Spin-Dependent Potentials in SU(3) Lattice Gauge Theory

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The tensor, spin-spin, and spin-orbit terms in the heavy-quark potential have been calculated, via Monte Carlo methods, in SU(3) lattice gauge theory on a $6^3\times 12$ lattice, at $\beta=6$, 7, 8, 9, and 10. The signs and dependence on quark separation of all terms are generally consistent with one-gluon exchange. There is some evidence for nonperturbative behavior in the spin-orbit and spin-spin potentials at $\beta=6.0$.

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Quark confinement is perhaps the most striking feature of QCD. As presently understood, confinement is (chromo)electric and spin independent in nature. Nevertheless, much of the rich structure of hadronic spectra and interactions is concerned with spin dependence and hence with (chromo)magnetic effects. Some time ago, Eichten and Feinberg[1] set up a general framework for describing the spin-dependent forces between heavy quarks, in which the needed potentials are expressed in terms of Wilson loops with electric (E) and magnetic (B) field-strength insertions. The tensor (V_3) and spin-spin (V_4) potentials are determined by

$$[(\hat{R}_i\hat{R}_j-\tfrac{1}{3}\delta_{ij})V_3+\tfrac{1}{3}\delta_{ij}V_4]=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}dT_1\int_{-T/2}^{T/2}dT_2\frac{\langle B_i(\mathbf{O},T_1)B_j(\mathbf{R},T_2)\rangle}{\langle 1\rangle}, \tag{1}$$

where $\mathbf{O}$ and $\mathbf{R}$ are the heavy-quark locations and $\langle 1\rangle$ is the conventional $R\times T$ Wilson loop. The potentials V_3 and V_4 describe the interactions between the magnetic moments of the heavy quarks. Spin-orbit terms are determined by

$$\hat{R}_k\frac{dV_1}{dR}=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}dT_1\int_{-T/2}^{T/2}dT_2\frac{T_1-T_2}{2}\epsilon_{ijk}\frac{\langle E_i(\mathbf{O},T_1)B_j(\mathbf{O},T_2)\rangle}{\langle 1\rangle} \tag{2}$$

and

$$\hat{R}_k\frac{dV_2}{dR}=\lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2}dT_1\int_{-T/2}^{T/2}dT_2\frac{T_1-T_2}{2}\epsilon_{ijk}\frac{\langle E_i(\mathbf{O},T_1)B_j(\mathbf{R},T_2)\rangle}{\langle 1\rangle}. \tag{3}$$

The spin-orbit terms describe the interactions between the magnetic moment of a heavy quark and the current produced by a quark in slow motion, which may be the same one (V_1), or the other quark, located a distance R away (V_2). We have written Eqs. (1)–(3) in the form appropriate for the Euclidean region. (E_i and B_j form a tensor $F_{\mu\beta}$, where $E_i=F_{41}$, $B_3=F_{12}$, etc.)

In this Letter, we report on our Monte Carlo calculations of the quantities needed in Eqs. (1)–(3), and the resulting estimates of the potentials.[2] The calculations were carried out for pure SU(3) lattice gauge theory on a $6^3\times 12$ lattice at $\beta=6/g^2=6$, 7, 8, 9, and 10. The lattice size used here was dictated by computer resources. This calculation is more CPU intensive and requires more memory than the central-potential calculation. The range of β values, broader than in central-potential studies, was chosen to investigate the overall systematics and magnitudes of spin-dependent effects, and to connect a range of couplings which is clearly nonperturbative ($\beta=6.0$) with one where it makes sense to compare with perturbation theory ($\beta=10.0$). Considering the range, remarkably little β dependence was found.

The choice of a lattice quantity to represent the field strengths in Eqs. (1)–(3) is highly nonunique. After some preliminary study, we finally settled on the simplest possibility: $(U_{\mu\beta}-U_{\mu\beta}^{\dagger})/2i=F_{\mu\beta}a^2+O(a^3)$ was used as a measure of field strength, where $U_{\mu\beta}$ is the product of links around a plaquette and a is the lattice spacing. The loop calculated with insertions made this way was averaged with one calculated by use of $U_{-\mu,-\beta}$, which is physically equivalent in the continuum limit.

In early runs, calculations were carried out for loop lengths $T=3$–7. The ratio of a Wilson loop with field-strength insertions to the conventional loop was found to be independent of T as long as the insertions were more than one lattice spacing from the loop ends.

