Energy-Level Quantization in the Zero-Voltage State of a Current-Biased Josephson Junction

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We report the first observation of quantized energy levels for a macroscopic variable, namely the phase difference across a current-biased Josephson junction in its zero-voltage state. The position of these energy levels is in quantitative agreement with a quantum mechanical calculation based on parameters of the junction that are measured in the classical regime.

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Do macroscopic variables obey quantum mechanics? This question, although central to the theory of measurement,[1] has only recently been addressed experimentally. An attractive candidate for such experimental investigation is the Josephson tunnel junction, a system in which thermal fluctuations and perturbations due to the environment can be made negligible. In the case of the current-biased junction, the macroscopic variable is the phase difference, δ, between the superconducting order parameters on either side of the barrier. The junction can be represented as a particle moving in a one-dimensional tilted cosine potential.[2] The zero-voltage state of the junction corresponds to the confinement of the particle to one well of this potential. After the particle escapes from this metastable state, it runs freely down the tilted cosine potential, and a voltage appears across the junction. For parameters of experimental interest, the potential well from which the particle escapes is represented, to a very good approximation, by a cubic potential[1,3] [Fig. 1(a)]. Previously, to demonstrate the quantum mechanical nature of δ, experiments[4,5] have been performed to investigate the escape rate from the metastable well via quantum macroscopic tunneling[3] at low temperatures. Related experiments[6,7] have been concerned with a superconducting ring interrupted by a Josephson tunnel junction.

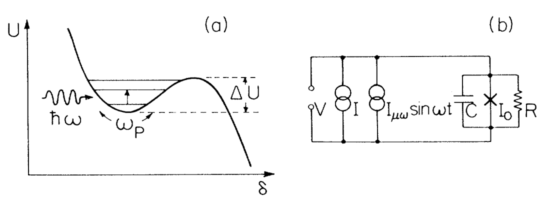

FIG. 1. (a) Cubic potential U vs phase difference δ showing three energy levels. Transition from the ground state to the first excited state induced by a photon of frequency $\omega/2\pi$ is shown. (b) Model of current-biased Josephson junction loaded with a resistor and irradiated with an external microwave current source.

In this Letter, we describe a novel experiment that demonstrates the quantum mechanical nature of the macroscopic variable through energy level quantization. We irradiate the junction with microwaves that, at appropriate frequencies, cause the particle to make transitions into excited states. Since the escape rate out of the well increases with increasing energy, by measuring changes in the escape rate we are able to demonstrate the existence of quantized energy levels spectroscopically in a way that does not require a precise knowledge of all the parameters of the junction.

We have designed the experiment so that the admittance across the Josephson junction due to its self-capacitance and current-voltage leads can be represented to a good approximation by a capacitance C and resistance R in parallel [see Fig. 1(b)]. For a junction with a critical current I_0 and for a bias current I close to I_0, the potential-barrier height ΔU and plasma frequency $\omega_p/2\pi$ (the oscillation frequency of the particle at the bottom of the well) are[8] $\Delta U = \frac{4}{3}\sqrt{2}U_0(1-I/I_0)^{3/2}$ and[1] $\omega_p = \omega_{p0}[1-(I/I_0)^2]^{1/4}$. Here,[1] $U_0 = I_0\Phi_0/2\pi$, $\omega_{p0} = (2\pi I_0/C\Phi_0)^{1/2}$, and $\Phi_0 = h/2e$. The dissipation is described through the damping factor $Q = \omega_p RC$. According to quantum theory, the energy levels in the well should be quantized as indicated in Fig. 1(a); because of the cubic term in the potential, the spacing between adjacent levels decreases with increasing energy in the well. We note that an increase in the bias current I decreases the spacings of energy levels.

For our experiments, the Nb-NbO_x-PbIn tunnel junctions were patterned photolithographically on Si chips in either a 10×10-μm^2 or a 80×10-μm^2 cross-strip geometry. Our experimental configuration for experiments in the ^{4}He temperature range has been described previously.[9] In the experiment, the junction and the last of a chain of low-pass filters for the bias circuitry were thermally anchored to the mixing chamber of a dilution refrigerator. The temperature of the mixing chamber was determined with a combination of a calibrated germanium thermometer and a ^{60}Co orientation thermometer. The accuracy of the temperature scale and the absence of a significant level of external noise were checked by use of the junction itself to measure temperature: By reducing the critical current with a magnetic field, we studied the junction in the classical limit $(k_BT/\hbar\omega_p > 1/2\pi)$[10] in which the zero-voltage state decays via thermal activation. Extensive measurements in this limit down to the lowest temperature attainable yielded escape rates that were in good agreement with those predicted theoretically[11] with use of the temperature determined by our thermometry.

