Squeezing of Quantum Fluctuations via Atomic Coherence Effects

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(Received 28 June 1985)

In the three-level medium a $\chi^{(3)}$ may be generated by atomic coherence effects which involve negligible saturation and, hence, are essentially "spontaneous-emission-free." This offers considerable experimental advantages for the generation of squeezed states of light via four-wave mixing or optical bistability.

There is currently an intensive effort being made to generate squeezed states of light.[1] Squeezed states have less quantum fluctuations in one quadrature than a coherent state. They have potential applications in the reduction of the quantum noise limits in optical detection and communication systems. Current experimental[2-4] efforts to generate squeezed states of light are focused on nonlinear optical interactions with either a second-order susceptibility $\chi^{(2)}$ (such as parametric amplification or second-harmonic generation) or a third-order susceptibility $\chi^{(3)}$ (such as four-wave mixing or dispersive optical bistability).

A semiclassical treatment of nonlinear optics expresses the polarization P of the medium in powers of the applied field E,[5]

$$P = \chi^{(1)}E + \chi^{(2)}E^2 + \chi^{(3)}E^3, \tag{1}$$

where $\chi^{(n)}$ is the nth-order optical susceptibility. Models for nonlinear interactions with E quantized and χ classical allow perfect squeezing of the electric field to be achieved in principle. Such models, however, neglect the quantum fluctuations which arise from a quantum treatment of the medium. We shall investigate how the quantum fluctuations from the medium limit the squeezing and how their effects may be minimized.

We shall restrict our attention to a model for $\chi^{(3)}$ which may be applicable to squeezing via four-wave mixing[6-8] or optical bistability.[9,10] A fully quantized treatment of $\chi^{(3)}$ for both of these problems using a two-level-atom model of the medium has been given by Reid and Walls.[8] The pump field is treated classically and the atoms are eliminated adiabatically. For degenerate four-wave mixing the following c-number stochastic differential equations are derived for the field amplitudes α_3 and α_4 of the signal modes:

$$\alpha_3 = -\gamma\alpha_3 + \chi\alpha_4^* + \Gamma_3(t),$$
$$\alpha_4 = -\gamma\alpha_4 + \chi\alpha_3^* + \Gamma_4(t). \tag{2}$$

Here γ is the loss coefficient and χ is the nonlinear gain coefficient. Both are functions of three medium-related parameters: I_p, the pump intensity normalized with respect to the saturation intensity; Δ, the detuning normalized with respect to the natural atomic linewidth; and a parameter α_0 proportional to the atomic density. The $\Gamma_3(t)$ and $\Gamma_4(t)$ are δ-correlated fluctuating forces also dependent on I_p, Δ, and α_0.

The effect of the atoms has been twofold. The first effect is the introduction of the nonlinearity χ and the associated contribution to the fluctuation correlations, $\langle \Gamma_3(t)\Gamma_4(t') \rangle \rightarrow \chi\delta(t - t')$. This feature is present in models[6] which are based on phenomenological Hamiltonians of the type

$$H = h\chi^{(3)}(E_p^{*2}a_3a_4 + E_p^2 a_3^{\dagger}a_4^{\dagger}), \tag{3}$$

where the pump field and the medium are treated classically. The second effect of the medium is the introduction of terms due to spontaneous emission which arise both in the loss parameter γ and also in extra contributions to the correlations of $\Gamma_3(t)$ and $\Gamma_4(t)$ at higher pump intensities approaching saturation.[8] The action of the atomic dipole has been to couple the pump mode and the two signal modes of the electromagnetic field. However, the atomic dipole has quantum fluctuations associated with it due to the spontaneous emission. The quantum fluctuations associated with the atomic dipole are then passed on to the signal modes of the field.

We shall consider the combined mode $e = (a_3 + a_4) = (X_1 + iX_2)e^{i\phi}$, where a_3 and a_4 are the annihilation operators of the signal modes and X_1 and X_2 are the amplitudes of the two quadrature phases at ϕ of the mode e. The variance of fluctuations in the optimal quadrature is

$$V(X_1) = 1 - \frac{A/|\chi|}{1 + \gamma/|\chi|}(1 - e^{-(\gamma + |\chi|)t}), \tag{4}$$

where squeezing is obtained for $V(X_1) < 1$.

