Elastic Scattering of α Particles and the Phase of the Nucleon-Nucleon Scattering Amplitude

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Glauber theory can describe elastic scattering of α particles by ^{4}He, ^{3}He, ^{2}H, and ^{1}H at 7 GeV/c if the phase of the nucleon-nucleon elastic-scattering amplitude varies with momentum transfer. The phase variation leads to diffraction patterns differing markedly from those typical of constant-phase calculations and greatly affects the magnitudes of the intensities. These changes are mainly due to changes in the interference between amplitudes for different orders of multiple scattering and to a decrease in their moduli.

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During the past twenty years the Glauber theory has been extremely successful in describing hadron-nucleus elastic scattering at energies of approximately 1 GeV or higher. This success has not been shared to the same degree in nucleus-nucleus ("heavy-ion") elastic scattering at corresponding energies of 1 GeV/nucleon or higher for several reasons. First, there has been a relative paucity of such measurements. Second, the extension of the theory to nucleus-nucleus collisions is significantly more complex and the computations are more difficult and lengthy so that fewer of these types of calculations exist.[1]

Recently a comprehensive set of measurements of elastic scattering of α particles by four very light nuclei (^{4}He, ^{3}He, ^{2}H, ^{1}H) was made at an incident α-particle momentum of 7 GeV/c over a range of $|t|$ values from ~ 0.07 to ~ 4 (GeV/c)^2. The cross sections fell from the barn to the nanobarn level. Such data, in which the intensities vary through so many orders of magnitude and over such a large range of momentum transfers, are extremely useful because they put enormous constraints on any theory. It is no longer sufficient to show that the theory describes measurements of collisions between just one given pair of nuclei. Now the theory must describe measurements between four different pairs of nuclei, and it must do so consistently. Whatever nucleon-nucleon (NN) elastic-scattering amplitude is used for one calculation should be used for the others as well. In addition, since these measurements have gone out to rather large momentum transfers, the calculated intensities will be much more sensitive to the NN elastic scattering amplitudes used as input.

The measurements for elastic scattering of α particles by the four light nuclei were accompanied[2] by theoretical analyses for the α-^{2}H, α-^{3}He, and α-^{4}He cross sections. These analyses were both by means of the so-called "rigid projectile approximation" and by means of the Glauber theory, with Gaussian densities for the nuclear ground states. The rigid-projectile approximation failed even qualitatively except at very small momentum transfers.[2] In the Glauber-theory calculations shown,[2] the broad qualitative trends of the data were to some extent roughly described. Quantitatively the results were in strong disagreement with the data, often being as much as an order of magnitude too low.

In the present analysis we have calculated the elastic-scattering differential cross sections for all four pairs of nuclei, evaluating the full multiple-scattering series (through sixteenth order multiple scattering for α-^{4}He collisions, twelfth order for α-^{3}He collisions, eighth order for α-d collisions, and fourth order for α-p collisions). We have used Gaussian densities for ^{3}He and ^{4}He, with 1.88 and 1.671 fm for their rms radii, respectively.[3] The resulting form factors for ^{3}He and ^{4}He are of the form $S=\exp(-R^2q^2/4)$ in which we have corrected R^2 for center-of-mass recoil and finite-proton-size effects. We have used[4] $\langle r_p^2\rangle = 0.775\,42$ fm^2, and we obtain $R^2=2.759$ fm^2 for ^{3}He and $R^2=1.793$ fm^2 for ^{4}He. For the deuteron we have used a sum of Gaussians which was fitted[5] to the deuteron ground-state form factor. The form factor used is given by[5]

$$S=0.34\exp(141.5t)+0.59\exp(26.1t)+0.08\exp(15.5t). \quad (1)$$

For the NN scattering amplitudes we have used the standard form

$$f=[ik\sigma(1-i\rho)/4\pi]e^{bt/2}, \quad (2)$$

with values of $\sigma=44$ mb, $\rho=-0.23$, and $b=6.0$ (GeV/c)$^{-2}$, which are the averages of the corresponding pp and np measured values.[6] The results are shown by the dashed curves ($\gamma=0$) in Fig. 1, together with the experimental data. In all four cases the agreement is poor, even qualitatively, except at very small momentum transfers. The disagreement is at times by an order of magnitude or more.