Consequently, in later runs only loops of length $T=6, 7$ were calculated. All results to be discussed here come from the case $T=7$, with insertions at least two lattice spacings from loop ends. Generating the loops with $R=1-4$, $T=6, 7$ took 35 sec of Cray-1S CPU time per configuration. We ran at least 3000 sweeps at each β, updating with the pseudo heat-bath algorithm.[3] Loops were measured every ten sweeps over the last 1000 sweeps, and put into ten bins for statistical analysis. To reduce the variance, an analytic[4] version of the multihit method[5] was applied to the independent timelike links in the loop sides. The systematic error remaining for $R=1$ should largely cancel in loop ratios.

We now turn to a discussion of the results, referring in all cases to the ratio of a loop with field-strength insertions to the loop with no insertions. To a good approximation, the results depend only on $t=T_1-T_2$, and so an average was performed over $2 \leqslant T_1, T_2 \leqslant T-2$, holding t fixed. The statistical error was less than 1% at $t=0$, $R=1$, but increases with t,R and is $O(100\%)$ for $|t|=3$, $R=4$. The results have many features in common with perturbation theory, and are compared in what follows to one-gluon exchange computed on a $6^3\times 12$ lattice.

For the case of two magnetic insertions we resolve the magnetic fields into longitudinal (L) and perpendicular (P) components with respect to $\mathbf{R}$. In perturbation theory, LL is positive at all t. In continuum perturbation theory, PP is negative at small $|t|$, and changes sign at $|t|=R$. On a $6^3\times 12$ lattice, this sign change occurs at $|t|>R$ for $R=1$, 2, and not at all for $R=3$. The Monte Carlo data agree in sign with perturbation theory, although the sign change for PP is only statistically significant for $R=1$. In Fig. 1, we show the LL and PP data for $R=2$ at $\beta=6.0$ and 10.0, where t and $-t$ have been averaged together, and the results plotted in lattice units for $t>0$. The solid line is perturbation theory on a $6^3\times 12$ lattice, for $\beta=10.0$.

For the mixed case of one electric and one magnetic insertion, the insertions may occur on the same or opposite sides, corresponding to Eqs. (2) and (3). The case of insertions on the same side vanishes to $O(g^2)$ in perturbation theory, and we found no measurable signal after extensive running at $\beta=6.0$ and 10.0. For insertions on opposite sides, the signal is large and measurable. In Fig. 2, we plot this case for the component of $\mathbf{E}\times\mathbf{B}$ which points along the interquark separation. Measurement of the electric field requires a space-time plaquette which connects two adjacent times. The average of these two times was assigned to the electric field, i.e., an electric-field measurement connecting links at $t=0$ and $t=1$ was assigned to $t=0.5$, etc. The solid line again represents one-gluon exchange on a $6^3\times 12$ lattice at $\beta=10.0$.

To obtain the potentials, we used simple discrete approximations to Eqs. (1)–(3), replacing the integrals with sums restricted to $|t|\leqslant 3$. This clearly introduces systematic errors. To estimate their size roughly, we computed the potentials in perturbation theory for the

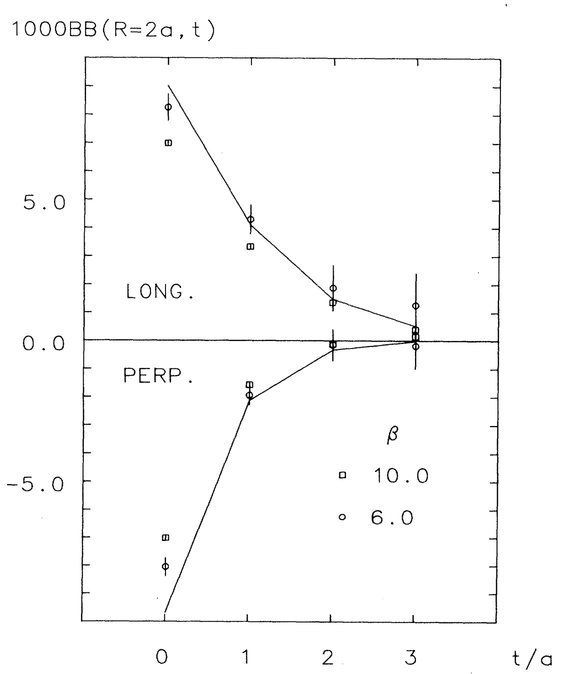

FIG. 1. LL and PP vs t at $R=2a$. The solid line is perturbation theory at $\beta=10.0$.

