Propagating Plasma Mode in Thin Superconducting Filaments

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In superconducting filaments of cross section smaller than about 10^{-14} m^2, in a large frequency range at all temperatures, collective density oscillations exist with a soundlike dispersion relation and weak damping. The restoring force is the Coulomb interaction. In the restricted geometry this force is proportional to the wavelength and hence does not shift the density mode to the bulk plasma frequency.

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We will report in this Letter on a new collective mode in superconductors with a soundlike linear dispersion relation. In contrast to neutral superfluids, where a large variety of such modes exists, in superconductors the Coulomb interaction shifts the frequency of all the density oscillations to the plasma frequency. So far only one propagating mode has been discovered.[1] The characteristic feature of this Carlson-Goldman (CG) mode is a balanced oscillation of normal and supercurrents, which avoids the buildup of space charge. Because of the Ohmic dissipation associated with the normal motion, the mode is overdamped except in a narrow temperature range near the transition temperature T_c.[2-4]

Our new mode is a charge-density oscillation. The essential feature is that in superconducting filaments with very small cross section, the Coulomb interaction is less effective so that the restoring force is proportional to the wavelength, which leads to a soundlike dispersion relation. The requirements on the small transverse dimensions are severe and can only be met with modern lithographic techniques. This may explain why this mode has not been detected or discussed in the past. In contrast to the CG mode our plasma mode exists at all temperatures below T_c; actually, the damping vanishes at zero temperature. Near T_c both modes coexist in different wave-vector regimes.

The propagating plasma mode involves oscillations of the phase of the superconducting order parameter and thus is the Goldstone mode associated with the corresponding degeneracy. Apart from being of conceptual and experimental interest, it may be of technical importance in understanding the behavior of small superconducting electronic circuits.

It is instructive to discuss first the plasma modes in restricted geometries for normal metals. We consider a filament or radius r_0 embedded in a medium with dielectric constant ϵ. In a very simple model, to be presented below, the density oscillations with wave vector k obey the dispersion relation

$$(\omega^2 + i\omega\tau_{\text{imp}}^{-1}) F(kr_0) - \omega_p^{(n)2} = 0. \tag{1}$$

Here $\omega_p^{(n)} = (4\pi ne^2/m)^{1/2}$ is the plasma frequency of the bulk normal metal and

$$F(y) = 1 + 2\epsilon K_1(y)/yK_0(y), \tag{2}$$

in which K_0 and K_1 are modified Bessel functions. It is assumed that the impurity-scattering time τ_{imp} is shorter than the inelastic-scattering time τ_{in}. For wave vectors and radii such that $kr_0 >> 1$ and $F=1$, we recover the standard bulk plasma oscillation. In contrast, for small radii and wave vectors such that $kr_0 << 1$ and $F^{-1} = \frac{1}{2}\epsilon^{-1}(kr_0)^2 \ln(1/kr_0)$, the electric field is mostly outside the filament. As a result we find a propagating plasma mode with the dispersion relation

$$\omega^2 + i\omega\tau_{\text{imp}}^{-1} - c_{pp}^{(n)2}k^2 = 0$$

and phase velocity

$$c_{pp}^{(n)2} = \tfrac{1}{2}\omega_p^{(n)2} r_0^2 \epsilon^{-1} \ln(1/kr_0). \tag{3}$$

We immediately notice that this mode is overdamped unless ω exceeds τ_{imp}^{-1}. Also, the propagating plasma mode only follows the above description if $c_{pp}^{(n)}$, as given by Eq. (3), is much smaller than the speed of light. Indeed this condition is required for consistency: When we derived Eq. (3) we ignored retardation effects, which would obscure the interesting features of our new mode. For typical metals with a plasma frequency of about 10^{16} s^{-1}, the radius r_0 has to be in the range of 10 nm or less. This, but even more so the requirement of high frequency ($\omega >> \tau_{\text{imp}}^{-1}$), makes it difficult to detect this normal-metal propagating plasma mode. In addition, in the normal metal, if the phase velocity exceeds the Fermi velocity the mode is "Landau damped" by the excitation of quasiparticles.[5]

In a superconductor it is much easier to detect the mode. The Ohmic damping is only associated with the normal current, and the gap in the excitation spectrum prevents or reduces the damping due to the excitation of quasiparticles. As will be shown below, the spectrum of collective modes in a superconducting filament follows from

$$(-i\omega+\tau_{\rm in}^{-1})[\omega^2F(kr_0)+i\omega 4\pi\sigma_{qp}-\omega_p^{(s)2}]-c_{\rm CG}^2k^2[-i\omega F(kr_0)+4\pi\sigma_{qp}]=0. \tag{4}$$

Here we introduced the plasma frequency associated with the superfluid density n_s, $\omega_p^{(s)}=(n_s/n)^{1/2}\omega_p^{(n)}$. The quasiparticle conductivity σ_{qp} has the well-known temperature dependence, reducing to the normal-state conductivity σ_n near T_c and vanishing exponentially near $T=0$. Finally $c_{\rm CG}$ is given by

$$c_{\rm CG}^2=n_s/m\chi_s \tag{5}$$

where χ_s is equal to $2N(0)f(\Delta/k_{\rm B}T)$. The limiting forms of f are $\pi\Delta/4k_{\rm B}T$ near T_c and 1 at $T=0$. Equation (4) is sufficient as long as $\omega<2\Delta/\hbar$. At larger frequencies a strong damping due to pair breaking will occur.