Microwave power of frequency $\omega/2\pi$ was injected via a separate, filtered coaxial line that was capacitively coupled to the leads of the junction. We used the technique of Fulton and Dunkleberger[8] to measure the escape rate of the junction out of the zero-voltage state as a function of the bias current, with and without microwave power. The microwave power P was adjusted to give a change in the escape rate Γ such that $[\Gamma(P)-\Gamma(0)]/\Gamma(0) < 2$; values of $\Gamma(0)$ were in the range 10^2 to 10^5 s^{-1}. We collected typically 10^5 switching events. After the junction had switched to the nonzero-voltage state, the bias current was turned off within 30 μs. We ensured that the elapsed time before the next switching event was sufficient for any rise in temperature that occurred in the dissipative state to have become negligible.

One expects the escape rate in the presence of microwaves to be resonantly enhanced over that in the absence of microwaves when the microwave frequency coincides with the spacing between two energy levels. In practice, we detect this resonance by varying the energy-level spacing with the bias current while keeping the microwave frequency fixed. Figures 2 to 4 illustrate typical results. In Fig. 2(a) we show the change in the escape rate due to 2.0-GHz microwave irradiation for a junction with parameters chosen so that the well contained several energy levels $(\Delta U/\hbar\omega_p \sim 6)$, there was significant population of the lower excited states $(k_BT/\hbar\omega_p \sim 0.3)$, and the damping was low enough $(Q \sim 80)$ to produce narrow resonances. We observe three peaks in $[\Gamma(P)-\Gamma(0)]/\Gamma(0)$, indicating that the escape rate is resonantly enhanced at certain values of the bias current. The two peaks at the higher bias currents are approximately Lorentzian. No further peaks were observed at values of bias current higher than those plotted in Fig. 2. These discrete resonances are characteristic of transitions between quantized energy levels.

To investigate the position of these peaks, we have computed the energy levels by solving the Schrödinger equation numerically,[12] using as input parameters the measured bias current, the value of $I_0 = 30.572 \pm 0.017$ μA determined from the current dependence of $\Gamma(0)$ at 28 mK, and the value of $C = 47.0 \pm 3.0$ pF measured at higher temperatures in the classical regime (see Ref. 9 for details of the last two determinations). The solid lines in Fig. 2(b) show the energy-level spacings $E_{n\to n+1}$ vs $(n=0,1,2)$. The dotted

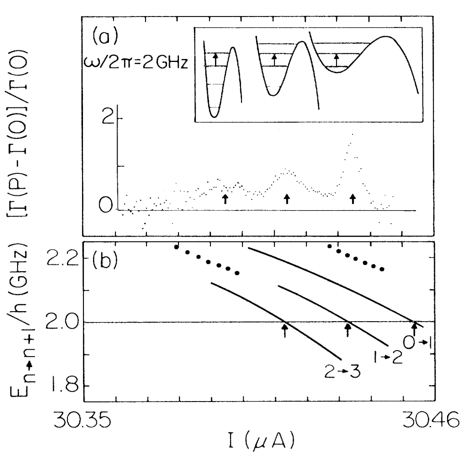

FIG. 2. (a) $[\Gamma(P)-\Gamma(0)]/\Gamma(0)$ vs I for a 80×10-μm^2 junction at 28 mK in the presence of 2.0-GHz microwaves $(k_BT/\hbar\omega = 0.29)$. Arrows indicate positions of resonances. Inset represents the corresponding transitions between energy levels. (b) Calculated energy-level spacings $E_{n\to n+1}$ vs I for $I_0 = 30.572 \pm 0.017$ μA and $C = 47.0 \pm 3.0$ pF. Dotted lines indicate uncertainties in the $E_{0\to1}$ curve due to errors in I_0 and C. Arrows indicate values of bias current at which resonances are predicted.

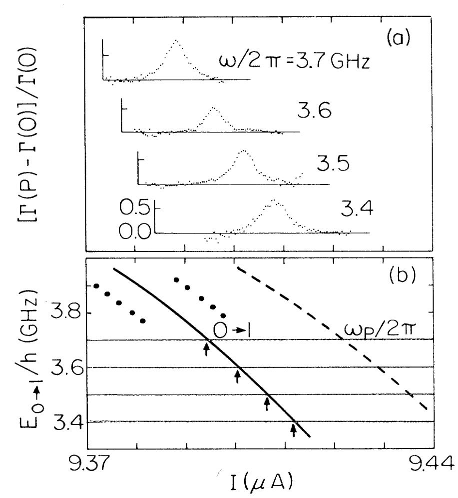

FIG. 3. (a) $[\Gamma(P)-\Gamma(0)]/\Gamma(0)$ vs I for a 10×10-μm^2 junction at 18 mK for four microwave frequencies. (b) Calculated energy-level spacing $E_{0\to1}$ vs I for $I_0 = 9.489 \pm 0.007$ μA and $C = 6.35 \pm 0.4$ pF. Dotted lines indicate uncertainties due to errors in I_0 and C. Arrows indicate values of bias current at which resonances are predicted. Dashed line indicates plasma frequency.

