{q})$'s computed for several temperatures at $\mu = 0$ are shown in Fig. 2, from which one can see the following points: (i) There is a sharp peak with a narrow width (10 ~ 60 MeV) above T_χ which implies the existence of (degenerate) elementary excitations, and the width vanishes at the critical point, (ii) the modes soften as the system approaches the critical point, and (iii) they well survive up to $T \simeq 200$ MeV, fairly above T_χ. The qualitative features do not change for $\mu \neq 0$. The enhancement of long-range correlations,

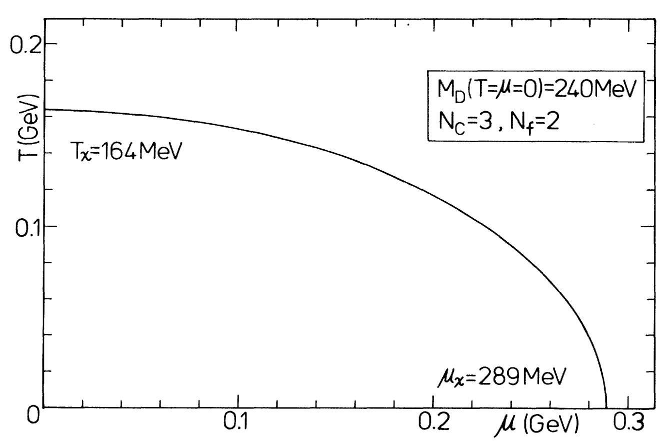

FIG. 1. Critical line calculated by use of the parameters which reproduce f_π (= 93 MeV) and m_π (= 140 MeV) in the lowest order of the chiral perturbation.

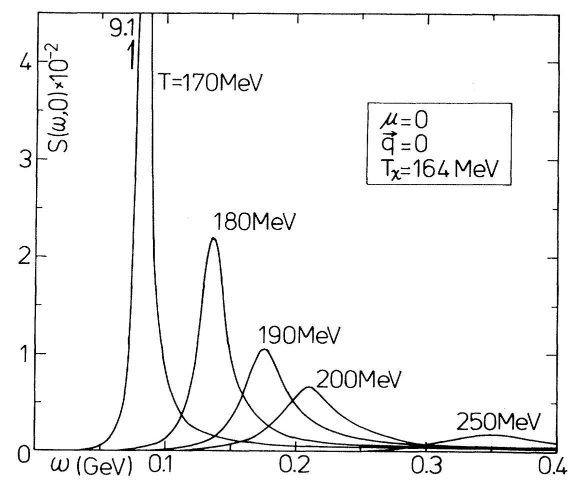

FIG. 2. Strength function at zero momentum transfer $(\mathbf{q}=0)$ above the critical temperature $T_\chi = 164$ MeV with $\mu = 0$. The shape and the peak position of $S(\omega, \mathbf{q})$ for $\mathbf{q} \neq 0$ as a function of $\omega^2 - \mathbf{q}^2$ hardly change from those of $S(\omega, \mathbf{0})$.

equivalently the existence of soft modes, is a direct reflection of large fluctuations of the pair operators near the critical point. The width of the modes, which arises within the (time-dependent) mean-field approximation, is caused by the Landau damping[10] and the width is found to be narrower than that of the soft modes in the $T=\mu=0$ world.[4] The narrowness can be accounted for by the Pauli blocking effect: The particle states of quarks and antiquarks into which collective modes might decay are partly occupied with $T\neq 0$ and/or $\mu \neq 0$ due to the thermal excitations and/or the finite density; hence the decay rates are suppressed.

To examine how finite $\hat{m}$ affects the above results, we show in Fig. 3 the dynamical mass $M=M_D(T,\mu)+\hat{m}$ calculated self-consistently and the real parts of the masses of σ- and π-like modes (i.e., the real parts of the poles of the retarded Green's functions for the respective channels) using the value $\hat{m}=(m_u+m_d)/2=5.5$ MeV.[11] The approximately degenerate modes at high temperature soften as temperature decreases and split into two branches near the critical point: The mass of the σ-like mode (m_σ) which keeps lying in the continuum has the minimum at $T=200$ MeV; the value may be adopted as a critical temperature (T_χ) when the explict symmetry breaking term $(\hat{m})$ exists. The mass of the pionlike mode (m_π) decreases monotonously with the decreasing temperature and comes out from the continuum at about T_χ defined above; below T_χ, the mode can be identified as pion, the mass of which is slightly modified by thermal effects.[12]

It is worth mentioning that the above soft modes with a narrow width are analogous to fluctuations in a superconductor above the critical temperature T_c[13]; the fluctuations give rise to precursory effects such as an enhancement of the electric conductivity and the magnetic susceptibility. In our case, the soft modes

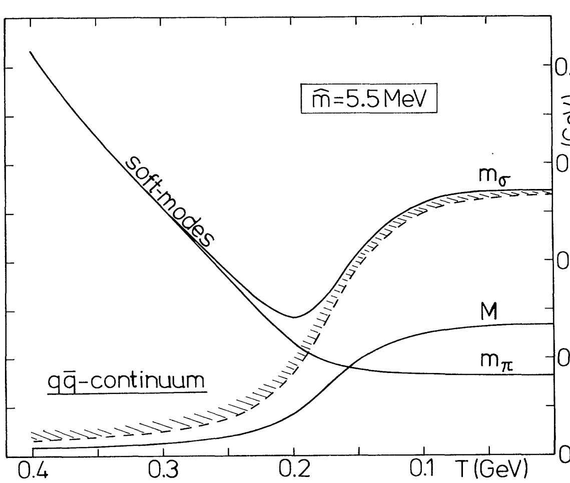

FIG. 3. Dynamical quark mass $M=M_D(T,\mu)+\hat{m}$, and the masses of σ mode (m_σ) and π mode (m_π). The dashed line denotes the $2M$ threshold from which the $q\bar{q}$ continuum starts.

above T_χ are carriers of chirality, isospin, and energy momentum, so it is likely that the modes affect various transport coefficients such as isospin conductivity, thermal conductivity, viscosity, and so on.