In the limit of large wave vectors or large radii, such that $kr_0>>1$, well-known results are reproduced by Eq. (4). At large frequencies we find the plasma oscillation of the superfluid component, which is not too relevant because of the simultaneous restriction $\omega<2\Delta/\hbar$. On the other hand, at small frequencies we recover the Carlson-Goldman mode as we know it from bulk samples, with the dispersion relation following from $\omega^2+i\omega(\tau_{\rm in}^{-1}+\omega_d)-c_{\rm CG}^2k^2=0$. The necessary counteroscillation of the normal component and the associated Ohmic dissipation make a large contribution to the damping, except in a region very close to T_c. The damping is given by the sum of the inelastic scattering rate and

$$\omega_d=\frac{\omega_p^{(s)2}}{4\pi\sigma_{qp}}=\frac{n_s}{n}\frac{\sigma_n}{\sigma_{qp}}\frac{1}{\tau_{\rm imp}}. \tag{6}$$

Near T_c in dirty samples the phase velocity is $c_{\rm CG}=(2D\Delta/\hbar)^{1/2}$ with D the diffusion constant.

In the other limit $kr_0<<1$, for thin wires and small wave vectors, Eq. (4) yields the following dispersion relation:

$$(-i\omega+\tau_{\rm in}^{-1})(\omega^2-c_{pp}^2k^2)+\frac{(c_{pp}^2-c_{\rm CG}^2)k^2}{\omega_d}[i\omega(-i\omega+\tau_{\rm in}^{-1})-c_{\rm CG}^2k^2]+\tau_{\rm in}^{-1}c_{\rm CG}^2k^2=0. \tag{7}$$

This is our main result. At small wave vectors it allows for a propagating plasma mode with a phase velocity given by

$$c_{pp}^2=c_{\rm CG}^2+(n_s/n)c_{pp}^{(n)2}. \tag{8}$$

In most cases the phase velocity c_{pp} is much larger than $c_{\rm CG}$, in which case the plasma oscillation follows from

$$\omega^2-c_{pp}^2k^2+i(\omega/\omega_d)c_{pp}^2k^2=0. \tag{9}$$

The damping of this propagating plasma mode is reduced when the temperature is lowered. This differs from the CG mode and is related to the fact that no counteroscillation of supercurrent and normal current is required. For the same reason also the conversion processes turn out to be rather ineffective as a damping mechansim. At large wave vectors we find again the Carlson-Goldman mode with its usual dispersion. It is unaffected by the restricted geometry.

The dispersion relations for both modes are depicted in Fig. 1. The temperature is close enough to T_c that the frequency window for the CG mode, $\omega_d<\omega<2\Delta/\hbar$, exists. In contrast, the plasma mode exists for all frequencies up to ω_d. The figure shows the typical situation where the phase velocities are very different (note the change of scale).

For thin superconducting films of thickness d, in particular in those with high sheet resistance, similar plasma oscillations are possible. Here the F function of Eq. (4) is $F=1+2\epsilon/kd$. Hence for $kd<<1$ the dispersion relation is $\omega\propto k^{1/2}$ and the damping can be very small. In normal-metal films a similar dispersion relation has been derived,[6] but here additional damping processes are present.

In Table I we list some explicit numerical values for two typical samples. The first is a conventional small strip; the second is approaching the limit of what can be made with confidence with present-day technology. The phase velocity c_{pp} depends on the size of the system, concentration, and temperature. For the examples chosen it is large, though small enough in comparison with the speed of light to allow us to ignore retardation effects. A consequence of the large value of c_{pp} is that the propagating plasma mode only exists at long wavelengths, larger than a value $\lambda_{\rm min}$ which follows from the restrictions on the existence of the mode posed by both the damping and the energy gap:

$$c_{pp}2\pi\lambda_{\rm min}^{-1}=\omega_{\rm max}=\min(\omega_d,2\Delta/\hbar). \tag{10}$$

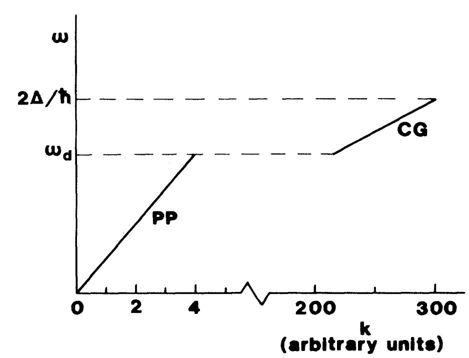

FIG. 1. Range of existence of propagating plasma (PP) and Carlson-Goldman (CG) modes near T_c. At lower temperatures ω_d exceeds $2\Delta/\hbar$ and no CG mode is possible.