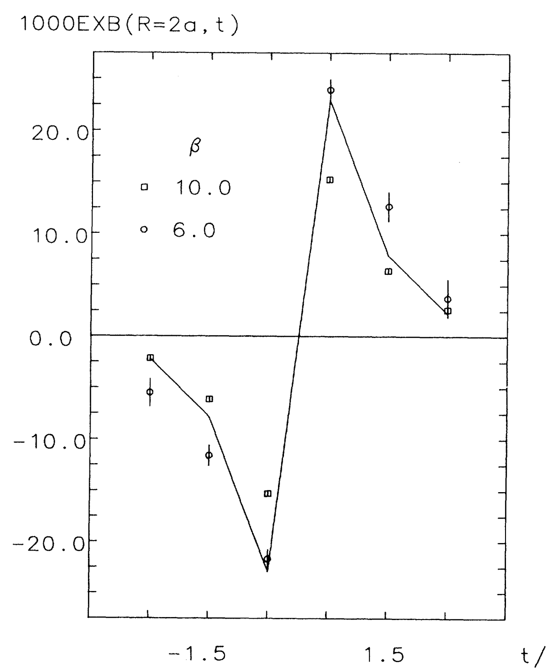

FIG. 2. $\mathbf{E}\times\mathbf{B}$ vs t at $R=2a$. The solid line is perturbation theory at $\beta=10.0$.

case of a $6^3 \times 12$ lattice with $|t| \leqslant 3$, and compared to the case of a 6^3 spatial lattice with continuous time, and t integrated from $-\infty$ to ∞. The result of this comparison suggests that for the worst case of the spin-orbit potential, the systematic error caused by the truncation $|t| \leqslant 3$ is roughly twice the statistical error in the present calculation.

In terms of the components L and P, V_3 is given by LL − PP, and V_4 by LL + 2PP. The spin-spin potential V_4 splits vector and pseudoscalar states in heavy-quark mesons and is proportional to $\delta^3(\mathbf{R})$ in lowest-order (continuum) perturbation theory. This gets smeared out in higher orders; thus more smearing should occur for stronger coupling. The Monte Carlo results show a hint of this. They are consistent with zero for $R > 1$, but for $R = 1$, there is a clear positive signal at $\beta = 6.0$, which falls by roughly a factor of 2 for each unit increase in β, becoming insignificant at $\beta = 10.0$. Even at $\beta = 6.0$, the spin-spin potential at $R = 1$ is small, 5% of the tensor.

Our major results are contained in Figs. 3 and 4, which show the tensor potential V_3 and the spin-orbit term V_2, with only statistical errors shown. In both cases, the solid line is one-gluon exchange for $\beta = 10.0$ computed for a 6^3 spatial lattice with continuous time. For the tensor potential, the R dependence is very similar to that in perturbation theory at all β values. For the spin-orbit term, although the errors are large, $\beta = 6.0$ appears to show a different R dependence from the other β values. In magnitude, the Monte Carlo results are consistently smaller than one-gluon exchange. With the values of the potentials at $R = 2$, the ratio of our calculated potentials to one-gluon exchange at each β ranges from 0.60 at $\beta = 6.0$ to 0.76 at $\beta = 10.0$ for the tensor potential. For the spin-orbit potential, the ratio varies from 0.77 at $\beta = 6.0$ to 0.72 at $\beta = 10.0$.

A calculation on a larger lattice would clearly be very desirable. However, some information of direct interest in phenomenology can be obtained from the present calculation. We found that the "same side" spin-orbit term V_1 vanishes for all practical purposes. A number of authors have argued that heavy-quark spectroscopy requires spin-orbit terms of unconventional sign, as would be produced by the Thomas precession for a Lorentz-scalar confining potential.[6] This could be imitated in the present formalism by a large V_1 term. Our calculations suggest strongly that there is no such term of appreciable size in pure SU(3) lattice gauge theory, and that the sign of the "opposite-side" term, V_2, agrees with one-gluon exchange.

