Comment on "Mass and Anomalous Magnetic Moment of an Electron between Two Conducting Parallel Plates"

Recently work has been published[1] dealing with the motion of an electron within a conducting cavity, with the Letter of this title[2] being a new addition. In order to simplify the calculations, the cavity has been replaced by a parallel pair of conducting plates a distance a apart. The plates alter the photon propagator. Of potential particular interest are the assertions that the corresponding alteration in the radiative correction to the magnetic moment of the electron produces a change in its g factor of the order $\delta g \sim (r_0/a)\ln(\lambdabar_c/a)$, where r_0 and $\lambdabar_c$ are respectively the classical radius and Compton wavelength of the electron. The University of Washington $g-2$ experiment utilizes a Penning trap whose conducting surfaces are about 1 cm apart. With $a\sim 1$ cm one would have a cavity shift $\delta g \sim 10^{-11}$ which is a substantial correction to the current experiment[3] whose precision in g is 4×10^{-12}. As emphasized elsewhere,[4] the work of Ref. 1 is in error since the result is not gauge invariant and thus depends upon an assigned "center" for a uniform magnetic field. A careful treatment for an arbitrary cavity[4] shows instead that $\delta g\sim 10^{-20}$ for the $g-2$ experiment. However, there may be a significant correction to the cyclotron frequency[4,5] whose measurement is necessary in the experimental determination of the g factor, but this is a separate issue.

The work of the recent Letter[2] is also in error. This is most easily seen by examining the "mass shift" $\Delta m(a) = -(e^2/2a)\ln(a/\lambdabar_c)$ presented in Eq. (8).[2] This formula makes no reference to the distance d of the electron from one of the plates. (The electron in the $g-2$ experiment is bound and well-localized.) Clearly as $d\rightarrow 0$, the electron interacts strongly with its adjacent image charge on the other side of the plate and the Coulomb energy diverges as $-(e^2/2)(1/2d)$. Thus, the mass shift must depend upon both plate spacing a and the position d of the average electron position. No dependence on d appears in the results of the Letter[2] because two physically incorrect idealizations were made.

First of all, the correct photon propagator $D_{\mu\nu}(x,x')$ within the plates is not translationally invariant and depends upon x_1+x_1' as well as x_1-x_1'. The nontranslationally invariant terms in the propagator depending upon x_1+x_1' displayed in Eqs. (1) and (3) of the Letter were, in fact, omitted in the actual calculation with the only effect of the plates being the simple replacement of the continuous integration over the conjugate wave number k_1 by a discrete mode sum. This procedure does not correctly model the physical effects of the plates; in particular, it cannot yield the necessary d dependence. The difficulty is explicitly illustrated by considering the modification of the classical Coulomb energy E_C of an electron with the two plates in more detail. The familiar image construction gives

$$E_C = -\frac{e^2}{4d} - \frac{e^2}{2a}\sum_{n=1}^{\infty}\left\{\frac{n}{n^2-(d/a)^2} - \frac{1}{n}\right\}. \quad (1)$$

Here, of course, the zeroth image corresponding to the (infinite) self-energy of the electron is deleted. The alteration in the signs of the image charges makes the sum over n converge. On the other hand, replacing the Coulomb Green's function for the plates with the free-space function altered only by the changing of the integration over k_1 in its Fourier representation to a sum over $k_1=\pi\nu/a$ (ν integer) produces a periodic function with periodic sources and thus corresponds to keeping only the even images of like sign. This corresponds to deleting the entire first term in the curly brackets in Eq. (1) (which carries the d dependence), resulting in a logarithmic divergence.

Secondly, the Letter represents the electron by a plane wave which is uniformly distributed between the plates. This, again, is an unphysical idealization. Indeed, a free electron placed within parallel conducting plates would be attracted to its nearest image charge and hit the plate. The correct procedure is to consider an electron which is closely bound to a center which is at a distance d from a plate. This is done properly in Refs. 4 and 5. There is no shift $\delta g(a) = -(r_0/a)\ln(a/\lambdabar_c)$ [Eq. (10) of the Letter] for the g factor of the electron in the University of Washington $g-2$ experiment.

David G. Boulware
Lowell S. Brown
Department of Physics, FM-15
University of Washington
Seattle, Washington 98195

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