TABLE I. Predicted values for two typical examples. Assumptions made, both samples: $T_c = 1.5$ K, $v_F = 1.3\times10^6$ m/s, $\mu_0 H_c(0) = 1.4\times10^{-2}$ T, $C' = 10^{-10}$ F/m; sample 1: $S = 3\times10^{-14}$ m^2, $R_n' = 10^7$ Ω/m, $\tau_{\rm imp} = 3\times10^{-16}$ s; sample 2: $S = 10^{-15}$ m^2, $R_n' = 10^9$ Ω/m, $\tau_{\rm imp} = 10^{-16}$ s.

	T/T_c	$2\Delta/\hbar$ (10^{10} s^{-1})	ω_d (10^{10} s^{-1})	c_{pp} (10^6 m/s)	$c_{\rm CG}$ (10^3 m/s)	$\lambda_{\min}$ (μm)	$(L'/C')^{1/2}$ (kΩ)	I_c (μA)	$\langle I_s^2\rangle^{1/2}$ (nA)
Sample 1	0.999	3.8	0.29	1.7	2.5	3700	5.9	0.035	1.8
	0.9	38	29	17	8.0	370	0.59	35	57
	0	69	$\cdots$	33	14	300	0.3	390	115
Sample 2	0.999	3.8	0.29	0.17	1.5	370	59	6.3×10^{-4}	(0.6)
	0.9	38	29	1.7	4.6	37	5.9	0.63	18
	0	69	$\cdots$	3.3	7.8	30	3.0	7.1	36

It is convenient to define parameters as the effective capacitance per unit length $C' = \epsilon/\{2ln(1/kr_0)\}$, the kinetic inductance per unit length $L' = m/(n_s e^2 S)$, and the normal-state resistance per unit length $R_n' = (\sigma_n S)^{-1}$ with S the cross section. The velocity of the plasma mode is essentially given by $c_{pp}^2 = (L'C')^{-1}$. If in addition R_{qp}' is defined as $R_n'\sigma_n/\sigma_{qp}$, the damping is characterized by $\omega_d = R_{qp}'/L'$. The introduction of electric circuit parameters makes the extension to noncylindrical geometries obvious. We ignore the weak dependence of C' on k. For the usual arrangement of a strip on a substrate, ϵ is given by the average of the values for substrate and vacuum. In Table I typical values for C' are chosen. Similar equivalent-circuit parameters for the CG mode have been indicated by Kadin, Smith, and Skocpol.[7]

We now will derive the relations given above. Both in the normal metal and in the superconductor we solve Poisson's equation in the cylindrical geometry to find the relation between the charge density and the electric field, considering a quasistationary situation. The charge density is composed of a bulk part that is homogeneously distributed over the cross section and a surface charge. We assume a harmonic variation along the z direction characterized by k. Then the radial distributions of ρ and V are given by

$$\rho(r) = \rho_0\left[\theta(r_0 - r) + \frac{\epsilon K_1(kr_0)}{kK_0(kr_0)}\delta(r - r_0)\right],$$

$$V(r) = \frac{4\pi\rho_0}{k^2}\left[\theta(r_0 - r) + \frac{K_0(kr)}{K_0(kr_0)}\theta(r - r_0)\right].$$

In the normal state, we combine these solutions with the continuity equation and the acceleration equation for the current,

$$\left(\frac{\partial}{\partial t} + \tau_{\rm imp}^{-1}\right)\mathbf{j} = -\frac{ne^2}{m}\nabla V.$$

Integration over the cross section yields Eq. (1).

In a superconductor we have both a quasiparticle current and a supercurrent. The latter can be parametrized by $j_s = en_s v_s$, where n_s is a temperature- and impurity-dependent transport coefficient and v_s satisfies the acceleration equation $\dot v_s = -m^{-1}\nabla\Phi_s$ where $\Phi_s = \mu_s + eV$ is the Cooper-pair electrochemical potential. The quasiparticle current j_n is driven by the electric potential, $j_n = -\sigma_{qp}\nabla V$, because the shifts in the normal chemical potential are small in comparison with eV. Thus the continuity equation can be written as $\mathrm{div} j_s - \sigma_n\nabla^2 V = -\dot\rho$ where ρ is the total charge density. The supercurrent and superfluid chemical potential also satisfy the relation

$$\mathrm{div} j_s + e\chi_s\left(\frac{\partial}{\partial t} + \tau_{\rm in}^{-1}\right)\mu_s = 0, \tag{11}$$

which has been derived in the appropriate limit from microscopic theory.[8] It can also be made plausible in the two-fluid-type description introduced by Pethick and Smith.[9] The essential feature there is to introduce two separate continuity equations for the superfluid and normal components which are couple by conversion processes:

$$\mathrm{div} j_n + \dot\rho_n = -Q^*/\tau_{Q^*}, \quad \mathrm{div} j_s + \dot\rho_s = Q^*/\tau_{Q^*}.$$