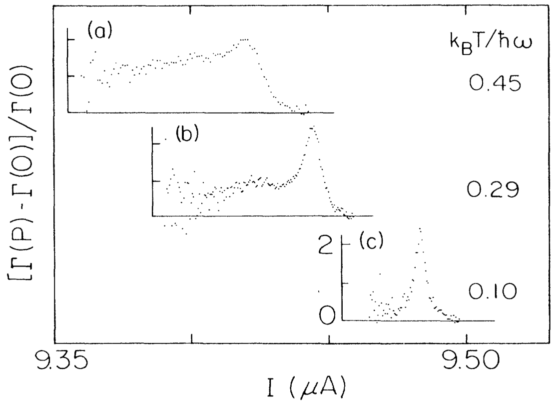

FIG. 4. $[\Gamma(P)-\Gamma(0)]/\Gamma(0)$ vs I for the junction of Fig. 3 with $I_0=9.57$ μA and $C\approx 6.35$ pF at three values of $k_BT/\hbar\omega$. The microwave frequencies are (a) 4.5 GHz, (b) 4.1 GHz, and (c) 3.7 GHz.

lines indicate the magnitude of the uncertainty in the position of the curve corresponding to the $E_{0\rightarrow 1}$ transition that arises from the estimated errors in I_0 and C. We note that a systematic error in I_0 or C will move all three curves by very nearly the same amount. The intersection of each of these curves with the horizontal line corresponding to a microwave frequency of 2.0 GHz is the prediction of the bias current at which the resonant peaks should occur. The absolute positions of the peaks with respect to the bias current agree with the predicted positions to within the experimental uncertainty. The separations of the peaks along the current axis are in excellent agreement with the predicted separations.

To study the dependence of the energy difference (E_1-E_0) on bias current, we made measurements on a second junction, with substantially lower values of I_0 and C, that was more strongly in the quantum regime ($k_BT/\hbar\omega\sim 0.1$, $\Delta U/\hbar\omega_p\sim 2$) at the values of I and T attainable in the experiment. Extensive measurements of I_0 and C in the classical regime yielded $I_0=9.489\pm 0.007$ μA and $C=6.35\pm 0.4$ pF. At fixed bias current over the temperature range from 18 to 25 mK, the escape rate was temperature independent[13] in the absence of microwaves, and in very good agreement with the predictions for macroscopic quantum tunneling.[3] Figure 3(a) shows resonances in $[\Gamma(P)-\Gamma(0)]/\Gamma(0)$ observed at 18 mK for four different microwave frequencies. In Fig. 3(b) we plot the predicted energy difference between the ground state and first excited state, together with the estimated uncertainty. The absolute measured positions of the resonances agree with the predictions to within the experimental uncertainties. The shift in the positions of the resonance as the microwave frequency is changed is in excellent agreement with the predicted shift. Furthermore, the measured positions of the resonances are clearly very different from a classical prediction for the resonant activation of the particle oscillating at the plasma frequency (dashed line).

To illustrate the evolution from quantum to classical behavior, in Fig. 4 we show the temperature dependence of the shape of the resonant response to microwaves for the junction studied in Fig. 3 with a slightly higher critical current. At the lowest temperature (c), at which the junction was firmly in the quantum regime, we observe a single, approximately Lorentzian resonance. At the intermediate temperature (b), a shoulder corresponding to the $E_{1\rightarrow 2}$ transition begins to appear. At the highest temperature (a), the resonance has become asymmetric, the peaks associated with individual transitions having merged into a continuum; this response is characteristic of that observed for the classical phenomenon of resonant activation.[9] A brief comment on the linewidth of the resonance is in order. Measurements of Q in the classical regime yield $Q=30\pm 15$ for the junction shown in Fig. 3. To our knowledge, there is no theory available for the effects of damping on the position and broadening of the energy levels of the excited states. However, one might reasonably expect the energy level of an excited state to be broadened by an amount $\Delta\omega$ so that the relative linewidth of the $E_{0\rightarrow 1}$ transition, $\Delta\omega/\omega$, is approximately equal to $1/Q$, assuming that the lifetime of the excited state against tunneling out of the well is sufficiently long. The data presented in Fig. 3(a) yield $\omega/\Delta\omega=50\pm 10$, in reasonable agreement with the value of Q measured in the classical limit. We note that the coherence of the ground and excited states must be maintained over time scales given by the lifetime of the excited state, approximately $2\pi Q/\omega_p\sim 14$ ns for the $E_{0\rightarrow 1}$ resonance in Fig. 3(a); for the same transition in Fig. 2(a), the coherence time is about 40 ns.

In summary, we have observed microwave-induced resonant enhancements of the rate at which a current-biased Josephson tunnel junction escapes from the zero-voltage state. The positions of these resonances are in excellent agreement with the predictions of a model in which the energy levels of the particle in the well are quantized, with no fitting parameters. These results provide very strong evidence for the quantum nature of the macroscopic variable δ, and imply that quantum coherence between an excited state and the ground state can be maintained for times as long as 40 ns.

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Note added.—Since this paper was submitted, two other papers concerned with experiments on macroscopic quantum tunneling have appeared.[14]

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