A is a function of the noise correlations. Provided

that we are in the dispersive regime ($\Delta \gg 1$) and are operating far below saturation ($I_p \ll \Delta^2$ and $10 I_p^2 < \Delta^3$), the correlations are those predicted by the ideal Hamiltonian Eq. (3), and $A \to 1$.[8] Loss is then the major barrier to squeezing and this can only be overcome at sufficiently high pump powers such that the ratio of gain to loss is enhanced, that is,

$$\gamma / |\chi| = \Delta / 2 I_p \to 0. \tag{5}$$

For a detuning $\Delta \sim 10^3$ in sodium atoms, this requires a laser intensity of at least 40 kW/cm^2. In order to achieve good squeezing we also require a large $|\chi|$ and long interaction time T. In the case of forward-propagating four-wave mixing, for example, this implies a long interaction length $L = cT$ (for example, an optical fiber[4]) and a high-density medium such that $\alpha_0 T > \Delta$. Yet collisions will destroy squeezing and impose an upper limit on the atomic density of 10^{12} cm^{-3}.[10] These conditions make the experimental task of generating good squeezing by degenerate four-wave mixing in a two-level atomic medium near the limits of current technology.

Ideally, one would wish to have nonlinearity without loss (that is, spontaneous-emission-free). As an alternative mechanism we shall consider the three-level atomic system shown in Fig. 1. Ground-state coherences are induced by the pump field and these lead to a nonlinearity via a nonabsorption resonance. This mechanism has been exploited as a source of non-linearity in a number of experimental situations including four-wave mixing[11-13] and optical bistability.[14-16]

We therefore consider degenerate four-wave mixing in a medium of three-level atoms. This gives equations for the field amplitudes α_3 and α_4 identical in form to Eq. (2) except that the coefficients α and χ and the noise correlations depend on α_0, I_p, Δ, the average one-photon detuning, and an additional parameter δ, the two-photon detuning normalized with respect to the natural atomic linewidth. The variance in X_1 is again given by Eq. (4).

In this paper we shall confine our attention to the contribution of spontaneous emission to the loss γ and assume that ideal noise correlations as predicted from the Hamiltonian Eq. (3) hold ($A \to 1$). This is equivalent, for example, to the procedure of Kumar

and Shapiro.[7] Calculations to determine the conditions for this assumption to be valid in the three-level case are in progress. In this limit the squeezing is determined by the ratio $|\chi|/\gamma$. Calculation of this ratio shows considerable enhancement at lower pump intensities than required for the two-level medium. For example, in the regime $\Delta \gg 1$, $\Delta \gg \delta$, $I_p^2 \ll 16\Delta^2\delta^2$, $I_p/\delta^2 \gg 1$, the ratio becomes

$$|\chi|/\gamma \to I_p/2\Delta + I_p^2/8\Delta\delta^2. \tag{6}$$

It is clear that this may be made large for small δ as a result of the second term which was not present in the two-level case. For example, $\Delta = 100$, $\delta = 0.5$, and $I_p = 50$ give the ratio $|\chi|/\gamma = 12$, while for the two-level medium the same parameters give $|\chi|/\gamma = \frac{1}{4}$. Thus in the three-level medium there is a substantial reduction in the intensity required to reach a gain-to-loss ratio favorable for squeezing. For the particular limits considered here, the χ for a two-level system is proportional to I_p/Δ^3, whereas in our three-level system χ is proportional to $I_p^2/\Delta^3\delta^2$. Thus we are able to get a large χ without saturation of the transition between the upper and lower levels in the three-level case. The three-level medium also allows a similar reduction in the magnitude of the atomic density α_0 required for a large $|\chi| t$. In addition we expect good squeezing to be possible at lower values of detuning Δ. These predictions have immediate experimental consequences. The lower values of parameters required puts such experiments well within the range of current technology.

The above discussion highlights a fundamental issue in atomic physics: how to produce a coherent atomic dipole without the accompanying quantum fluctuations associated with spontaneous emission. In a two-level atom the nonlinearity is generated via saturation, and spontaneous emission is only avoided by going far from resonance. In a three-level atom the nonlinearity may be generated through atomic coherences with negligible atomic saturation and thus offers an alternative and attractive means of producing a coherent dipole.
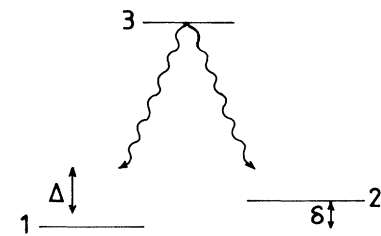
FIG. 1. Three-level atomic system.

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