Is it possible to obtain reasonable agreement with the data without modifying the usual Glauber theory? We note that the NN scattering amplitude, Eq. (2), has

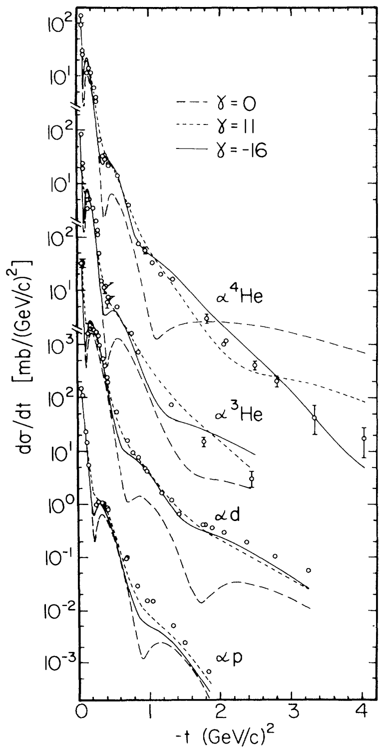

FIG. 1. Differential cross sections for elastic scattering of α particles by ^{4}He, ^{3}He, ^{2}H, and ^{1}H, at 7.0 GeV/c [data of Satta *et al.* (Ref. 2)]. The dashed curve is the constant-phase result ($\gamma = 0$). The dotted and solid curves are calculations with phase variations in the NN amplitude [$\gamma = 11$ and -16 (GeV/c)^2, respectively, in Eq. (3)].

a constant phase when b is taken to be the slope of the NN forward differential cross section, as it almost always is. However, it is perhaps not unreasonable to expect that over the rather larger range of momentum transfers represented by the data the phase will vary appreciably. A simple phase variation can be effected by taking the NN amplitude to have the form

$$f = [ik\sigma(1 - i\rho)/4\pi]e^{(b+i\gamma)t/2}. \qquad (3)$$

One of us (VF) used such a phase variation many years ago to describe hadron-deuteron scattering[7] and also suggested this phase variation for quark-quark amplitudes in early hadron-hadron multiple-scattering analyses using quark models.[8] This parametrization does not affect the NN differential cross section and b is still the measured NN slope parameter. Now, however, there is one adjustable parameter, γ, since the NN phase variation cannot be measured directly from NN scattering. But the *same* value of γ must be used in describing all four α-nucleus measurements since it is independent of the nucleus.

In Fig. 1 we show our calculations for α-^{4}He, α-^{3}He, α-d, and α-p elastic scattering with $\gamma = 11$ (GeV/c)$^{-2}$ and also with $\gamma = -16$ (GeV/c)$^{-2}$. (When ρ is small in magnitude there generally are two values of γ of roughly equal magnitude but opposite signs which yield somewhat similar results at large momentum transfer.) The agreement with the data is much improved in all cases, and in some cases it is rather good.

While the t dependence of the phase variation of the NN amplitude may not be as simple as that assumed in Eq. (3), the marked improvement in these light-ion results strongly indicates the presence of a phase variation. Furthermore, while it is also true that the NN phase we obtain varies by a few cycles between the forward direction and $-t \approx 4$ (GeV/c)^2, the modulus of the NN amplitude given by Eq. (3) varies by more than 5 orders of magnitude over the same range of momentum transfer. With such a large modulus variation it is not unreasonable to expect a large phase variation. In addition, at values of t where nth-order multiple scattering dominates, the intensity often depends on the NN amplitude at t/n^2. For example, in α-p collisions near $-t = 1.8$ (GeV/c)^2 the dominant amplitude is the quadruple-scattering amplitude which depends mainly on the NN amplitudes at the much smaller value $-t \approx 0.1$ (GeV/c)^2.

Why is there such a marked change in the differential cross sections when the NN amplitude has a phase variation? For the lightest ions and at momentum transfers which are not too large the answer is to be found mainly in the resulting phase relations among the amplitudes corresponding to the various orders of multiple scattering. Let us consider, for example, α-d scattering. In Glauber theory the collision is described in terms of scattering amplitudes for single scattering, double scattering, and so on, up through octuple scattering. If there is no phase variation ($\gamma = 0$) and if ρ is small in magnitude compared with unity, amplitudes for any two successive orders of multiple scattering will be $\sim \pi - \rho$ radians out of phase with each other, i.e., their phases will be totally correlated and will differ by nearly π. At $t = 0$ the moduli of these amplitudes decrease with increasing order of multiple scattering, but as $-t$ increases the moduli of the higher-order amplitudes decrease more slowly. Consequently for small $|t|$ values the single-scattering amplitude is largest, at somewhat larger $|t|$ values the double-scattering amplitude is largest, and so on. This very regular relationship among the moduli *and among the phases* is conducive to very strong destructive interference among the amplitudes for the various orders of multiple scattering, leading to the appearance of minima in the cross sections. For example, the minimum near $-t = 1.73$ (GeV/c)^2 shown in Fig. 1 ($\gamma = 0$) occurs as a result of very strong cancellations between the amplitudes for odd-order multiple scattering and those for even order. In Fig. 2(a) we show in the complex plane the largest four amplitudes at

