Comment on "Electrical-Conductivity Fluctuations near the Percolation Threshold"

Recently, Chen and Chou[1] have reported the results of a careful study of $1/f$ noise in three-dimensional (3D) carbon-wax mixtures near the percolation threshold p_c. Close to p_c, both the resistance R and the power spectrum S_R have been shown to diverge in perfect agreement with the predictions of Rammal and co-workers.[2] In the notations of Ref. 2, $R \sim (\Delta p)^{-t}$ and $S_R/R^2 \sim (\Delta p)^{-\kappa}$, the measured exponents are respectively $t = 2.3 \pm 0.4$ and $\kappa = 5 \pm 1$. The direct plot of S_R vs R leads to $S_R \sim R^Q$ where $Q = 2 + \kappa/t = 3.7 \pm 0.2$ is the noise-versus-resistance exponent. Similar measurements of Q on 2D films have also been performed by two other groups.[3,4] In clumped evaporated gold films[3] subjected to ion milling, a large value $Q = 4$ has been obtained. Much larger values, $5.4 \leqslant Q \leqslant 8.1$, were obtained[4] in a large number of metallic films (Al, Cr, In) where the metal was removed by sandblasting. These deviations from the lattice percolation predictions can be attributed to continuum corrections[4-6] which may be at the origin of the enhancement of both t and κ. The purpose of this Comment is to show that the 3D data are probably the first quantitative confirmation of this idea.

First let us mention that the measured exponents are actually outside the bounds found for Q in the lattice percolation theory. With use of the known bounds[2,5] for the exponent $b = d - \kappa/\nu = d - t(Q-2)/\nu$, $-\beta_L \leqslant b \leqslant -2\beta_L - 1/\nu$, it is easy to obtain $2.82 \leqslant Q \leqslant 3.05$ in 2D ($\nu = \frac{4}{3}$, $-\beta_L = 0.973$) and $2.84 \leqslant Q \leqslant 2.85$ in 3D ($\nu = 0.88$, $-\beta_L = 1.16$). Here $\beta_L = d - 2 - t/\nu$, d is the Euclidean dimension, and ν is the correlation-length critical exponent. The effective-medium theory (EMT) gives[2] the value $Q_m = 3$ for Q.

As was pointed out by various authors, transport exponents such as t and κ can be modified in continuum percolation models[4-6] in contrast with static exponents (e.g., ν). The simplest model is provided by the following probability distribution $p(g)$ of bond conductances in the equivalent lattice model: $p(g) = (1-p)\delta(g) + ph(g)$. Here $h(g)$ is a continuous normalized function. The "Swiss-cheese" class of models is actually a possible realization[6] of $p(g)$, with an anomalous distribution $h(g) \sim g^{-\alpha}$ ($\alpha < 1$) near $g = 0$. For this class of models, the conductivity exponent is given by $t(\alpha) = (d-2)\nu + 1/(1-\alpha)$ for $0 \leqslant \alpha < 1$ and $t(\alpha) = (d-2)\nu + 1$ for $\alpha \leqslant 0$. This result is implicitly contained in the work of Ben-Mizrahi and Bergman,[7] coincides with the large-d limit,[8] and was rederived[9] recently by use of an $\epsilon = 6 - d$ expansion technique. The simplest derivation of this result is probably the following argument. The conductance $g = (\sum_l g_l^{-1})^{-1}$ of L conductances $\{g_l\}$, $1 \leqslant l \leqslant L$, taken from $h(g)$ and combined in series is given, at $g \sim 0$, by $\min\{g_l, 1 \leqslant l \leqslant L\}$. A simple calculation[6] yields $g \sim L^{1/(\alpha-1)}$ ($0 \leqslant \alpha < 1$), $g \sim L^{-1}$ ($\alpha \leqslant 0$), and leads to the desired results $-\nu\beta_L = 1/(1-\alpha)$ and 1, respectively. Note that $t(\alpha)$ so obtained differs from its EMT value[8]: $t_m = t(\alpha) - \nu(d-2)$.

The calculation of κ can be carried out similarly, with use of the series composition rule[2] for the relative noise: $s = \sum_l s_l (g/g_l)^2$. For the sake of simplicity we consider the class of "Swiss-cheese" models and assume[5] $g_l \sim \delta_l^u$, $s_l \sim \delta_l^{-v}$ where $u = 1/(1-\alpha)$ and v is the exponent relating the relative noise s_l of bond l to the neck width δ_l. Depending on the values of u and v, one obtains three distinct relevant cases: (a) $\kappa = d\nu + v$ for $u > 1$, $v + 2u > 1$; (b) $\kappa = d\nu + (2u + v - 2)$ for $u < 1$, $v + 2u > 1$; and (c) $\kappa = d\nu - 1$ for $u < 1$ and $v + 2u < 1$. These expressions are actually different from the EMT results[5] $Q_m = 2 + v/u$ at $u > 1$ and $Q_m = 3 + (v-1)/u$ at $u < 1$, which are expected to be correct[2] far from p_c.

Within the framework of "Swiss-cheese" models, where $u = d - \frac{3}{2}$ and $v = d - \frac{1}{2}$, one obtains the following results. At $d = 2$ (case b), $t = t_m = 1$, $\kappa = 3.16$, $\kappa_m = 2$, and $Q = 5.16$, $Q_m = 4$. At $d = 3$ (case a), $t = 2.38$, $t_m = 1.50$, $\kappa = 5.14$, $\kappa_m = 2.5$, and $Q = 4.16$, $Q_m = 3.67$. Clearly, the data of Ref. 1 fit nicely with these estimations. The results of Ref. 3 seem to follow the EMT[5] estimates, whereas those of Ref. 4 are definitely larger than the above estimates. Note that, in all cases, an enhancement of Q is obtained. However, a definitive comparison in 2D would require the simultaneous measurement of both exponents, t and κ. Furthermore, the measurement of S_R at different temperatures would be useful for the identification of the conduction mechanism and the microscopic origin of the observed noise.

R. Rammal
Centre de Recherches sur les Tres Basses Temperatures
Centre National de la Recherche Scientifique
38042 Grenoble Cedex, France

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