The magnitudes we have found for the spin-orbit and tensor potentials are quite small, approximately 70% of simple (unrenormalized) perturbation theory. This is at least three times smaller than the magnitudes used in phenomenology, where one-gluon exchange with a nominal value of the running coupling corresponding to $\alpha = g^2/4\pi = 0.3$ is typical.[1] (Note that at $\beta = 6.0$, the unrenormalized value of α is 0.08.) However, this discrepancy is likely to be due to lattice

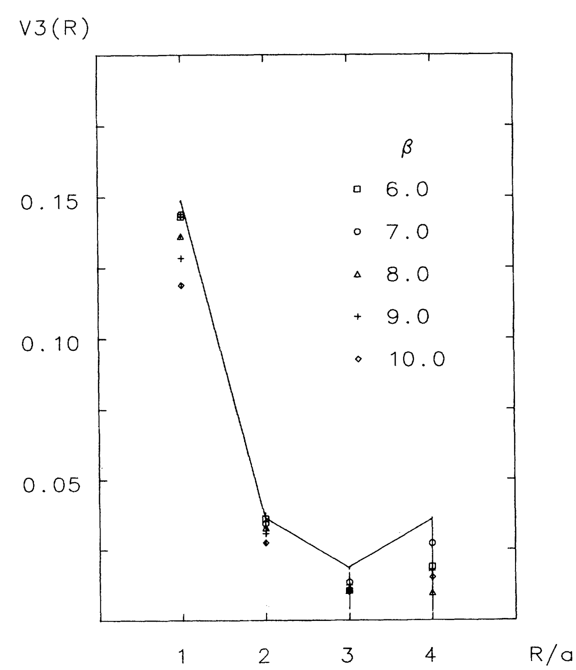

FIG. 3. Spin-spin potentials vs R at $\beta = 6$, 7, 8, 9, and 10. The solid line is perturbation theory at $\beta = 10.0$.

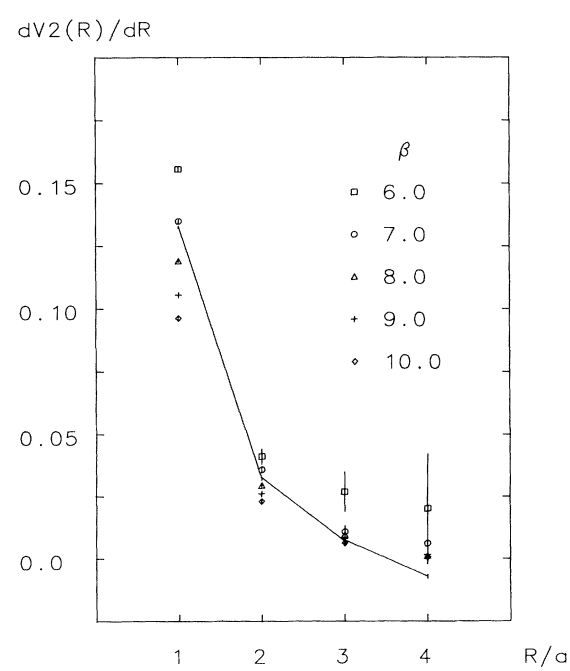

FIG. 4. Spin-orbit potential vs R at $\beta = 6$, 7, 8, 9, and 10. The solid line is perturbation theory at $\beta = 10.0$.

artifacts in the present calculation. (It is much too large to be accounted for by the neglected interval $|t| > 3$.) Dividing the Wilson loop with field-strength insertions by the conventional loop removes the lattice artifacts associated with the corners and the perimeter of the large $(T\times R)$ loop. However, this does not touch the lattice artifacts associated with the insertion itself which has its own perimeter and corners and forms another corner where it attaches to the main loop. These effects would show up as an unwanted function of β, multiplying the desired physical answer. (We note in passing that the naive procedure of dividing by a factor of the 1×1 Wilson loop for each insertion, analogous to dividing by the $T\times R$ loop, does enhance the magnitudes of V_2 and V_3 to much more reasonable values.) We leave for the future a detailed investigation of how to construct a quantity which is free of lattice artifacts in the continuum limit. This may be subtle, since it is known that in perturbative QCD, the spin-dependent terms depend on $\log(mR)$, where m is the heavy-quark mass.[7] To take account of this, it may be necessary to consider field strength insertions that range over an area $O(1/m)$ in size.

In this calculation, we have established that these rather delicate spin effects are measurable. At the distances we could explore here, one-gluon exchange gives a surprisingly good overall description, although the spin-spin potential and the spin-orbit potential point to nonperturbative effects at $\beta = 6.0$. We are planning to go on to a high-statistics calculation on a very large $(24^3\times 48)$ lattice where a central-potential calculation has previously been done.[8] This will allow us to probe a range of quark separations, time extent of loops, field-strength insertion areas, etc., which were inaccessible in the present work.

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