The normal and superfluid "charge densities" are related to the chemical potential shifts by $\rho_n = e\chi_n\mu_n$ and $\rho_s = e\chi_s\mu_s$ where the χ's are generalized susceptibilities satisfying $\chi_n + \chi_s = 2N(0)$ and the limiting forms of χ_s were given before. The sum of ρ_n and ρ_s is the total charge density. The conversion is proportional to the charge imbalance $Q^* = e\chi_n(\mu_n - \mu_s)$. The conversion rate $1/\tau_{Q^*}$ is given by $(\chi_s/\chi_n)\tau_{\rm in}^{-1}$, reducing to $\pi\Delta/4k_{\rm B}T\tau_{\rm in}$ near T_c. From these relations it is straightforward to obtain Eq. (11).

Finally, we close these relations by the solution to the Poisson equation, as given above, and obtain the complete dispersion relation of the superconducting filament, Eq. (4).

The derivation outlined above correctly reproduces the essential features of the collective mode. The more detailed microscopic analysis shows that there exist corrections of order $\hbar\omega/2\Delta$ and additional damping mechanisms due to the excitation of quasiparticle degrees of freedom. In the case of the Carlson-Goldman mode this analysis has been performed in the past[8] and quantitative modifications of the dispersion relation have been found. On the other hand, this analysis showed that as long as we can ignore nonlinear effects and as long as the frequency is not close to or larger than the gap, $\hbar\omega << 2\Delta$, these corrections are small. In particular there exists no strong damping due to the excitation of particle-hole pairs.

There are several ways in which the existence and the properties of the propagating plasma mode can be investigated experimentally. Because the mode contains an oscillation of the total current and charge density—in contrast to the Carlson-Goldman mode where the total charge density remains constant—it should be relatively easy to couple to. As noted before, the wavelengths are relatively long so that the finite length of the sample in many cases will limit the number of allowed modes. We can put this to our advantage by measuring resonance frequencies as a function of sample length to determine c_{pp}. The modes can be excited and detected by means of small local probes which act as ac current or voltage sources. Josephson coupling or capacitive coupling can be used. As the plasma mode also involves oscillations of the phase, the detection method as used by Carlson and Goldman is conceivable too. Their CG mode could only be observed through the phase, by coupling to the phase of another superconductor.

In a bulk superconductor phase fluctuations are suppressed because of the associated charge fluctuations and Coulomb interactions. In superconducting thin films there is a possibility of enhanced phase fluctuations associated with the formation of vortex pairs, unbinding at the Kosterlitz-Thouless transition.[10] In the filament we also have significant phase fluctuations, which stimulate the loss of phase coherence by local phase slippage. We can estimate the amplitude of current and voltage fluctuations simply by assuming that for each k mode the energy is $k_B T$ at finite temperatures and $\frac{1}{2}\hbar\omega(k)$ at $T=0$. Locally the expectation value of the current due to the propagating plasma mode follows from

$$\langle I_s^2\rangle_{pp} = \frac{1}{\pi}\frac{1}{L'c_{pp}}k_B T\omega_p, \quad T\sim T_c,$$
$$= \frac{1}{\pi}\frac{1}{L'c_{pp}}\frac{\hbar}{4}\left(\frac{2\Delta}{\hbar}\right)^2, \quad T=0.$$

Results are shown in the table. Notice that in the sample with small cross section $\langle I_s^2\rangle^{1/2}$ becomes comparable to I_c, in particular close to T_c.[11] Near T_c, the current fluctuations due to the CG mode are much larger, in particular because $c_{\rm CG}$ is considerably smaller than c_{pp} for practical samples. The power spectrum of the voltage fluctuations is of the Johnson-Nyquist type with an equivalent resistance $(L'/C')^{1/2}$. This value is also quoted in the table. It is independent of the length of filament considered, and one might think that it could exceed the normal-state resistance if that length is very short. However, a length A imposes a minimum frequency $\omega_{\min}=2\pi c_{pp}/A$. The white noise occurs in the frequency range between $\omega_{\min}$ and ω_d. As A is decreased, $\omega_{\min}$ increases to become equal to ω_d just when $(L'/C')^{1/2}$ approaches the normal-state resistance AR_n'.

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