To see an example of the phenomena induced by the soft modes, let us consider the cooling of the droplet of QGP produced by the ultrarelativistic heavy-ion collisions. The cooling mechanisms proposed so far[14] are the hydrodynamical expansion and the pion evaporation on the surface through a formation of the color flux tube, although Banerjee, Glendenning, and Matsui[15] have pointed out that the latter mechanism is not so important compared with the former. Here we argue that the soft modes provide us with another mechanism of the QGP cooling: Our low-energy soft modes which are color-singlet will be thermally excited inside the droplet, then they turn into pion at the surface where $T \cong T_\chi$ with no need for the formation of the flux tube and transmit the energy momentum outside, which causes the cooling of the droplet.

We have investigated in this Letter the fluctuations of the order parameter in the Wigner phase of chiral symmetry and shown that they become elementary excitations near the critical point in addition to the quarks and gluons. Finally, we make a brief comment: In our model interaction with $N_f=2$, the chiral transition is second order. For the larger numbers of flavor, the order of the transition may depend on the effective interaction one adopts. However, in the case of a weak first-order transition, soft modes around the chirally symmetric vacuum exist and give rise to the fluctuation effects discussed above. For instance, in the effective potential $V_{\rm eff}(\sigma)$ calculated by Goldberg[16] who used the SU(3) ⊗ SU(3) linear σ model with a determinantial interaction, one can see the weakening of the restoring force around the symmetric vacuum $(\sigma=0)$ as the system approaches the critical point, which implies an existence of the soft modes.

We acknowledge the interest of the members of the Nuclear Theory Group at Kyoto University in this work. The computer calculation has been financially supported by the Institute for Nuclear Study, The University of Tokyo.

[1]See the reviews, E. V. Shuryak, Phys. Rep. **61**, 71 (1980), and **115**, 151 (1984).

[2]Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961), and **124**, 246 (1961).

[3]G. 't Hooft, Phys. Rev. D **14**, 3432 (1976); C. G. Callan, R. Dashen, and D. J. Gross, Phys. Rev. D **16**, 2526 (1977), and D **17**, 2717 (1978). The determinantial interaction we adopt in the text is related also to the quark version of the $U_A(1)$ anomaly term in the chiral Lagrangean; see C. Rosenzweig, J. Schechter, and C. G. Trahern, Phys. Rev. D **21**, 3388 (1980). The use of the NJL-type Lagrangean as effec-

tive for QCD is discussed in A. Dhar and S. R. Wadia, Phys. Rev. Lett. **52**, 959 (1984).

[4]T. Hatsuda and T. Kunihiro, Kyoto University Report No. KUNS778, 1985 (to be published), and Phys. Lett. **145B**, 7 (1984); T. Kunihiro and T. Hatsuda, Prog. Theor. Phys. **71**, 1332 (1984).

[5]A. Manohar and H. Georgi, Nucl. Phys. **B234**, 189 (1984).

[6]A. L. Fetter and J. D. Walecka, *Quantum Theory of Many-Particle Systems* (McGraw-Hill, New York, 1971).

[7]T. Matsubara, Prog. Theor. Phys. **14**, 351 (1955); C. W. Bernard, Phys. Rev. D **9**, 3312 (1974); L. Dolan and R. Jackiw, Phys. Rev. D **9**, 3320 (1974); S. Weinberg, Phys. Rev. D **9**, 3357 (1974).

[8]J. Kogut *et al.,* Phys. Rev. Lett. **50**, 393 (1983).

[9]A. A. Abrikosov, L. P. Gorkov, and I. E. Dzyaloshinski, *Methods of Quantum Field Theory in Statistical Physics* (Prentice-Hall, Engelwood Cliffs, N.J., 1963).

[10]E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981).

[11]J. Gasser and H. Leutwyler, Phys. Rep. **87**, 77 (1982).

[12]We have used the cutoff $\Lambda_3 = 825$ MeV which is derived from the lowest order of the chiral perturbation (see Ref. 4). Therefore, the pion mass m_π at $T = \mu = 0$ shown in Fig. 3 which is the exact solution of the dispersion equation becomes slightly larger than 140 MeV.

[13]L. G. Aslamazov and A. I. Larkin, Fiz. Tverd. Tela (Leningrad) **10**, 1104 (1968) [Sov. Phys. Solid State **10**, 875 (1968)]; K. Maki, Prog. Theor. Phys. **39**, 897 (1968), and **40**, 193 (1968); see also, M. Tinkham, *Introduction to Superconductivity* (McGraw-Hill, New York, 1975).

[14]See, e.g., M. Gyulassy, Nucl. Phys. **A418**, 59c (1984), and Ref. 1.

[15]B. Banerjee, N. K. Glendenning, and T. Matsui, Phys. Lett. **127B**, 453 (1983).

[16]H. Goldberg, Phys. Lett. **131B**, 133 (1983).