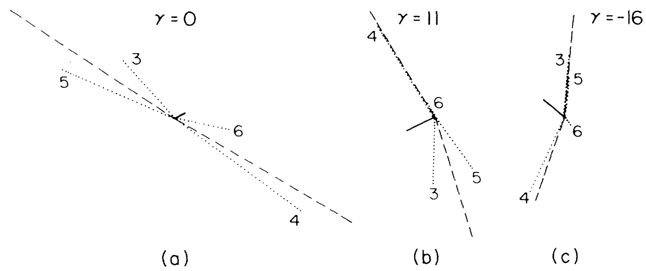

FIG. 2. Amplitudes for multiple scattering in the complex plane, for α-d scattering at $-t = 1.726$ (GeV/c)^2. The dotted lines represent amplitudes for triple (3), quadruple (4), quintuple (5), and sextuple (6) scattering. The dashed lines represent the sum (3) + (5) and the sum (4) + (6). The solid line represents the sum (3) + (4) + (5) + (6). (a) No phase variation, $\gamma = 0$. (b) $\gamma = 11$ (GeV/c)^2. (c) $\gamma = -16$ (GeV/c)$^{-2}$. The units are arbitrary but the same in (a), (b), and (c).

$-t = 1.726$ (GeV/c)^2 (the dotted lines, which are labeled 3, 4, 5, and 6 corresponding to triple through sextuple scattering). First, it is clear that although the above minimum is the third minimum (see Fig. 1), it does *not* arise only (or even mainly) from the interference between the triple- and quadruple-scattering amplitudes, as the usual arguments regarding the Glauber multiple-scattering series would lead one to believe. In fact, the quintuple-scattering amplitude is larger than the triple-scattering amplitude. Second, all four amplitudes shown are appreciable. The triple plus quintuple amplitude and the quadruple plus sextuple amplitude (shown dashed) are nearly equal and opposite. Their resultant is given by the short solid line at $\sim 30°$ with the horizontal. In this case, the amplitude for sextuple scattering (not shown) is nearly equal to this resultant in magnitude and is at $181°$, leading to an actual final resultant that is half as large as that shown. It is this almost perfect cancellation among the *third- through seventh*-order multiple-scattering amplitudes that leads to the minimum near $-t = 1.73$ (GeV/c)^2.

What happens when there is a phase variation? The moduli of the amplitudes for all orders except single scattering decrease compared to those obtained with no phase variation. The larger the order of multiple scattering, the greater the decrease. But more important at momentum transfers which are not too large, the phase relations among the various amplitudes are greatly changed. Amplitudes for successive orders of multiple scattering no longer always have phase differences close to π, nor do they even have constant phase differences. Consequently, very strong destructive interferences will be much less likely, especially away from the very small momentum transfers. Therefore, despite *smaller* individual amplitudes for double through octuple scattering, the resultant scattering amplitudes and corresponding differential cross sections will generally be *larger* than for the case of no phase variation.

In Figs. 2(b) and 2(c) we show the largest four amplitudes for $\gamma = 11$ (GeV/c)$^{-2}$ and $\gamma = -16$ (GeV/c)$^{-2}$ at $-t = 1.726$ (GeV/c)^2. We see that the lack of total correlation among the phases leads to a much smaller degree of cancellation and therefore to a much larger resultant amplitude. The resulting differential cross section (see Fig. 1) consequently exhibits only a weak shoulder and is much larger than that obtained with no phase variation. This argument is not valid for the larger systems at the very large momentum transfers. In those cases *many* higher-order multiple-scattering amplitudes are significant and the decrease in the moduli of these amplitudes relative to those obtained with no phase variation is *very great.* This large decrease more than offsets any tendency for strong cancellation among the amplitudes obtained with no variation, and consequently leads to cross sections which are *smaller* than those obtained with $\gamma = 0$. [For example, see the α-^{4}He results for $-t \gtrsim 3$ (GeV/c)^2 in Fig. 1.]

The above discussion shows that the presence of a phase variation in the *NN* elastic-scattering amplitude leads to large changes in the α-p, α-d, α-^{3}He, and α-^{4}He elastic-scattering differential cross sections, and brings the Glauber theory into agreement with the measurements.

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