

Cates Responds: Havlin's Comment¹ on my Letter² raises two main points. The more important point concerns the interpretation of the fractal Einstein relation. In Ref. 2, I stated and used the results (a) for any fractal of $d_s < 2$,

$$d_w = d_f + x, \quad (1)$$

where x is the exponent describing the resistance between points (see below), and (b) for $d_s > 2$, this resistance is dominated by local (short-cutoff-dependent) contributions, so that $x = 0$, and Eq. (1) does not apply.

Although I believe that these results are widely known, their statement without proof in Ref. 2 may have caused some confusion. A derivation is as follows. Let us write the average frequency-domain diffusion propagator $P(r, r'; \omega)$ between points (r, r') on the fractal in the scaling form

$$P(r, r'; \omega) \sim |r - r'|^b f(|r - r'|/\xi_\omega), \quad (2)$$

where the diffusion length $\xi_\omega = \omega^{-1/d_w}$. Conservation of probability then requires that $\int d\omega dr r^{d_f-1} \times e^{i\omega t} P(r', r; \omega) = \text{const}$ (for any r') and hence

$$d_w = d_f + b \quad (\text{for all } d_s). \quad (3)$$

Now consider the electrostatic potential, $V(r'')$, at some point r'' on the fractal in the presence of a steady unit current source at r , with a unit sink at r' . Because of the equivalence between Kirchoff's laws and the steady-state diffusion equation, $V(r'') \propto P(r, r''; 0) - P(r', r''; 0)$. Hence the resistance between points r and r' , $\Omega(r, r') \propto V(r) - V(r')$, obeys

$$\begin{aligned} \Omega(r, r') &\propto 2P(r, r'; 0) - P(r, r; 0) - P(r', r'; 0) \\ &\sim \text{sgn}(b) [|r - r'|^b - \Lambda^b], \end{aligned}$$

where Λ is a short cutoff. Defining the exponent x by $\Omega(r, r') \sim |r - r'|^x$ (with $|r - r'| \gg \Lambda$) we see that $x = b$ (if $b > 0$) and $x = 0$ (with Ω dependent on Λ) if $b < 0$. These results, along with Eq. (3), complete the confirmation of statements (a) and (b) above.

Of course, Havlin is correct in saying that one can define a new exponent, x' say, for which $d_w = d_f + x'$ for all d_s . It is clear from Eq. (3) that any consistent definition of x' (as might be obtained by consideration of the resistance between bars) must then amount to identification of $x' \equiv b$ in Eq. (2). However, because b does not (for $d_s > 2$) determine the resistance between points, the more general version of the Einstein relation [Eq. (3)] has no bearing on any of the calculations or discussion in my Letter.²

The other main point raised by Havlin concerns my remark² that "as $d_s \rightarrow 2$, $d_f \rightarrow \infty$; the structure is collapsed in all dimensions." This remark was intended to refer to the ideal (Gaussian) structure of a polymer-

ic fractal (pf). It does not apply in the presence of excluded volume, as may be seen by letting $d_s \rightarrow 2$ in Eq. (3) of Ref. 2, nor, of course, to fractals which are not polymeric. I remarked further that any ideal pf with $d_s > 2$ should also have a saturating radius ($d_f = \infty$).

Havlin appears, at first sight, to have found counterexamples to both of these remarks, in the form of a family of trees without dead ends. Taking every sequence of bonds as a random walk, he finds^{1,3} $d_f = 2(\alpha + 1)$ while $d_s = \alpha + 1$, where α is a parameter. For $\alpha \geq 1$, these results, if correct, would imply d_f finite for certain ideal pf's with $d_s \geq 2$. Much more important, for $0 < \alpha < 1$, they would contradict a central result of Ref. 2, that $d_f = 2d_s/(2 - d_s)$ for all ideal pf's with $d_s < 2$.

This apparent discrepancy is resolved by noting that Havlin's trees are not of self-similar connectivity and are therefore not fractals in the usual sense. Instead, each is built around a well-defined and completely atypical center (at $l=0$). In fact, because the cluster extends to $l=\infty$ in all directions, and the average chemical separation between branches increases like α/l as one moves away from this center, *any randomly chosen piece of the cluster of finite chemical size contains no branches, with probability 1*. Thus, adopting the conventional site-averaged⁴ definitions of d_s , d_f , and d_w , one finds $d_w = 4$, but (in contrast with the calculations of Refs. 1 and 3) $d_s = d_l = 1$ and $d_f = 2$, for any α . These exponents are the same as those of an ordinary linear polymer, and so there is no conflict with any of my remarks in Ref. 2.

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¹S. Havlin, preceding Comment [Phys. Rev. Lett. 55, 130(C) (1985)].

²M. E. Cates, Phys. Rev. Lett. 53, 926 (1984).

³S. Havlin, R. Nossal, B. Trus, and G. H. Weiss, to be published.

⁴One should (for example) average over starting points (as well as end points) of a walk when calculating d_w . This is done (in effect) in Ref. 3 to obtain the result $d_w = 4$, but the same step is omitted when determining d_f . It is possible (though rather misleading) to define new "effective" values of the exponents, calculated by using the atypical center as an origin and not averaging. If one chooses to do this, Havlin's value of $d_f = 2(\alpha + 1)$ is correct, but the result $d_w = 4$ is not. Instead, $d_w = 2(\alpha + 2)$ and $d_s = 2(\alpha + 1)/(\alpha + 2) < 2$, so once again there is no conflict with Ref. 2.